Reliably Estimating the Diffusion Orientation Distribution Function from High Angular Resolution Diffusion Imaging Data

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Abstract

This paper describes an automated model selection method for analysing the relationship between the order of the spherical harmonic basis functions used to fit high angular resolution diffusion imaging (HARDI) data and the accuracy of the fitting results. The method performs statistical inference on the spherical harmonic expansion coefficients and uses a backward elimination procedure to remove those basis functions that contribute the least to explaining the data. The proposed method improves the accuracy of higher order spherical harmonic expansion while preserving its shape adaptation properties.

1 Introduction

Diffusion-weighted (DW) imaging of the brain uses water molecule diffusion in the tissue to introduce contrast to images. Water diffusivity can be analysed to study brain connectivity in vivo [14]. DW imaging can calculate a single scalar diffusivity, which depends on the direction of the applied magnetic field gradient. Diffusion tensor imaging (DTI) [4, 5, 19] extends this one-dimensional technique and reconstructs the three-dimensional diffusion profile. Due to the assumption of a single fibre bundle within a voxel, DTI calculates only the dominant fibre direction for the voxel. Regions of complex anatomy though, require more advanced reconstruction techniques such as high angular resolution diffusion imaging (HARDI) [17] methods based on apparent diffusion coefficient (ADC) analysis [2, 6], diffusion spectrum imaging [18], generalized diffusion tensor imaging [9, 11], q-ball imaging [12, 13, 16], spherical deconvolution [3, 15] and persistent angular structure (PAS) MRI [7].

Most HARDI methods rely on extracting from the diffusion signal a probability density function called the orientation distribution function (ODF), which can be processed further to produce fibre ODFs (fODF). Such probability functions can be used to estimate connectivity of different brain regions. The ideal fODF is a set of delta functions, and is obtained by convolution of the diffusion weighted signal with the response function for a single fibre. A sharp and noise suppressed representation of the fODF, with the capacity for resolving multiple fibre bundle configurations depends on an accurate modelling of the diffusion signal.

In order to estimate ODFs and fODFs from HARDI data the signal profile is usually expressed with a set of orthogonal basis functions such as spherical harmonics (SHs). SHs can be seen as the extension of Fourier basis functions to the sphere. They are the solutions of Laplace's equation and are given by:

$$Y_l^m(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(cos\theta)\exp(im\phi) \tag{1}$$

with P_l^m being the associated Legendre function of order l and degree m. We define a single integer $j = l^2 + l + m$ to enumerate a SH series.

In the first attempt to utilize the SH framework the ADC profile was modelled using a linear combination of SHs [2]:

$$\mathbf{f} = \alpha\mathbf{Y} \tag{2}$$

where α is a vector with SH coefficients, $\mathbf{Y}$ is a matrix with the elements $Y_{ij} = Y_j(\theta_i, \phi_i)$ and $\mathbf{f}$ is a vector with either a diffusion coefficient d or diffusion signal S. The relation between the d and S is given by:

$$d = -\frac{1}{b}\log(S). \tag{3}$$

where b is a diffusion-weighting factor.

The advantage of this approach is that the calculation of the coefficients requires a relatively small number of samples and a linear least squares fitting. However, with this scheme it is not initially obvious how many SHs should be used. Further it is not obvious which of the SHs should be included in the expansion, leading to the use of a relatively large number of SHs to represent the signal. To our knowledge, no detailed experimental analysis of this model selection problem has been published.

Diffusion-weighted signals are particularly affected by noise (Figure 1 image *a*) due to the high degree of signal attenuation needed for a suitable diffusion weighting. By using too many basis functions to explain the signal the ODF will fit to the acquisition noise (Figure 1 image *c*) whereas by using too few it will not resolve the underlying signal accurately.

In this paper we present an approach that uses a backward elimination process to find those SHs that significantly contribute, in a statistical sense, to explaining the measured signal, and to remove those that merely fit to the noise. This will improve the robustness of the SH representation of the diffusion signal to acquisition noise.

2 Model selection by backward elimination

For a sufficiently high signal to noise ratio, the acquisition noise is (to a reasonable approximation) normally distributed with a variance σ^2. Consequently statistical inference can be performed on the estimated expansion coefficients.

