Comparison of Least Squares and Maximum Likelihood for Apparent Diffusion Coefficient Estimation in Prostate DW-MRI

Valentin Hamy[1] (valentin.hamy.09@ucl.ac.uk)
Simon Walker-Samuel[2] (simon.walkersamuel@ucl.ac.uk)
Shonit Punwani[1] (shonit.punwani@uclh.nhs.uk)
David Atkinson[1] (D.Atkinson@ucl.ac.uk)

[1] Centre for Medical Imaging, University College London, London, UK
[2] Centre for Advanced Biological Imaging, University College London, London, UK

Abstract

The problem of apparent diffusion coefficient (ADC) estimation from Rician distributed diffusion-weighted magnetic resonance (DW-MR) data is addressed. The least squares (LS) algorithm, widely used in clinical practice, is known to produce biased estimates as it considers the noise as normally distributed. Maximum likelihood (ML) can provide a more robust alternative. In this study, simulated data based on real prostate cancer DW-MR scans were used to compare LS and ML efficiency, for signal to noise ratios typical of the different types of tissue. The ML approach provided significantly less biased estimates than the LS, potentially allowing better accuracy in prostate cancer grading from MR images.

1 Introduction

Prostate cancer is the most common cancer among men, representing a fifth of all cancer diagnosed in men in 2006 [3]. It has been shown that diffusion weighted magnetic resonance imaging (DW-MRI) used in combination with T2-weighted MRI is of interest to diagnose prostate cancer [1]. In particular, Verma *et al.* [6] have shown that apparent diffusion coefficient (ADC) values are negatively correlated to Gleason grades in the prostate peripheral zone. However the creation of ADC maps from diffusion weighted images is usually achieved using the least squares fitting method, which does not account for the noise in magnitude MR data. An alternative approach using maximum likelihood for Rician distributed data points was introduced by Sijbers *et al.* [5]. This estimation scheme was then applied to DW-MR images of mice prostate cancer by Walker-Samuel *et al.* [7]. In the latter, maximum likelihood yielded unbiased ADC estimates in the case of late stage necrotic tumour tissue.

The purpose of this study is to compare the performance of least squares (LS) and maximum likelihood approaches for ADC estimation in human prostate cancer DW-MRI. The objective here is to state which approach should be preferred for early stage tumour grading.

2 Material and Methods

2.1 Rician Noise in Diffusion MR

The MR signal magnitude in diffusion weighted imaging decays exponentially with increasing B-values.

$$S(Bvalue|ADC, S_0) = S_0 \exp(ADC \times Bvalue) \quad (1)$$

where ADC is the apparent diffusion coefficient of the tissue, and S_0 is the signal magnitude obtained without applying any diffusion gradient.

Thermal agitation causes normally distributed noise in both the imaginary and real signal components from which the modulus is taken to produce the output magnitude signal. As a consequence, noise in magnitude data follows a Rice distribution [5] modelled by the following probability density function:

$$p(M|S,\sigma_R) = \frac{M}{\sigma_R^2}\exp(-\frac{M^2+S^2}{2\sigma_R^2})I_0(\frac{MS}{2\sigma_R^2}) \tag{2}$$

where M is the noisy magnitude MR data, S is the true magnitude data, σ_R is the Rician noise parameter, corresponding to the standard deviation of the underlying Gaussian distribution, and I_o is the 0th order modified Bessel function of the first kind.

2.2 Parameter Estimation

The least squares (LS) estimate consists in approximating the signal parameters ADC and S_0 by minimizing the sum of square differences between the noisy data points M_i acquired for each B-value $Bvalue_i$ and the model given in (1). However, LS provides an accurate estimate only when the noise is Gaussian distributed. Sijbers *et al.* defined another approach using maximum likelihood (ML) to estimate MR signal intensity corrupted with Rician noise [5]. Following that work, Walker-Samuel *et al.* [7] applied the maximum likelihood approach to mouse diffusion weighted MR data. In the latter study, the Likelihood function is defined as follows:

$$L(ADC,S_0;M,\sigma_R) = \prod_{i=1}^{N} p(M_i|S_i,\sigma_R) \tag{3}$$

Where N is number of B-values. Then by taking the negative logarithm of the Likelihood function and combining equations 2 and 3:

$$-\log(L(ADC,S_0;M,\sigma_R) \propto \sum_{i=1}^{N}\frac{S(Bvalue_i|ADC,S_0)^2}{2\sigma_R^2} - \sum_{i=1}^{N}\log(I_0(\frac{S(Bvalue_i|ADC,S_0)M_i}{\sigma_R^2})) \tag{4}$$

