Convolutional Sparse Coding-based Image Decomposition

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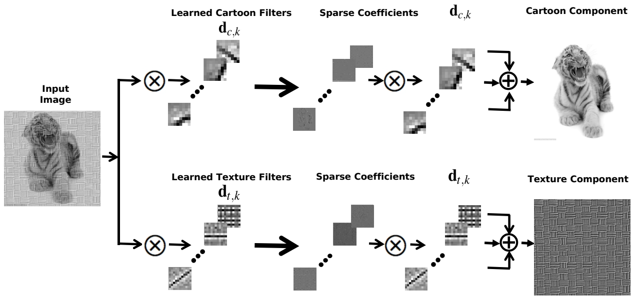

Figure 1: An overview of the proposed CSCD method for image decomposition.

Algorithm 1: The CSCD Algorithm for Image Decomposition.

Input: $\{\mathbf{d}_{c,k}\}_{k=1}^{K_c}$, $\{\mathbf{d}_{t,k}\}_{k=1}^{K_t}$, $\mathbf{y}$, λ_c, λ_t, L
Initialization
for $i = 1 : L$
  Obtain $\hat{\mathbf{x}}_{c,k}$ by solving (2) when fixing $\hat{\mathbf{x}}_{t,k}$.
  Obtain $\mathbf{y}_c$ by applying the Haar threshold.
  Use $\mathbf{y}_c$ to replace $\sum_{k=1}^{K_c} \mathbf{d}_{c,k} * \hat{\mathbf{x}}_{c,k}$ in (2).
  Obtain $\hat{\mathbf{x}}_{t,k}$ by solving (2) when fixing $\hat{\mathbf{x}}_{c,k}$.
end for
$\hat{\mathbf{y}}_c$=$\mathbf{y}_c$;
$\hat{\mathbf{y}}_t = \sum_{k=1}^{K_{l,t}} \mathbf{d}_{t,k} * \hat{\mathbf{x}}_{t,k}$,
Output: $\hat{\mathbf{y}}_c$, $\hat{\mathbf{y}}_t$

Contribution: Even though dictionary-based image decomposition methods have been proposed in the literature for the purpose of decomposing an image into cartoon and texture parts, most of the existing approaches are patch-based and features learned with these methods often contain shifted versions of the same features [1]. To overcome this issue, we propose a novel sparsity-based method for cartoon and texture decomposition based on Convolutional Sparse Coding (CSC). Our method first learns a set of generic filters that can sparsely represent cartoon and texture type images. Then using these learned filters, we propose a sparsity-based optimization framework to decompose a given image into cartoon and texture components. Figure 1 gives an overview of the proposed images separation method. By working directly on the whole image, the proposed image decomposition algorithm does not need to divide the image into overlapping patches for leaning local dictionaries. Extensive experiments indicate the proposed method performs favorably compared to state-of-the-art image decomposition methods.

Proposed Method: Following the mixture model

$$\mathbf{y} = \mathbf{y}_c + \mathbf{y}_t, \tag{1}$$

given an input image $\mathbf{y}$, our goal is to separate it into a cartoon part $\mathbf{y}_c$ and a texture part $\mathbf{y}_t$. Assume that we have already learned the convolutional filters corresponding to $\mathbf{y}_c$ and $\mathbf{y}_t$ by solving the CSC problem for the cartoon and the texture components separately. That is, we have learned $\{\mathbf{d}_{c,k}\}_{k=1}^{K_c}$ and $\{\mathbf{d}_{t,k}\}_{k=1}^{K_t}$ such that $\mathbf{y}_c = \sum_{k=1}^{K_c} \mathbf{d}_{c,k} * \mathbf{x}_{c,k}$ and $\mathbf{y}_t = \sum_{k=1}^{K_t} \mathbf{d}_{t,k} * \mathbf{x}_{t,k}$, where $\mathbf{x}_{c,k}$ and $\mathbf{x}_{t,k}$ are the sparse coefficients that approximate $\mathbf{y}_c$ and $\mathbf{y}_t$ when convolved with the filters $\mathbf{d}_{c,k}$ and $\mathbf{d}_{t,k}$, respectively. We propose to estimate $\mathbf{y}_c$ and $\mathbf{y}_t$ via $\mathbf{x}_{c,k}$ and $\mathbf{x}_{t,k}$ by solving the following CSC-based optimization problem

