Efficient Video Summarization Using Principal Person Appearance for Video-Based Person Re-Identification

BMVC 2017 Submission # 392
Supplementary Material

1 Derivation of the solution in Section 3.2, equation (9)

In this additional section, we describe details of the derivation for equation (9)∼line[238] from equation (7)∼line[225] in Section 3.2, **Update** $\mathbf{L}_2^{t+1}$.

Here, we represent again equation (7)∼line[225]:

$$\mathbf{L}_2^{t+1} = \arg\min_{\mathbf{L}_2} \ \lambda_L \|\mathbf{L}_2\mathbf{P}\|_F^2 + \frac{\mu_2}{2}\|\mathbf{L}_2 - \mathbf{L}_3^t - \frac{1}{\mu_2}\mathbf{Y}_2^t\|_F^2. \tag{1}$$

As we mentioned on the primary paper, the equation (1) is simplified by replacing $\mathbf{L}_3^t + \frac{1}{\mu_2}\mathbf{Y}_2^t = \mathbf{Q}$ as follows:

$$\mathbf{L}_2^{t+1} = \arg\min_{\mathbf{L}_2} \ \lambda_L \|\mathbf{L}_2\mathbf{P}\|_F^2 + \frac{\mu_2}{2}\|\mathbf{L}_2 - \mathbf{Q}\|_F^2. \tag{2}$$

Now, we want to solve (2) by a closed form based on the element-wise approach. The element of $\mathbf{L}_2\mathbf{P}$ can be expressed as $[\mathbf{L}_2\mathbf{P}]_{i,j} = \sum_{k=1}^{N_s} l_{i,k} \cdot p_{k,j}$. Note that the square of Frobenius norm for a matrix is expressed as $||\mathbf{A}||_F^2 = \sum_i \sum_j |[\mathbf{A}]_{i,j}|^2$. Then, for the element-wise minimization, the necessary condition for an arbitrary element $l_{i,j}$ can be represented as follows:

$$\begin{aligned} 0 &= \frac{\partial}{\partial l_{i,j}} \left[\lambda_L \sum_{r=1}^{d} \sum_{c=1}^{N_s-1} |\sum_{k=1}^{N_s} l_{r,k} \cdot p_{k,c}|^2 + \frac{\mu_2}{2} \sum_{r=1}^{d} \sum_{c=1}^{N_s} |l_{r,c} - q_{r,c}|^2 \right], \\ &= \frac{\partial}{\partial l_{i,j}} \left[\lambda_L \sum_{r=1}^{d} \sum_{c=1}^{N_s-1} |\sum_{k=1}^{N_s} l_{r,k} \cdot p_{k,c}|^2 \right] + \mu_2 (l_{i,j} - q_{i,j}), \\ &= \frac{\lambda_L}{\mu_2} \cdot \frac{\partial}{\partial l_{i,j}} \left[\sum_{c=1}^{N_s-1} |\sum_{k=1}^{N_s} l_{i,k} \cdot p_{k,c}|^2 \right] + l_{i,j} - q_{i,j}. \end{aligned} \tag{3}$$

The final equation of (3) becomes definitely same with equation (8)∼line[231]. The first term of the final equation in (3) is the element-wise derivative of $\|\mathbf{L}_2\mathbf{P}\|_F^2$ with respect to $\mathbf{L}_2$, and it can be reformulated as follows:

$$\frac{\partial}{\partial l_{i,j}}\left[\sum_{r=1}^{N_s-1}|\sum_{c=1}^{N_s} l_{i,c}\cdot p_{c,r}|^2\right] = \frac{\partial}{\partial l_{i,j}}\left[|\sum_{c=1}^{N_s} l_{i,c}\cdot p_{c,1}|^2+\cdots+|\sum_{c=1}^{N_s} l_{i,c}\cdot p_{c,N_s-1}|^2\right]. \quad (4)$$

Recall the condition of elements of **P** in equation (2)∼line[167] to make it simpler, before solving differential equation for (4). Here we show again the condition for an arbitrary element $p_{i,j}$ in **P** as follows:

$$p_{i,j} = \begin{cases} -1 & if\ i=j, \\ 1 & if\ i=j+1, \quad \forall i \in \{1,...,N_s\},\ \forall, j\in\{1,...,N_s-1\}. \\ 0 & \text{otherwise}, \end{cases} \quad (5)$$

Then the equation (4) can be represented again as follows:

$$\begin{aligned} &\frac{\partial}{\partial l_{i,j}}\Big[|l_{i,1}\cdot p_{1,1}+l_{i,2}\cdot p_{2,1}|^2+|l_{i,2}\cdot p_{2,2}+l_{i,3}\cdot p_{3,2}|^2+|l_{i,3}\cdot p_{3,3}+l_{i,4}\cdot p_{4,3}|^2+\cdots+ \\ &\qquad |l_{i,N_s-2}\cdot p_{N_s-2,N_s-2}+l_{i,N_s-1}\cdot p_{N_s-1,N_s-2}|^2+|l_{i,N_s-1}\cdot p_{N_s-1,N_s-1}+l_{i,N_s}\cdot p_{N_s,N_s-1}|^2\Big] \\ &= \frac{\partial}{\partial l_{i,j}}\Big[|l_{i,2}-l_{i,1}|^2+|l_{i,3}-l_{i,2}|^2+|l_{i,4}-l_{i,3}|^2+\cdots+|l_{i,N_s-1}-l_{i,N_s-2}|^2+|l_{i,N_s}-l_{i,N_s-1}|^2\Big]. \end{aligned} \quad (6)$$

From equation (6), we can find the completed form of the element-wise derivative of $\|\mathbf{L}_2\mathbf{P}\|$ corresponding to the value of j:

$$\frac{\partial}{\partial l_{i,j}}\left(\|\mathbf{L}_2\mathbf{P}\|_F^2\right) = \begin{cases} 2(l_{i,j}-l_{i,j+1}), & if\ j=1, \\ 2(2l_{i,j}-l_{i,j-1}-l_{i,j+1}), & if\ 1<j<N_s, \\ 2(l_{i,j}-l_{i,j-1}), & if\ j=N_s. \end{cases} \quad (7)$$

Finally , by substituting (7) into the first term of (3), the closed form on a coordinate decent manner can be directly derived as the solution of equation (9)∼line[238]:

$$l_{i,j}^{(t+1)} = \begin{cases} \dfrac{2\tau\cdot l_{i,j+1}^{(t)}+q_{i,j}}{2\tau+1}, & if\ j=1, \\ \dfrac{2\tau\cdot(l_{i,j-1}^{(t)}+l_{i,j+1}^{(t)})+q_{i,j}}{4\tau+1}, & if\ 1<j<N_s, \\ \dfrac{2\tau\cdot l_{i,j-1}^{(t)}+q_{i,j}}{2\tau+1}, & if\ j=N_s. \end{cases} \quad (8)$$