Point-Placement Error in Shape Models

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Abstract

Here we analyse the effects of the error involved in manually placing "mark-up" points in active shape models. We demonstrate that the magnitudes of eigenvalues retained in principal components analysis (PCA) are increased by point-placement error. We present a revised covariance matrix that reduces or completely removes the effects of point-placement error. We find results for "predicted" placement errors that are in excellent agreement with the "set-up" parameters of simulations for an elliptical shape. We find that increasing point-placement error in the original "training set" increases point-to-line and point-to-point errors in ASM image searches and adversely affects convergence.

1 Introduction

Active shape models (ASMs) and active appearance models (AAMs) [1, 2, 3, 4] have been used extensively in image processing to carry out the segmentation of features from (especially medical) images (see Fig. 1). However, the effects of inaccuracy in the placement of "mark-up points" used in ASMs and AAMs have not been considered in great detail. Here we use "measurement models" [5] in order to analyse and account for such sources of error.

2 Method

A 2D shape may be represented by a vector $\mathbf{z}_{ij}$ of size $2n$, where n is the number of mark-up points. The index i refers to a specific image in the training sample of size N, and the index j refers to a specific observer (or equivalently to a specific replication of the mark-up points by the same observer). The "mean shape" over the N images in our "sample" for observer/replication j is given by $\overline{\mathbf{z}}_j = \frac{1}{N}\sum_{i=1}^{N}\mathbf{z}_{ij}$. We define additionally: $\delta\mathbf{z}_{ij} = \mathbf{z}_{ij} - \overline{\mathbf{z}}_j$. The covariance matrix is given by

$$\mathrm{Cov}(\mathbf{z}'_{j_1}, \mathbf{z}_{j_2}) = \frac{1}{N-1}\sum_{i=1}^{N}\delta\mathbf{z}'_{ij_1}\;\delta\mathbf{z}_{ij_2} \quad . \tag{1}$$

The l^{th} eigenvalue of Eq. (1) is denoted λ_l and its (normalised) eigenvector is denoted by $\hat{\mathbf{u}}_l$. (The symbol $'$ indicates the transpose of a vector.) A new shape is represented by

$$\mathbf{z}^{\text{new}} = \overline{\mathbf{z}} + \sum_{l=1}^{M} b_l \hat{\mathbf{u}}_l \quad . \tag{2}$$

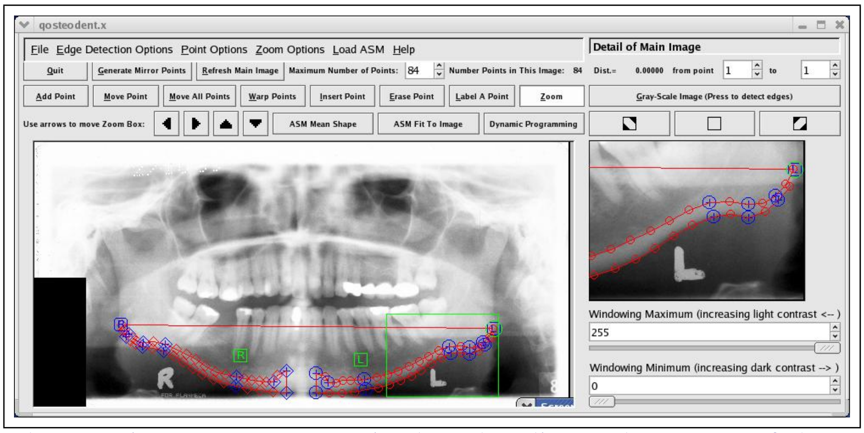


Figure 1: Mark-up points on a panoramic dental radiograph as part of the OSTEODENT project (see Ref. [4]). It was remarked anecdotally during the process of "marking-up" these images by clinicians that placement of points parallel to strong edges was much more difficult than placement of these points perpendicular to these edges, e.g., vertically at the left- and rightmost points of the jaw. However, the effects of placement errors on the application and use of ASMs (e.g., in image searches) is not well-understood.