If the measured diffusion signal is $\mathbf{S}$ and the fitted signal obtained by expanding N volumes over L spherical harmonics is $\mathbf{S}_L$ then an unbiased estimate of σ^2 (Equation 4) and the t distribution with $N-L$ degrees of freedom under the null hypothesis (Equation 5) can be computed as:

$$\hat{\sigma}^2 = \frac{\Sigma(\mathbf{S}-\mathbf{S}_L)^2}{N-L} \tag{4}$$

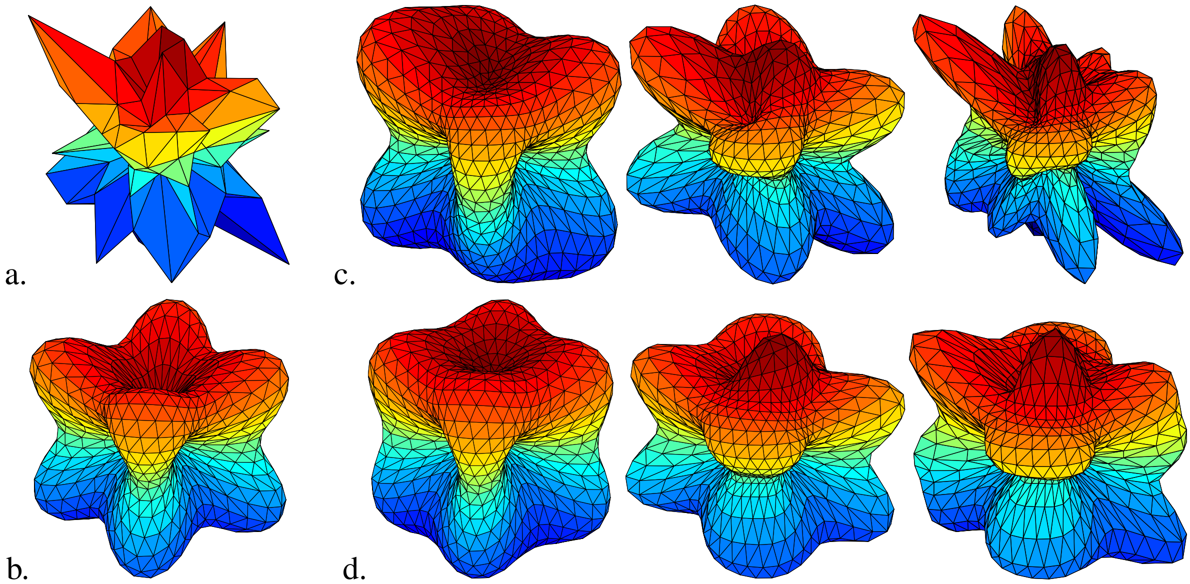

Figure 1: ADC profiles of a synthetic 3 fibre configuration. Visualisation of: noisy 60-volume scan (a), ground truth (b) and 4th, 6th and 8th order SH expansion of noisy data both without (c) and with (d) backward elimination model selection.

$$t_i = \frac{|\alpha_i|}{\hat{\sigma}\sqrt{M_{ii}}} \tag{5}$$

where M_{ii} is the ith diagonal element of matrix $\mathbf{M} = (\mathbf{Y}^T\mathbf{Y})^{-1}$.

To establish whether the expansion coefficient α_i contributes, in a statistical sense, to explaining the diffusion signal, rather then simply to noise, t_i should be compared with the t_{N-L} distribution. If t_i is lower than some critical value (CV) the coefficient can be considered as statistically insignificant and will be removed from the expansion of Equation 2. Since the estimate of σ^2 depends on the number of coefficients being estimated, the removal of the coefficients is done iteratively through a process called backward elimination. The algorithm can be summarised as:

1. Estimate the current set of expansion coefficients and the variance σ^2.
2. Compute the t value for each of the coefficients.
3. Eliminate from the expansion the SH function with the smallest t value.
4. Repeat from step 1 until only significant expansion coefficients are left.

