Measurement of Aortic Pressure Wave Velocity by 4D Image Registration

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Abstract

Aortic pulse wave velocity is an important diagnostic measure and has been associated with significant cardiovascular mortality. Our hypothesis is that this parameter can be measured using displacement fields obtained by registration of time-series images. This paper presents a method in which a form of the wave equation is used as a regularization term in the registration process. It is demonstrated that the method can capture the pulse wave velocity from numerical phantom images, even when noise is added to these images. It is further demonstrated that the method yields plausible results when operated on a real 4D image dataset.

1 Introduction

Aortic pressure wave velocity (PWV), or simply wave velocity, has been identified as an important parameter in the diagnosis of cardiovascular disease and risk. The 'gold standard' invasive measurement for the aortic wave velocity involves direct measurement of pressure using two catheter tipped cannulae inserted via the femoral artery and positioned at known positions in the aorta. If the difference in arrival times of the pressure wave between these two points can be determined and the distance between them is knownthe average wave velocity between these two points can be computed. The usual analysis, the 'foot–to-foot' method, uses a measurement of the position of the initial upswing of the pressure wave as a timing point [1]. The assumption is that this measurement is relatively uncorrupted by the presence of any reflected wave. For a distance of 0.2 m between the measurement points and a wave velocity of 5m/s the measured delay will be 40 ms.and therefore high temporal resolution is required. This method is invasive. Non-invasive methods which have been proposed include Doppler ultrasound [2] and velocity-encoded CMR images, including transit-time, flow area and cross-correlation methods [3]. These methods generally use a waveform analysis similar to the invasive approach.

This paper presents a method in which wave velocity is computed from time series (4D) images of the aorta using constrained image registration. The constraint is derived from the physics of wave transmission in a uniform elastic cylinder. Advantages of this method are that all of the local information in the image set contributes to the computation of the wave velocity parameter, high temporal resolution is not required and the method is not sensitive to the presence of wave reflections. Typically the length of aorta which can be imaged from the valve plane to the femoral bifurcation is ~30-40 cm.

2 Theory

For a cylindrical axi-symmetric vessel the variation of pressure in the vessel with time (*t*) and position along the vessel axis (*z*) obeys the wave equation

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial z^2} = 0 \qquad (1)$$

if the effects of viscosity can be ignored, which they generally can for flow in the aorta. In this equation c is the wave velocity in the vessel. Clinically we do not have direct access to the pressure in the vessel but it can be shown that the pressure is given by

$$p(z,t) - p_{ref} = -\frac{Eh}{\left(1-\nu^2\right)R_{ref}(z)} r(z,t) \qquad (2)$$

E is the Young's modulus of the vessel wall material, h is the wall thickness, v is Poisson's ratio and R_{ref} is a reference vessel radius (typically the average radius over the cardiac cycle). p_{ref} is the pressure required to statically expand from the unpressurised radius R_o to R_{ref}. and $r(z,t)$ is the fractional change in radius as the pressure in the vessel changes from $p(z,t)$ to p_{ref} p_{ref} is taken to be independent of t and z. In the present work we can take R_{ref} to be independent of z since we are concerned with cylindrical vessels of constant radius. The variable r can then be used as a surrogate for pressure in the wave equation.

We take cross-sectional images normal to the vessel axis at several equally spaced points along the vessel axis and across the cardiac cycle. For our ideal vessel these images will be circular and will expand and contract uniformly as the pressure in the vessel changes. The expansion of the vessel relative to a reference cross-sectional image can be obtained by image registration. $f(x,y,z)$ is the cross-sectional image at the point z along the vessel axis when the pressure is p_{ref} and $m(x,y,z,t)$ is the image at the same point z when the pressure is $p(z,t)$,

The fractional change in radius, along with possible displacements $d_x(z,t)$, $d_y(z,t)$ along the x and y axes can be obtained by image registration of f and m. Based on the approach to image registration presented in [4,5] we can show that for small values of $r(z,t)$, $d_x(z,t)$, and $d_y(z,t)$ the relationship between the intensities of the two images at the common co-ordinate x,y is given by

$$m - f = \frac{r(z,t)}{2}\left(x\left(\frac{\partial f}{\partial x} + \frac{\partial m}{\partial x}\right) + y\left(\frac{\partial f}{\partial y} + \frac{\partial m}{\partial y}\right)\right) + \frac{d_x(z,t)}{2}\left(\frac{\partial f}{\partial x} + \frac{\partial m}{\partial x}\right) + \frac{d_y(z,t)}{2}\left(\frac{\partial f}{\partial y} + \frac{\partial m}{\partial y}\right) + .. \quad (3)$$

There is one of these equations for each pixel. Writing the equations in matrix form we have equation 4a. One of these equations can be solved for each cross-section. However we also know that the values of ***r*** taken along the *z* axis should satisfy the wave equation. This equation can be used as a constraint in the registration to produce a set of r values consistent with the image data and the wave equation. In matrix form the constraint can be written as equation 4b where $\boldsymbol{L}_z$ and $\boldsymbol{L}_t$ are discrete second order differentials along the *z* and *t* axes

$$\boldsymbol{f} - \boldsymbol{m} = \boldsymbol{T}[\boldsymbol{r} \quad \boldsymbol{d}_x \quad \boldsymbol{d}_y]^t) \quad (4a)$$

$$(\boldsymbol{L}_t - c^2\boldsymbol{L}_z)\boldsymbol{r} = 0 \quad (4b)$$

$$C = (\boldsymbol{m} - \boldsymbol{f})^t(\boldsymbol{m} - \boldsymbol{f}) + \lambda \boldsymbol{r}^t(\boldsymbol{L}_t - c^2\boldsymbol{L}_z)^t(\boldsymbol{L}_t - c^2\boldsymbol{L}_z)\boldsymbol{r} \quad (4c)$$

