ACD

Making Rotation Averaging Fast and Robust with Anisotropic Coordinate Descent

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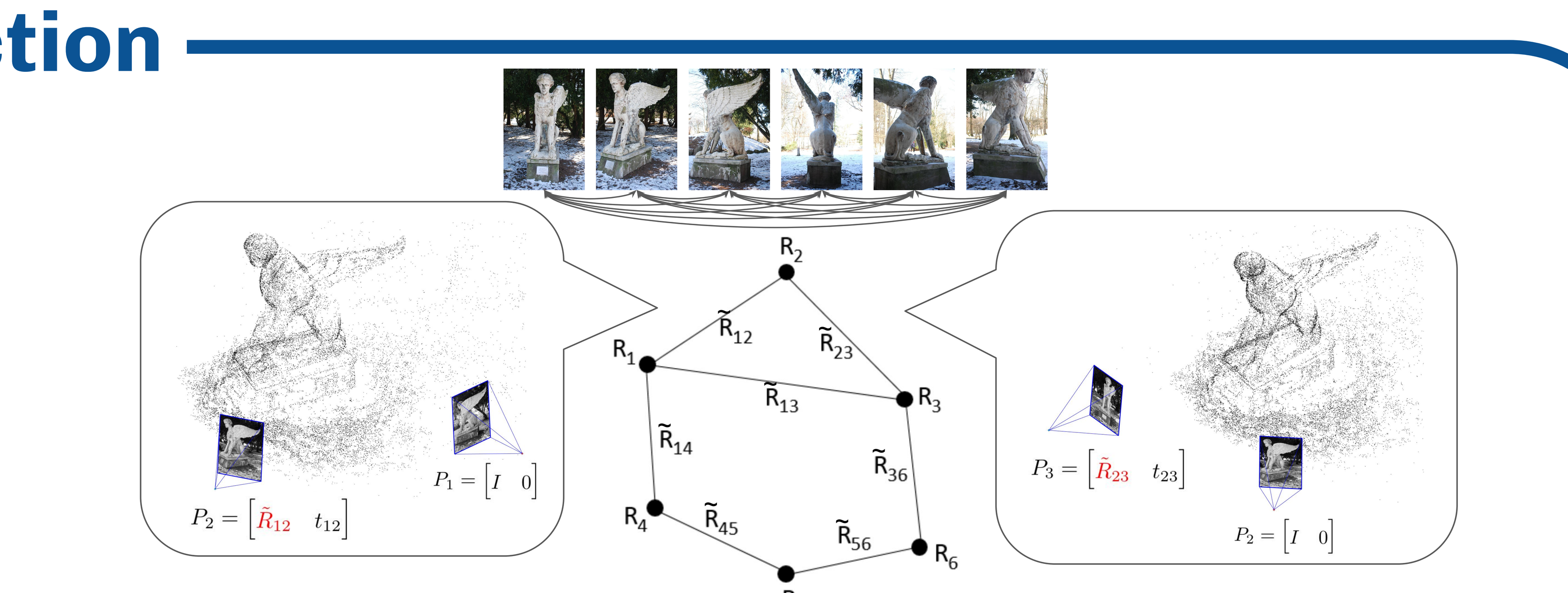
Introduction

Rotation averaging is a key subproblem in SLAM/global SfM. The problem is sketched to the right.

We address the problem of **fast**, **accurate** and **robust** anisotropic rotation averaging. **Anisotropic** means propagating pairwise **uncertainties**, which can potentially improve accuracy.

cSO(3) [1] is the first formulation able to recover global optima in outlier-free scenario. It uses generic semidefinite programming (SDP) solver that has limited scalability.

Robust local methods [2,3] are fast but sensitive to initialization.



Given: relative rotations $\{\tilde{R}_{ij}\}$ (edges) — relative pose solutions (local coordinate system)

Find: camera orientations $\{R_i\}$ (nodes) — rotations in global coordinate system.