3 Experimental setup and results

A backward elimination model selection procedure was applied to a synthetic HARDI datasets. Data for each voxel was generated using a multi-Gaussian model [1, 6]:

$$S(\bar{g}_i) = S_0 \sum_{k=1}^{n} v_k \exp(-b\bar{g}_i{}^T \mathbf{D}_k \bar{g}_i). \tag{6}$$

where S_0 is a T2 image, v_k is a volume fraction ($\sum_{k=1}^{n} v_k = 1$) and D_k a diffusion tensor ($\lambda_1 = 0.0015$, $\lambda_2 = \lambda_3 = 0.0005$) of the kth fiber, b is a constant (1500) and $\bar{g}_i$ is a diffusion

	4^{th}	6^{th}	8^{th}	CV
noiseless	7.0936e-03	6.0679e-05	3.3518e-07	-
SNR 20	0.6398	1.1882	1.9114	-
	0.6241	1.021	1.6716	90%
	0.6181	0.8943	1.408	95%
	0.6202	0.7856	1.1018	97.5%

Table 1: Mean error of 4^{th}, 6^{th} and 8^{th} order SH expansion of random 3 fibre orientations. Same data with and without added noise (row 1 and 2) and with a backward elimination model selection applied to the noisy data (row 3, 4 and 5).

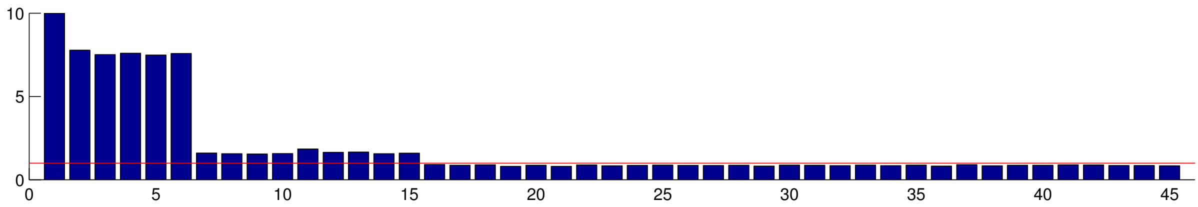

Figure 2: Frequency (in thousands) of how often a basis function was statistically significant. 3 random fibre orientations, $CV = 97.5\%$. Red line marks 1000 (10% of cases).

encoding direction (60 directions in total). To simulate a real world measurement with a signal-to-noise ratio of 20 a normally distributed noise with the standard deviation of 0.05 was added.

Table 1 shows the comparison of results obtained using full and reduced SH expansion. The test case included 10000 voxels with 3 random volume fractions and fibre orientations, each expanded using 4th, 6th and 8th order SH. For backward elimination model selection the CV was set to 90%, 95% and 97.5%.

4 Discussion and further work

We have shown that the ODF of any 3 fibre configuration can be accurately modelled with a 4th order SH (Table 1 row 1). The estimation accuracy can be improved by applying a backward elimination model selection procedure (Table 1 rows 3–5 and Figure 1 image *d*) to the expansion. Although we have only investigated the SH expansion, the proposed backward elimination procedure can be applied to other basis functions as well. The important advantage of this method is that it can also be used for other DW-MRI data.

It was observed that when expanding a noisy HARDI dataset (SNR of 20) the higher order basis functions rarely (once for every ten cases) contribute to explaining the signal (Figure 2) and instead they mostly interpolate noise. It was also observed that in the case of expanding the signal with a 4th order SHs the highest CV (97.5%) performs (on average) worse than the lower (95%). To minimize the effect of the type 2 statistical error a CV should be chosen based on the order of the SHs used.

In this research each voxel was solved independently. However, even if the fitting procedure minimises the least squares error function, real data is inherently noisy and may still cause errors. Spatial regularisation within the neighbouring voxels can produce smoother diffusion signals. Incorporating regularisation into the backward elimination method should improve the accuracy of the model and produce a better results.

Though SHs have been widely used for data modelling, they are the extension of Fourier basis functions to the sphere, globally supported and therefore cannot represent high-frequency signals efficiently. They also suffer from some difficulties, such as ringing. Spherical wavelets [8] and spherical ridgelets [10] have been used to estimate and sharpen ODF. In future work we will investigate the application of the proposed model selection procedure to these promising approaches.

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