The b-coefficients are (scalar) coefficients and M is the number of the eigenvectors used in PCA in descending order of λ_l. Constraints on these coefficients are that $|b_l| \leq 3\sqrt{\lambda_l}$ so that we never stray too far away from a "sensible solution." The cumulative amount of variability A_M of M of the eigenvalues/vectors is given by $A_M = \frac{\sum_{l=1}^{M}\lambda_l}{\sum_{l=1}^{2n}\lambda_l}$.

A "measurement model" [5] for the mark-up points for a given image i and observer/ replication j,

$$\mathbf{z}_{ij} = \eta_i + \varepsilon_{ij} \quad , \tag{3}$$

where η_i indicates the "true" value and ε_{ij} is the "placement" or "measurement" error. (η and ε are also vectors of size $2n$.) Errors ε are assumed uncorrelated with the "true" values for the points η and they are also uncorrelated between different observers/replications, i.e., for different values of j. (Here there are no "systematic biases" dependent on observer.) From these discussions, we see now that

$$\mathrm{Cov}(\mathbf{z}'_{j_1}, \mathbf{z}_{j_2}) = \mathrm{Cov}(\eta', \eta) + \mathrm{Cov}(\varepsilon'_{j_1}, \varepsilon_{j_2})\delta_{j_1,j_2} \quad , \tag{4}$$

where δ_{j_1,j_2} is equal to 1 if $j_1 = j_2$ and it is equal to 0 otherwise. We use Eq. (4) in order to form an "error-corrected" version of the covariance matrix that should reduce or remove the effects of point-placement or "measurement" error. For the simplest case of two observers/replications, we see that $\mathrm{Cov}(\mathbf{z}'_1, \mathbf{z}_2) = \mathrm{Cov}(\mathbf{z}'_2, \mathbf{z}_1) = \mathrm{Cov}(\eta', \eta)$; which ought to be correct in the asymptotic limit $N \rightarrow \infty$. However, this is clearly impossible in a practical study and so we note that we might have problems with small sample sizes leading to a (marginally) non-symmetric covariance matrix. We propose that a reasonable approximation to might be given by

$$\mathrm{Cov}(\eta', \eta) = \frac{\mathrm{Cov}(\mathbf{z}'_1, \mathbf{z}_2) + \mathrm{Cov}(\mathbf{z}'_2, \mathbf{z}_1)}{2} \quad . \tag{5}$$

We estimate placement errors by finding $\mathrm{Cov}(\varepsilon'_j, \varepsilon_j) = \mathrm{Cov}(\mathbf{z}'_j, \mathbf{z}_j) - \mathrm{Cov}(\eta', \eta)$. The variance at each mark-up point is given by the diagonal of this matrix and so the predicted error

	No Placement Error	With Placement Error	"Error-Corrected" Matrix of Eq. (5)
1	0.125	0.131	0.125
2	0.059	0.063	0.060
3	0.009	0.023	0.010
4	0.008	0.020	0.009
5	0.002	0.018	0.002
6	0.000	0.018	0.002

Table 1: The first six eigenvalues for the elliptic shape with $n = 20$ and $N = 1000$.

(i.e., the standard deviation) is found readily. An estimate of the reliability for a specific observer/replication is given by $\kappa = \text{Var}(\eta)/\text{Var}(\mathbf{z})$. A value of κ near to zero indicates low reliability and a value near to 1 indicates good reliability.

An elliptical shape was used in the simulation study. The basic equations are given by

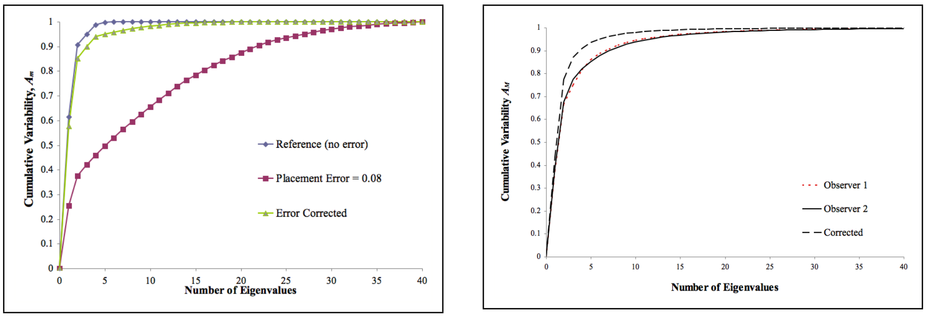
$$\begin{aligned} x(\theta) &= a\cos(\theta)\cos(\phi) - b\sin(\theta)\sin(\phi) \\ y(\theta) &= a\cos(\theta)\sin(\phi) + b\sin(\theta)\cos(\phi) \ . \end{aligned} \tag{6}$$