The constrained solution of equations 4a and 4b minimizes a cost function of the form of equation 4c. There are two parameters in this equation, *c* and *λ*. *λ* balances the relative importance given to the data and the constraint when equation 4c is minimised. *c* is initially unknown, although clinical values of wave velocity typically lie within a known range (~5-10 m/s).. The solution we propose to find *c* is as follows. A suitable value of *c* is chosen i.e. a value within the known clinical range of values, and equations 4 solved. The values of ***r*** are then inserted into the equation

$$\boldsymbol{L}_z^+\boldsymbol{L}_t\boldsymbol{r} - \boldsymbol{c}^2\boldsymbol{L}_z^+\boldsymbol{L}_z\boldsymbol{r} = 0 \quad (5)$$

which can then be solved for c^2. The matrix $\boldsymbol{L}_z^+$ is the pseudo-inverse of $\boldsymbol{L}_z$; solution of equation 5 produces a more robust value of *c* than solution of equation 4b. This new value is then inserted back in to equations 4 and a new ***r*** vector computed.. The process is iterated until a stable value of c^2 is obtained. Experimentally the algorithm is insensitive to the chosen starting value of wave velocity.

3 Materials

3.1 Simulated data

The wave equation has an analytical solution for a general cosine pressure wave. The solution for a general cyclic pressure wave can be obtained by summing up the solutions for the individual Fourier components of the pressure wave. 40 components were used. A program was written to compute the pressure within a cylindrical vessel as a function of time *t* and axial position z for a typical cardiac pressure input. Vessel dimensions (radius and wall thickness) and material properties (Young's modulus, blood density and viscosity) were specified. Vessel radius as a function of time and axial position was computed. These radii were converted into cross sectional images. The length of the vessel was 300 mm, the radius 10mm and the wall thickness 0.5 mm. The spacing between slices was 10mm and the timestep 20ms.

Material properties were chosen to give a wave velocity of 7 m/s. In practice wave reflections will occur within the aorta and pressure waves travelling in both directions will be present. The theory still holds for such a situation since the wave equation is satisfied by waves travelling in either direction The data set used here contains a backward wave with amplitude ~ 25% of the forward wave.

3.2 Physical phantom

A physical phantom consisting of a water filled cylindrical silicone tube through which water was pumped using a realistic pressure cycle was imaged in an MR scanner with a cardiac cine sequence. The images of the vessel were segmented to produce a set of cross sectional images.. The scanned length was 29 cm. Physical dimensions were similar to the aorta. The Young's modulus for the silicone was 1.01 MPa, Poisson's ratio was 0.45 and wall thickness 1mm. The calculated wave velocity was 7.8 m/s.

4 Methods and Results

4.1 Calculation of wave velocity: simulated data.

The wave velocity was calculated for different values of true wave velocity and λ. Values of λ (0,0.1,1,10) were used. Table 1 shows the calculated wave velocities.

Table 1

λ **Wave velocity** (m/s)	0	0.1	1	10
5	4.96	4.95	4.95	4.94
7	6.80	6.79	6.80	6.80
10	10.07	10.07	10.07	10.07

In practice the vessel data is to be extracted from clinical images which generally contain noise. In order to assess the sensitivity of the algorithm to image noise, Gaussian noise of standard deviation 10% of the maximum image amplitude was added to the cross sectional images. 30 data sets were produced. Values of c were computed from each of these data sets and mean and standard deviation computed. These are shown in Table 2 for four values of λ.

Table 2 Noise standard deviation 10% of maximum image amplitude. n = 30

Wave velocity m/s

λ	0	0.1	1	10
Mean (sd)	6.651 (0.233)	6.693 (0.163)	6.672 (0.150)	6.674 (0.153)

4.2 Calculation of wave velocity: physical phantom.

Calculation of wave velocity from the physical phantom data for a range of λ are shown in Table 3. The calculated measured wave velocity from the image data was 6.32 m/s with a λ of 100. No direct pressure measurements were available and so an accurate assessment of wave velocity using the foot-to-foot method was not possible. In this case increasing λ moved the computed value closer to the expected value.

Table 3

λ	0.1	1	10	100
Wave velocity m/s	5.47	5.53	5.67	6.32

5 Discussion and conclusions.

We have described an algorithm based on constrained image registration for extracting estimates of pulse wave velocity from images of a cylindrical vessel based on constrained image registration and have applied it to simulated and physical data. The method is shown to be capable of calculating wave velocity with reasonable accuracy even in the presence of significant noise levels. In the absence of noise the constraint has little effect as far as ideal simulated data is concerned, as might be expected. The computed value of wave velocity is close to the true value. With no constraint the effect of noise on the accuracy of the estimate of wave velocity appears to be greater than with the constraint, although this trend does not appear to be particularly sensitive to the value of λ chosen. The method has also been used with image data from the physical phantom. The results obtained are consistent with an estimate of wave velocity derived from the physical dimensions and the material properties of the phantom. In this case the effect of the constraint appears more significant. No independent direct measurement of wave velocity was available.

In practice real aorta are not axially symmetric cylindrical vessels. Extraction of equally spaced cross sections normal to the vessel axis, so simulating a cylindrical vessel, is fairly straightforward. However, such vessels are also significantly tapered and this may require a modification of the constraint if the algorithm is to work on such data. This is being explored and we intend to extend the method to clinical data.

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