Prior work and analysis

Isotropic case: SDP relaxation O(3) [4]:

$$\min_{\mathbf{X}\succeq 0} -\langle \tilde{\mathbf{R}}, \mathbf{X}\rangle \quad \text{s.t. } \mathtt{X}_{ii} = \mathtt{I}$$

sought matrix
if rank($\mathbf{X}$)=3 → $\mathbf{X}=\mathbf{R}\mathbf{R}^\top$
$\mathbf{R} = [R_1^\top \ R_2^\top \ \ldots]^\top$

blocks of relative rotations ($\mathbf{0}_{3\times 3}$ if N/A)

$$\tilde{\mathbf{R}} = \begin{pmatrix} 0 & \tilde{R}_{12}^\top & \tilde{R}_{13}^\top & \ldots & \tilde{R}_{1n}^\top \\ \tilde{R}_{12} & 0 & \tilde{R}_{23}^\top & \ldots & \tilde{R}_{2n}^\top \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{1n} & \tilde{R}_{2n} & \tilde{R}_{3n} & \ldots & 0 \end{pmatrix}$$

[4] proposes block-coordinate descent-based approach:

1) fix all but one block-column of unknowns in $\mathbf{X}$
2) solve a sub-problem, update the unknown block

RCD [5] proposes to keep track of "effective" variables $\mathbf{R}$ reasoning that optimal $\mathbf{X}=\mathbf{R}\mathbf{R}^\top$. We show that throughout iterations the resulting $\mathbf{R}$ always stays within SO(3)^n.

Anisotropic case: SDP formulation cSO(3) [1]:

$$\min_{\mathbf{X}\succeq 0} -\langle \mathbf{N}, \mathbf{X}\rangle \quad \text{s.t. } \mathtt{X}_{ii} = \mathtt{I} \text{ and } \mathtt{X}_{ij} \in \operatorname{convhull}(\mathrm{SO}(3))$$

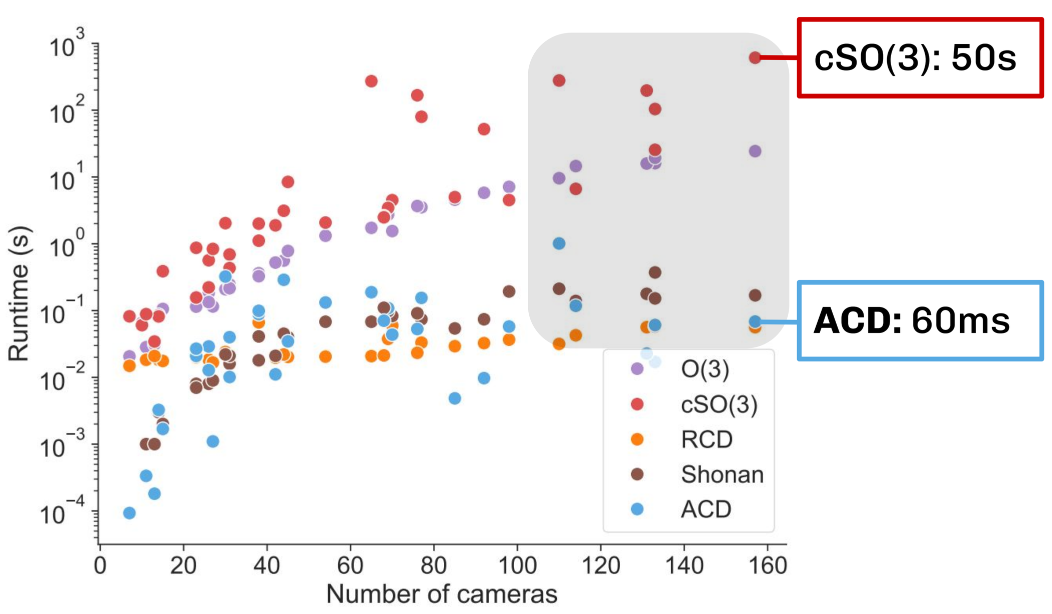
blocks of relative rotations pre-multiplied with matrices encoding uncertainty ($\mathbf{0}_{3\times 3}$ if N/A)

No dedicated solver exists. We propose a sub-problem where SO(3) constraint is enforced. It leads to a much simpler closed-form solution — see below

Contributions / Main results

I simple iterative solver propagating uncertainties (**ACD**)
- key operation highlighted

```
for t ∈ {1,...} do
    for k ∈ shuffle({1,...,n}) do
        R_i^(t+1) ← R_i^(t)   (i ≠ k);
        R_k^(t+1) ← project_SO(3)(N_k^T R^(t));
    end
end
return R
```