The coefficients a and b were both set to a value of 0.4 (standard deviation=0.1), and ϕ was set to zero (standard deviation=0.025 rads.). Two sets of mark-up points were generated automatically with equal amounts of uncorrelated random (Gaussian) error added to the placement of the points. Points on the left and the right of the shape had more random error in the y-direction than those near to x=0. Points near to x=0 had more random error in the x-direction than those at the left and right edges. Mark-up points were placed on the cortical bone edges of panoramic mandible images [4] as part of the OSTEODENT study by two experts (separately) using a custom-written graphical user interface (again, see Fig. 1 above). $N = 133$ images were used in this study and there were n=84 mark-up points in total.

3 Results

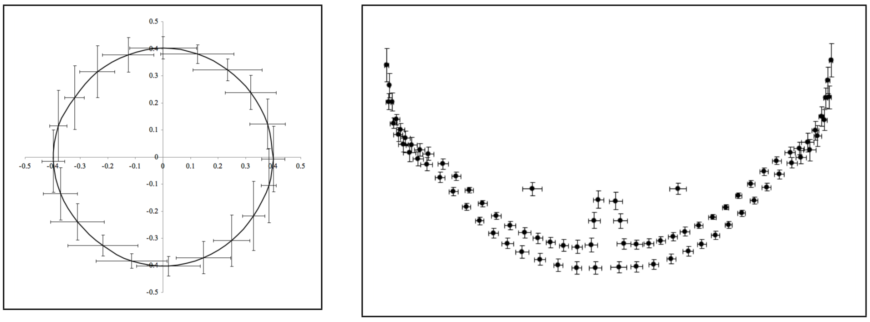
Results for the first six eigenvalues for the elliptic shape for the case of $n = 20$ (d.o.f. = 2×20) and $N = 1000$ are shown in Table 1. We see from this table that the eigenvalues for the cases with random placement error compare less well to a "reference" case with zero placement error than the corrected case of Eq. (5). We see that the effect of placement error is to increase the eigenvalues. The cumulative amount of variability A_M is shown in Fig. 1 for the elliptic shape with $n = 20$ (d.o.f. = 2×20) and $N = 1000$, and also for the OSTEODENT data. We see that the effects of placement error (magnitude 0.08 in both the x- and y-directions separately) for the simulated elliptical shape is to reduce the cumulative amount of variability A_M at every value of M when compared to the "reference" case with no placement error. The curve for the "corrected" covariance matrix of Eq. (5) lies quite close to the "reference" case with zero placement error. The results for the corrected covariance matrix of Eq. (5) for the OSTEODENT data are higher than those curves for the two independent observers in Fig. 2b. These results indicate that the corrected covariance matrix of Eq. (5) is probably going some way to removing the effects of point-placement error.

The mean shapes for the "elliptical" simulations ($n = 20$, $N = 1000$) and the OSTEODENT data are shown in Fig. 3. We see that the mean shapes are circles for the "elliptical" simulations and that the values for a and b are near to their "true" values of 0.4, as expected. The average *predicted* errors (both in the x- and y-directions separately) were 0.080($\pm$0.001),



(a) (b)

Figure 2: Cumulative variability A_M for the (a) simulated elliptic shapes ($n = 20$, $N = 1000$) and (b) the OSTEODENT data



(a) (b)