$$\begin{aligned}\hat{\mathbf{x}}_{c,k}, \hat{\mathbf{x}}_{t,k} = \arg\min_{\mathbf{x}_{c,k}, \mathbf{x}_{t,k}} \frac{1}{2} \left\| \mathbf{y} - \sum_{k=1}^{K_c} \mathbf{d}_{c,k} * \mathbf{x}_{c,k} - \sum_{k=1}^{K_t} \mathbf{d}_{t,k} * \mathbf{x}_{t,k} \right\|_2^2 \\ + \lambda_c \sum_{k=1}^{K_c} \left\| \mathbf{x}_{c,k} \right\|_1 + \lambda_t \sum_{k=1}^{K_t} \left\| \mathbf{x}_{t,k} \right\|_1 + \beta TV \left(\sum_{k=1}^{K_c} \mathbf{d}_{c,k} * \mathbf{x}_{c,k} \right).\end{aligned} \tag{2}$$

Once, $\mathbf{x}_{c,k}$ and $\mathbf{x}_{t,k}$ are estimated, the two components can be obtained by $\hat{\mathbf{y}}_c = \sum_{k=1}^{K_c} \mathbf{d}_{c,k} * \hat{\mathbf{x}}_{c,k}$ and $\hat{\mathbf{y}}_t = \sum_{k=1}^{K_t} \mathbf{d}_{t,k} * \hat{\mathbf{x}}_{t,k}$. We propose an iterative method for solving the above optimization problem. The overall CSCD algorithm is summarized in the Algorithm 1, where λ_c and λ_t are the changing parameters corresponding to the two parts, L is the total iteration number.

Experiments: We present the results of our proposed CSCD algorithm for image decomposition and compare them with the sparsity-based MCA method [4], adaptive dictionary learning-based MCA (A-MCA) method [3], and a recent stat-of-the-art Block Nuclear Norm (BNN) based image decomposition method [2]. In these experiments, we use the Peak Signal to Noise Ratio (PSNR) to measure the performance of the routines tested (See Figure 2). Various experiments show the significance of our CSC-based image separation method over the other methods.

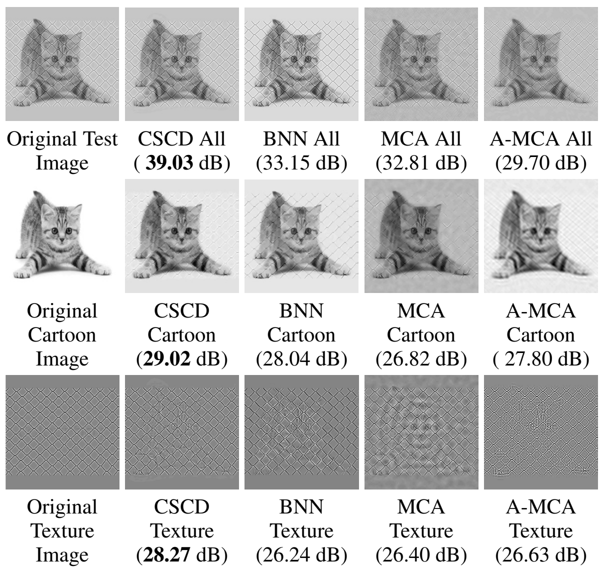

Figure 2: Image decomposition results on the *Cat+Cage* image. We compare the performance of our method with that of BNN, MCA and A-MCA.

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