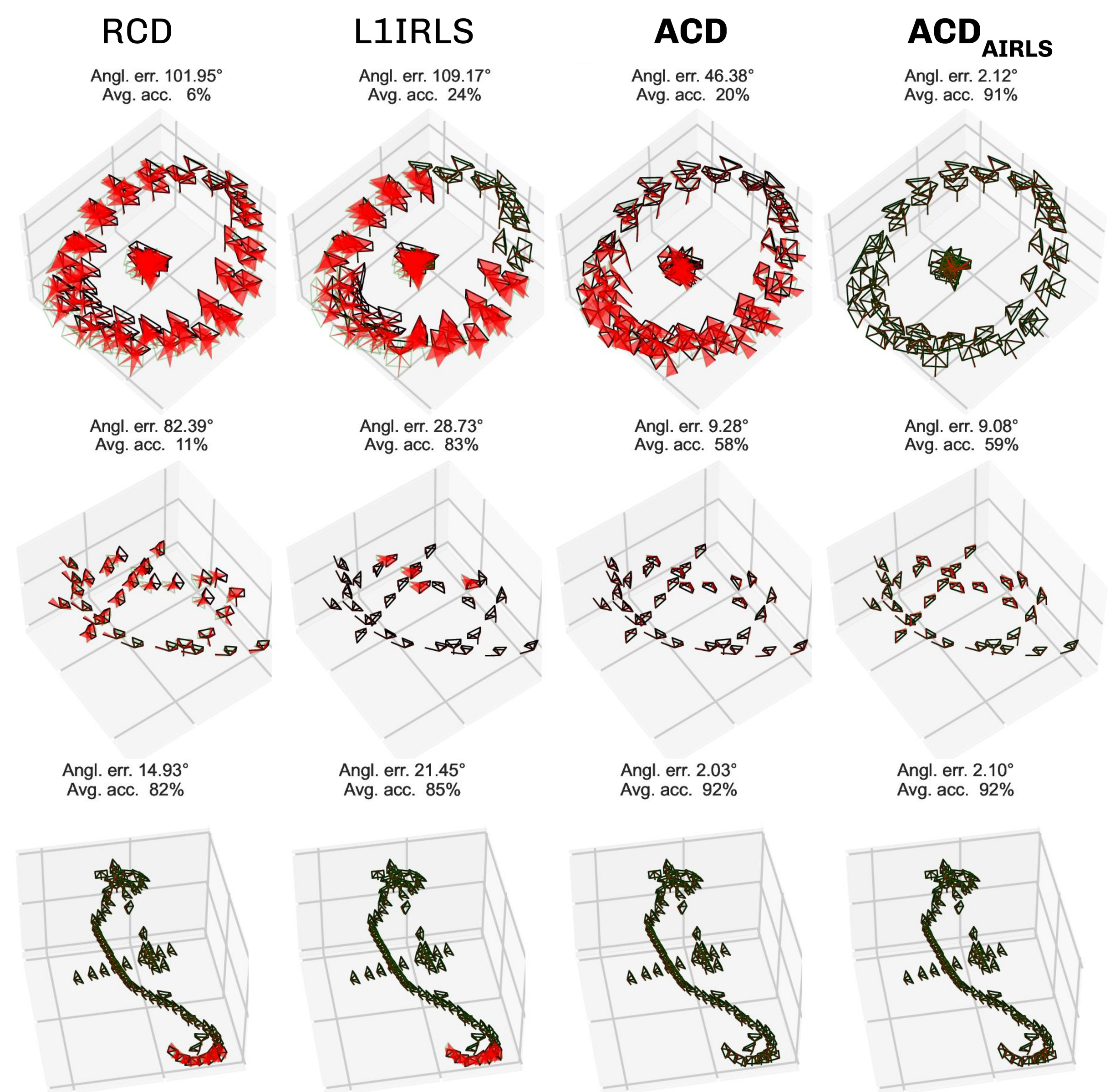
II yields large improvements in runtime

III Anisotropic extension for robust refinement ($_{\text{AIRLS}}$)

$$\min_{\{\Delta\hat{\omega}_k\}} \sum_{(i,j)\in\mathcal{E}} \rho\left(\left\|(\Delta\hat{\omega}_j - \Delta\hat{\omega}_i) - \omega(\mathtt{R}_j^\top \tilde{\mathtt{R}}_{ij} \mathtt{R}_i)\right\|_{\mathtt{H}_{ij}}\right)$$