This negative log-likelihood function can be minimized with respect to ADC and S_0, yielding the most likely value of ADC given the data, and Rician noise model. Note that terms independent of S have been omitted in (4).

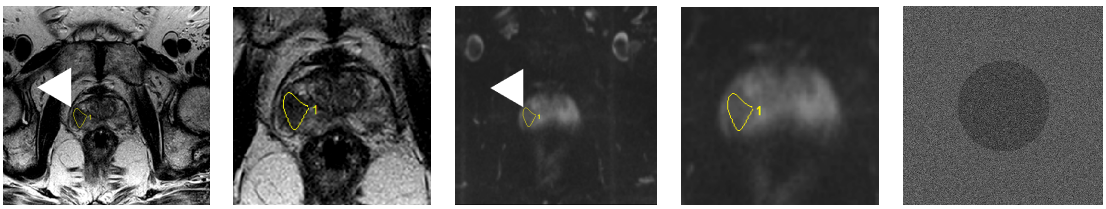
2.3 Monte Carlo Simulation

1-Dimension Diffusion MR signals were simulated in order to test the accuracy of both methods over a range of realistic values of ADCs and SNRs as presented in [7]. For this first experiment S_0 was chosen equal to one. For ADC$\in$ [0.9;3] ($\times 10^-3 mm^2/s$) and SNR$\in$ [1;10], N = 10000 simulations were run using the model in (1) with the following B-values: [0 , 50 , 150 , 500 , 1000] (mm^2/s). Every simulated signal was corrupted with Rician noise and passed as input to the LS and the ML algorithms for ADC estimation. The value of SNR in simulated signals is defined here as the ratio of S_0 divided by σ_R. The accuracy of each method was then evaluated by computing the median deviation from the true ADC value (expressed in percent), along with the standard deviation of the estimates over the N simulations.

2.4 Phantom Simulation

Further assessment of the two methods was achieved using a phantom simulating a field of view (FOV) with two types of tissue: Prostate peripheral zone (PZ) tissue and tumour. The objective of this experiment was to evaluate the variation in estimates with the size of the tumour region of interest (ROI). Tumour within the phantom was designed as a disk with varying diameter ($5 < D < 30$ pixels). Phantom images were corrupted with Rician noise and given as input to the two algorithms. Estimations were repeated $N = 150$ times using the same value of σ_R for noise distribution. The accuracies of the LS and ML algorithms were determined using the mean and standard deviation (for N simulations) of median deviation from the true ADC value for regions of interest corresponding to the two types of tissue in the phantom.

Phantoms were created by generating a prior ADC map, used as references for further accuracy assessment. ADC values chosen for tumour ($0.9 \times 10^{-}3mm^2/s$) and peripheral zone ($1.5 \times 10^{-}3mm^2/s$) were based previous studies by deSouza *et al.* [2]. S_0 values used in phantoms were calculated from real prostate scans analysed by a radiologist, using mean values of ROIs delineated within the noisy S_0 image, as shown in figure 1. This resulted in a peripheral zone S_{0PZ} of 0.34 ± 0.024, and a tumour S_{0T} of 0.26 ± 0.085. The resulting S_0 and ADC maps were then used to generate a set of diffusion images, using the same B-values as previously. A similar method to that used to obtain realistic S_0 values was used for the



(a) (b) (c) (d) (e)