Figure 3: Mean shape and predicted placement errors (standard deviations) for the (a) simulated elliptic shapes ($n = 20$, $N = 1000$) and (b) OSTEODENT data (expert 1)

which also compares well to the "true" value of 0.08 in these simulations. The predicted errors in the y-direction also increased near to the right and left of the figure and those in the x-direction at the top and the bottom, also as expected from the set-up of this simulation. Results for the predicted errors for the OSTEODENT study indicated that errors were greatest along the direction of shape edges (e.g., at the left- and rightmost points of the jaw) and also at the "mental foramen" [4]. Also, expert 1 had a mean(s.d.) predicted placement error of 74(17) pixels and expert 2 had a mean(s.d.) error of 59(22) pixels for the OSTEODENT data. The result for the reliability for the simulated elliptical shapes was $\kappa = 0.44$ ($n = 20$, $N = 1000$; average placement error in the x- and y-directions = 0.08). Results for the reliabilities of experts 1 & 2 for the OSTEODENT data were $\kappa = 0.786$ and 0.845, respectively.

A simple ASM image search code was written for the elliptical simulated shapes with $n = 40$ and $N = 500$. Initial results for the point-to-line (P-2-L) and point-to-point (P-2-P) errors and the percentage (%C) of successfully converged ASM image searches (to 3 d.p.s. in max. 200 iterations) for "new" shapes under the same conditions as for the "training set" are shown in Table 2. The "reference" case with zero placement error demonstrated the smallest amount of error, and convergence was found to be very good. (400 "test" cases were used in order to determine these quantities.) The cases with error added in the "training set" showed increased point-to-line and point-to-point errors and also poorer convergence properties. Interestingly, this became more apparent for increased numbers of eigenvectors retained, M. The predicted mark-up points from ASM image searches were more prone to "wander" along the edge of the image boundary for larger values of M. We suspect that this is because these eigenvalues/vectors for larger M corresponded purely to random "noise"

	$M = 10$			$M = 20$			$M = 40$		
	P-2-L	P-2-P	%C	P-2-L	P-2-P	%C	P-2-L	P-2-P	%C
"Reference"	0.027(8)	0.10(3)	99%	0.026(8)	0.10(3)	100%	0. 026(8)	0.10(3)	99%
Error=0.1	0.033(7)	0.10(2)	99%	0.044(7)	0.16(3)	91%	0.042(6)	0.25(4)	79%
Error=0.2	0.036(7)	0.10(3)	100%	0.048(7)	0.16(3)	95%	0.046(6)	0.25(3)	76%
"Corrected"	0.037(7)	0.12(2)	99%	0.046(8)	0.17(4)	94%	0.044(6)	0.21(4)	80%

Table 2: Mean point-to-line (P-2-L) and point-to-point (P-2-P) errors evaluated over 400 ASM image searches for the elliptic shapes with $n = 40$ and $N = 500$. The percentage of successfully converged searches (%C; 3 d.p.s in 200 iterations) is shown also. ("Reference"=no placement-error in the underlying shape model; Error=0.1,0.2 has some placement-error; "Corrected" uses Eq. (5). Errors w.r.t. the last decimal place shown are in brackets.)

from the placement error in the underlying shape model. Finally, results for the point-to-point distance for the "corrected" covariance matrix of Eq. (5) (arguably) improved slightly on those results with random placement-error added (error=0.1,0.2) for $M = 40$ in these very initial studies, although not for lower M.

4 Conclusions

Measurement models allow us to predict where placement errors are smallest and "reliability" is greatest geographically on a (e.g., mean) shape. This information might be used to to decide where to extract biometric parameters or which set of mark-up points one ought to employ, where more than one set of mark-up points is available. Placement error in the "training set" for the shape model was seen to increase point-to-line and point-to-point errors in ASM image searches and to adversely affect convergence. A revised covariance matrix that reduces or removes the effects of placement error was tested: cumulative variability curves were shifted upwards nearer to the "reference" case with zero error; and, a slight reduction in point-to-point errors for larger values of M was seen in ASM image searches in very initial tests. However, more testing is needed to establish firmly whether the "corrected" covariance matrix is a useful and practical method for feature segmentation via ASMs.

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