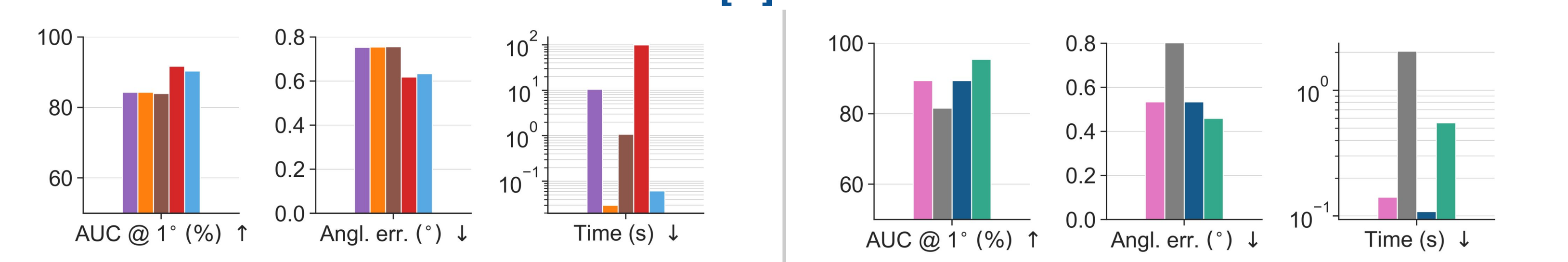
robust kernel | unknown axis-angle updates | current rotational residual | uncertainty

Evaluation

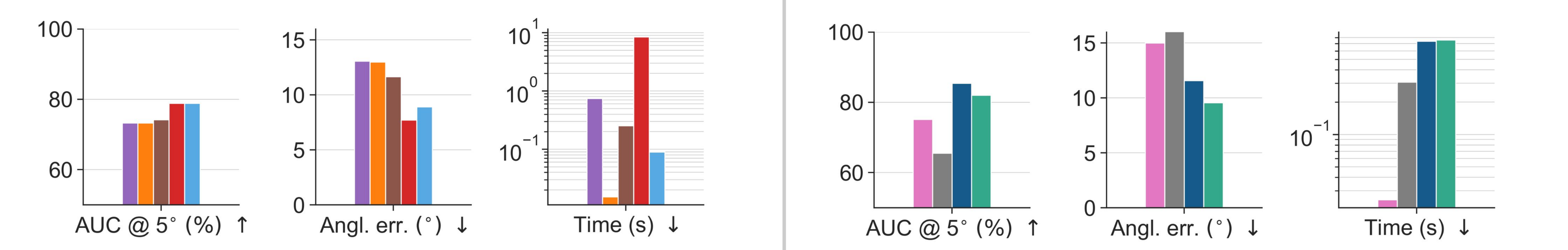


Non-robust methods: O(3), RCD, Shonan, cSO(3), **ACD**
Robust methods: L1IRLS, Zhang, **ACD$_{\text{IRLS}}$**, **ACD$_{\text{AIRLS}}$**

SfM [4] datasets



ETH3D MVS [5] datasets with SIFT features



Overall:

1) anisotropic RA/**ACD** gives more accurate solutions
2) **ACD** is significantly faster than SDP solver cSO(3) [1]
3) **ACD$_{\text{IRLS/AIRLS}}$** often improve the results further

Code, paper etc

ylochman.github.io/acd

References

[1] Olsson et al. Certifiably optimal anisotropic rotation averaging. ICCV'25
[2] Chatterjee, Govindu. Efficient and robust large-scale rotation averaging. ICCV'13
[3] Zhang et al. Revisiting rotation averaging: Uncertainties and robust losses. CVPR'23
[4] Eriksson et al. Rotation averaging and strong duality. CVPR'18
[5] Parra et al. Rotation coordinate descent for fast globally optimal rotation averaging. CVPR'21

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