LieMorph: Transformer-based Image Registration Using Flows on Lie Groups

MIC Institute of Mathematics and Image Computing

Johannes Bostelmann, Jan Lellmann

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1. Motivation

Image Registration Problem:

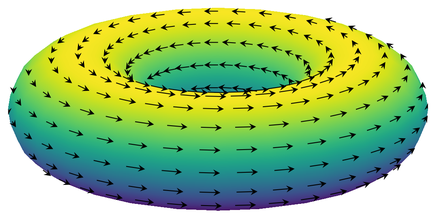
Given two images $I_1, I_2 : \mathbb{R}^n \to \mathbb{R}$, align corresponding structures by finding a "reasonable" ***deformation function*** $\phi : \mathbb{R}^n \to \mathbb{R}^n$ that maps between corresponding points, so that

$$I_1(x) \approx I_2(\phi(x)).$$

$\to$ ***ill-posed,*** heavily underdetermined.

Contribution:

- We augment the TransMorph architecture with an SVF-based representation using ***matrix-valued velocity fields***.
- By regularizing the velocity field on a chosen Lie group, this allows us to precisely control which parts of the deformation should not be penalized.



An invariant vectorfield on $SO(\mathbb{R}, 2) \times SO(\mathbb{R}, 2)$.

2. Formulation

Generalized flow equation:

Let $\Omega \subset \mathbb{R}^n$ be a domain, and consider a matrix Lie group G acting on $\mathbb{R}^n$. We extend the ***flow equation***:

$$\frac{\partial \phi}{\partial t}(x,t) = v(\phi(x))$$
$$\phi(x,0) = x$$

to a matrix-valued formulation:

$$\frac{\partial M}{\partial t}(x,t) = \nu(M(x,t)x)_{M(x,t)}$$
$$M(\cdot,0) = id,$$

where $M : \Omega \times [0,1] \to G$ is a matrix-valued field, and the velocity $\nu : \mathbb{R}^n \to \mathfrak{g}$ the control term of the ODE with values in $\mathfrak{g}$ the space of right invariant vector fields on G.

- The final deformation is then parametrized by $\phi(x) = M(x,1)x$.
- Right-invariant vector fields X on G are uniquely described by their value at the identity $X_{id} \in T_{id}G$.
  $\to$ They can be parametrized with a finite-dimensional vector.

Benefits:

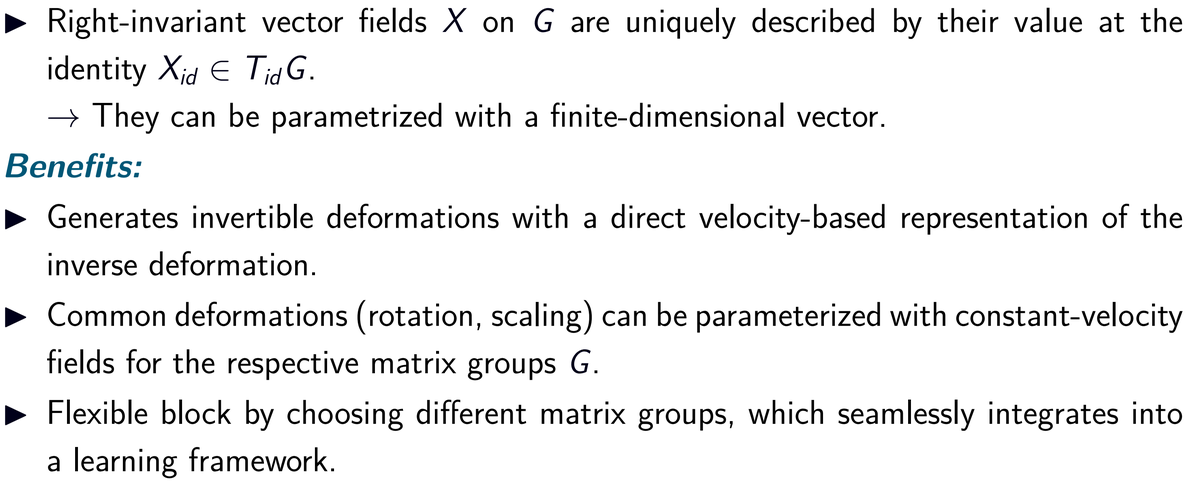
- Generates invertible deformations with a direct velocity-based representation of the inverse deformation.
- Common deformations (rotation, scaling) can be parameterized with constant-velocity fields for the respective matrix groups G.
- Flexible block by choosing different matrix groups, which seamlessly integrates into a learning framework.

3. Enhancing Smoothness - Velocity-Based Regularizers

Parameterizing deformations with matrix-valued velocity fields ν naturally yields a family of penalty terms:

$$R(\nu) := \int_\Omega \|\mathrm{D}\nu_{id}(x)\|_A^p dx.$$

- Deformations ϕ that are actions of elements in G can be parametrized with constant velocity fields and are, therefore, not penalized.
- Results on the task of denoising synthetic displacement fields:



input | result $SE(3)$ | ground truth

error, using $\bar{R}$ [2] | ours, R_G on SE(3) | ours, R_G on SIM(3)

4. Numerical Results

- TransMorph architecture as in [Chen22] with SWIN Transformers as encoders and U-net-like convolutional decoders.
- $\to$ generates velocity fields on matrix groups SE(3) and SIM(3) in our setting.
- $\to$ fully differentiable integration layer based on scaling and squaring for ϕ [Teed21].