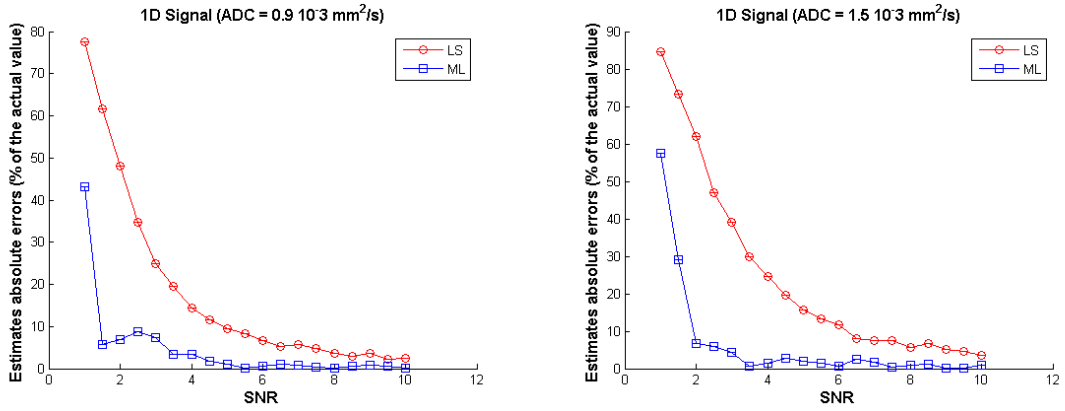
Figure 1: Example of real prostate MR data with delineated regions of interest used for S_0 values approximation. T2-MR 1(a) and B0 DW-MR 1(c) images with complete FOV (tumour location indicated by the arrow head). And zoom of the T2 1(b) and B0 1(d) images showing the ROI more clearly. An example of a phantom B_0 image for a tumour diameter of 15 pixels is showed in 1(e).

choice of σ_R. Background - *i.e.* noise only - pixels were taken from patients data and fitted to a Rayleigh distribution providing an estimate of the Rician distribution parameter. As the Rice and Rayleigh distributions are equivalent at $SNR = 0$ [4] (the resulting values for σ_R were such that $\sigma_R \in [0.05, 0.07]$).

3 Results

3.1 Monte Carlo Simulation

Comparison of median errors from LS and ML estimates, shows that the ML is on average more accurate which is consistent with results obtained in [7]. In general, the LS tends to underestimate ADCs resulting in a bigger absolute error (see figure 2). However, the standard deviation of ML estimates is large for low SNRs (< 4), as illustrated in figure 3.

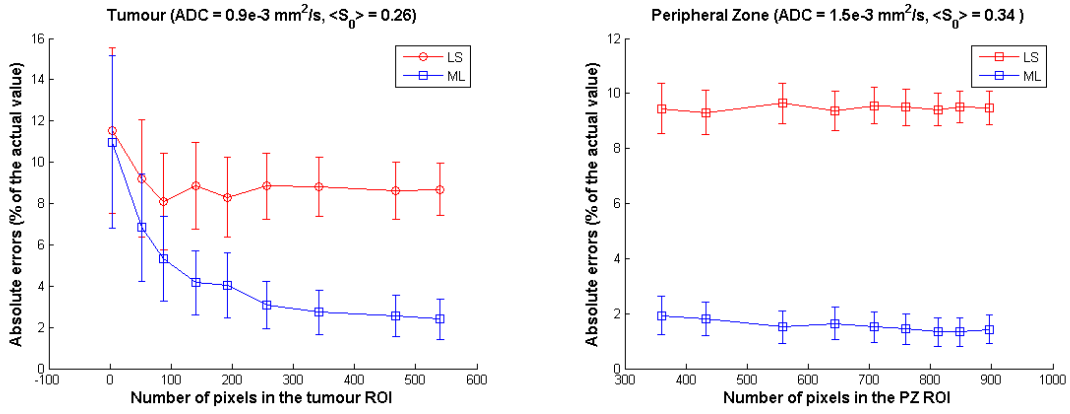


(a) (b)

Figure 2: Monte carlo simulations at ADC values of $9 \times 10^{-}4mm^2/s$ (left) and $1.5 \times 10^{-}3mm^2/s$ (right). These graphs show the median of absolute error of estimates compared to the real ADC value, obtained with both LS and ML for various SNR values. Results are presented as a percentage of the real ADC value

3.2 Phantom Simulation

Tumour S_0 is lower than PZ resulting, in a higher SNR in the peripheral zone than in the tumour region for constant σ_R ($SNR_T \approx 3.7$ and $SNR_{PZ} \approx 4.8$ for $\sigma_R = 0.07$). As a consequence, a bigger variability can be observed in the tumour region, especially for small ROIs, whereas estimates in the peripheral zone are more stable. Despite this effect, it is clear that ML median estimate of ADC in the two type of tissue is much more accurate than that of the LS: with an average median error difference of 4.3% in the tumour region and 7.9% in the PZ region.



(a) (b)

Figure 3: Result estimates of phantom experiment at $\sigma_R = 0.07$: The graph show the mean and standard deviation (over the N = 150 simulations) of the median estimates for pixels in the tumour in 3(a) and peripheral zone 3(b) obtained with both LS and ML. The number of pixels in the Tumour (respectively PZ) increases (respectively decreases) as the tumour diameter increases.

4 Discussion and Conclusion

Our results indicate that ML estimates may be more useful than LS for assessment of prostate tumours. The variability of ML may be further reduced using Bayesian inference along with Random Markov Field as suggested by Walker-Samuel *et al.* [8]. Limitations of our method include estimating S_0 from noisy data and no consideration of the spatially dependent noise that may occur when using multi-coil data. However, for a study focused on a small organ such as the prostate, the noise distribution can be assumed to remain unchanged over the studied ROI, this is why it was considered as uniform in the presented experiments.

In this paper the estimation of ADC using maximum likelihood as an alternative to the least squares algorithm was discussed. It was shown, based on simulated data with values (of ADC and S_0) taken from real prostate DW-MR scans, that ML median estimates are more accurate for realistic level of noise and ROI size. These results suggest that ML should be preferred for tumour grading in clinical practice. Future work could highlight quantitative correlation between ADC and Gleason grades for prostate cancer in the peripheral zone.

References

[1] E. M. Charles-Edwards and N. M. deSouza. Diffusion-weighted magnetic resonance imaging and its application to cancer. *Cancer Imaging*, 6:135–143, 2006.

[2] N. M. deSouza, S. F. Riches, N. J. Vanas, V. A. Morgan, S. A. Ashley, C. Fisher, G. S. Payne, and C. Parker. Diffusion-weighted magnetic resonance imaging: a potential non invasive marker of tumour aggressiveness in localized prostate cancer. *Clinical Radiology*, 63:774–782, 2008.

[3] J. Ferlay, P. Autier, M. Boniol, M. Heanue, M. Colombet, and P. Boyle. Estimates of the cancer incidence and mortality in europe in 2006. *Annals of Oncology*, 18:581–592, 2007.

[4] H. Gudbjartsson and S. Patz. The rician distribution of noisy MR data. *Magnetic Resonance in Medicine*, 34:910–914, 1995.

[5] J. Sijbers, A. J. Den Dekkler, P. Scheunders, and D. Van Dyck. Maximum-likelihood estimation of rician distribution parameters. *Annals of Oncology*, 18:581–592, 2007.

[6] S. Verma, A. Rajesh, H. Morales, L. Lemen, G. Bills, M. Delworth, K. Gaitonde, J. Ying, R. Samartunga, and M. Lamba. Assessment of aggressiveness of prostate cancer: Correlation of apparent diffusion coefficient with histologic grade after radical prostatectomy. *American Journal of Roentgenology*, 196:374–381, 2011.

[7] S. Walker-Samuel, M. Orton, L. McPhail, and S. Robinson. Robust estimation of the apparent diffusion coefficient (adc) in heterogeneous solid tumors. *Magnetic Resonance in Medicine*, 62(2):420–429, 2009.

[8] S. Walker-Samuel, M. Orton, J. Boult, and S. Robinson. Improving apparent diffusion coefficient estimates and elucidating tumor heterogeneity using bayesian adaptive smoothing. *Magnetic Resonance in Medicine*, 65(2):438–447, 2011.