Atlas-based Registration on IXI Dataset

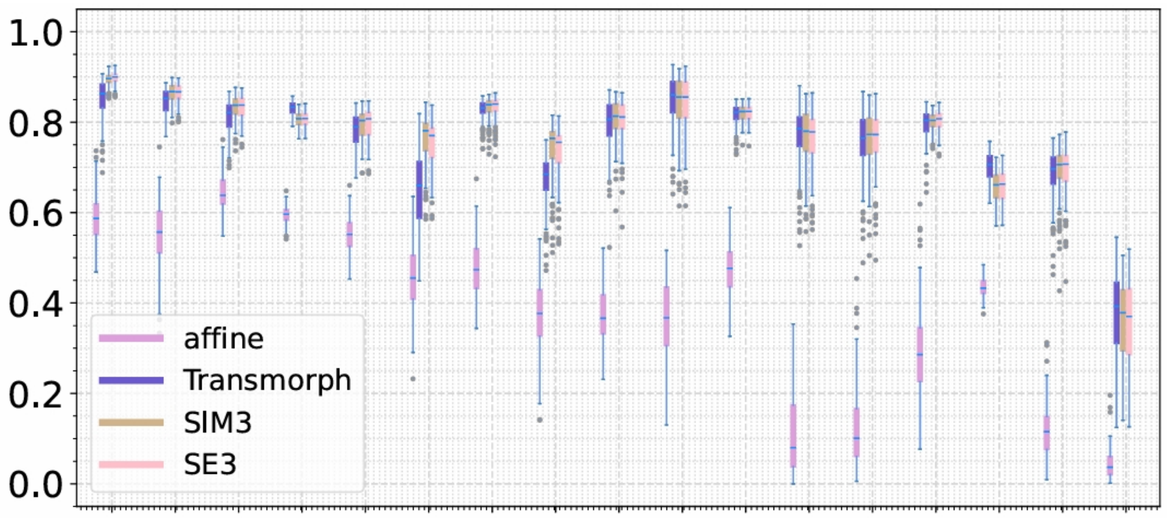
- Windowed NCC as image similarity term, velocity-based regularization on the corresponding tangential space.

$$\mathcal{L}(I_1, I_2, ; f_\theta(I_1, I_2)) := NCC(I_1, I_2 \circ S(f_\theta(I_1, I_2))) + \lambda R_G(f_\theta(I_1, I_2))$$

- Improvement on the mean Dice score with nearly no folding.

method	Dice score (nearest)	Dice score (trilinear)	$\det(J_\phi) \leq 0$
initial	0.406 ± 0.035	0.406 ± 0.035	-
TransMorph [2]	0.744 ± 0.029	0.751 ± 0.030	0.015 ± 0.003
LieMorph SE	$\underline{0.756 \pm 0.025}$	$\underline{0.763 \pm 0.025}$	**0**
LieMorph SIM	$\mathbf{0.757 \pm 0.024}$	$\mathbf{0.764 \pm 0.025}$	**0**

Unsupervised atlas-based registration on the IXI dataset.



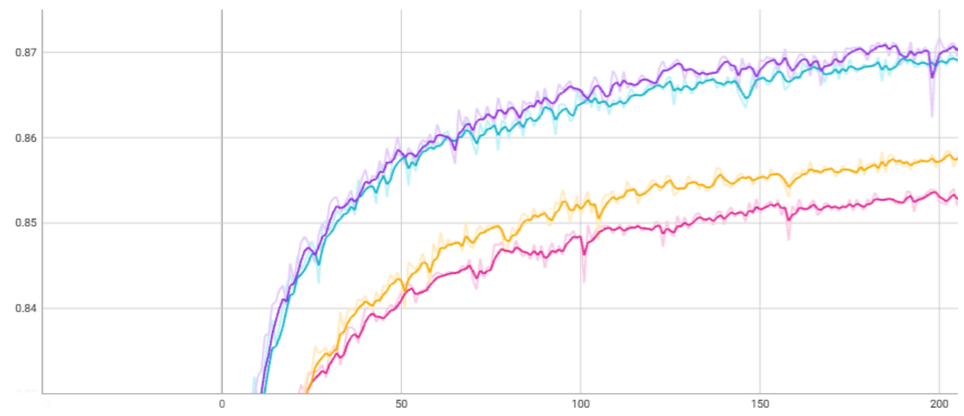
Dice Scores evaluated on 17 different anatomical regions.

Inter Patient Registration on the OASIS dataset

- Semi-supervised setting using segmentations on the training set.
- An additional segmentation term was added to the loss used for the IXI Dataset.
- We improve on the plain TransMorph model, and compare favorably even against a larger architecture.

method	Dice score (nearest)	Dice score (trilinear)	$\det(J_\phi) \leq 0$
initial	0.5718 ± 0.0517	0.5718 ± 0.0517	-
LieMorph SE	$\mathbf{0.8717 \pm 0.0129}$	$\mathbf{0.8789 \pm 0.0130}$	$\underline{1.548 \cdot 10^{-6}}$
LieMorph SIM	$\underline{0.8699 \pm 0.0141}$	$\underline{0.8772 \pm 0.0136}$	$\mathbf{1.050 \cdot 10^{-6}}$
TransMorph large	0.8623 ± 0.0144	0.8695 ± 0.0146	0.0080
TransMorph	0.8575 ± 0.0145	0.8645 ± 0.0141	0.0084

Weakly-supervised inter-patient registration on the OASIS dataset.



Validation score on the semi-supervised registration task on the OASIS dataset. LieMorph with the matrix groups SE(3) (cyan) and SIM(3) (violet) speeds up training compared to plain TransMorph (pink) and TransMorph-large (yellow).

- The implementation is available at `https://github.com/sennhoj321/Liemorph`.

Literature

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