Fast Eigen Matching

Accelerating Matching and Learning of Eigenspace method

Yusuke Sekikawa
ysekikawa@d-itlab.co.jp
Koichiro Suzuki,
ksuzuki,@d-itlab.co.jp
Kosuke Hara
khara@d-itlab.co.jp
Yuichi Yoshida
yyoshida@d-itlab.co.jp
Ikuro Sato
isato@d-itlab.co.jp

DENSO IT Laboratory, Inc. Japan.

Abstract

We propose a method for accelerating the matching and learning processes of the eigenspace method for rotation invariant template matching (RITM). To achieve efficient matching using *eigenimages*, it is necessary to learn 2D-Fourier transform of *eigenimages* before matching. Little attentions has been paid to speeding up the learning process, which is important for applications in which a template changes frame by frame. We propose two key ideas: First, to further speedup the matching process using FFT, we decompose rotated templates to orthogonal *fast-eigenimages* using Fourier basis by utilizing the circularity of rotated templates. Second, to speedup the learning process, we compute 2D-Fourier transform of the *fast-eigenimages* in polar coordinates using Hankel transform[11]. Proposed learning method is equivalent to but considerably faster than that existing method, i.e., rotated template generation, SVD and 2D-FFTs in Cartesian coordinates. Experiments revealed that the learning, matching and the total processes becomes respectively 120, 3, and 36 times faster while keeping comparable detection rate compared to existing method utilizing SVD in Cartesian coordinates. The algorithm was successfully applied to global localization of mobile robot where online learning is required.

1 Introduction

Correlation-based template matching is one of the basic techniques used in computer vision. Among them, rotation invariant template matching (RITM), which locates a known template in a query irrespective of the template's translation and orientation, has been widely put to use in many industrial applications. A naive implementation of RITM requires intensive computation since one needs to correlate query f with N rotated templates $\boldsymbol{T}$ (Fig.1 left). Eigenspace methods takes advantage of the fact that a set of correlated images $\boldsymbol{T}$ can be

Pages 16.1-16.12
DOI: https://dx.doi.org/10.5244/C.30.16

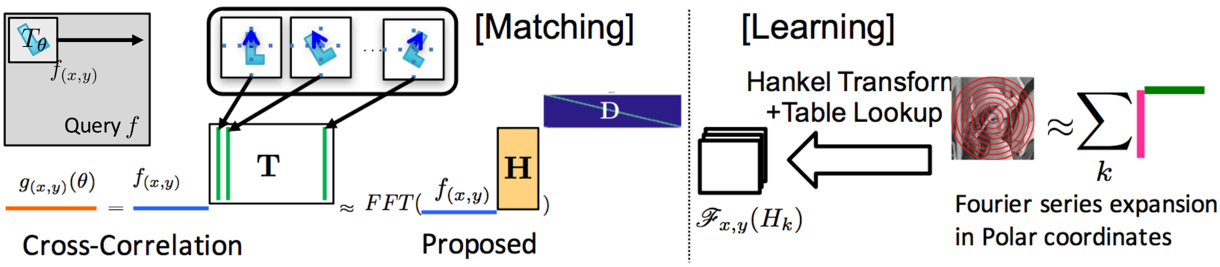

Fig. 1: Overview of the proposed method

approximately represented by a small set of *eigenimages*. Once *eigenimages* and it's 2D-Fourier transform are computed, RITM can be performed very efficiently using these 2D-Fourier transform*ed eigenimages*[6].

There have been efforts to accelerate the matching process; however, little attention has been paid to the speeding up of the learning process to build 2D-Fourier transform of *eigenimages*. It is also important to speedup the learning process, especially for applications such as global robot localization, where a template changes frame by frame and efficient online learning is required. The existing eigenspace methods are not feasible for problem settings of this kind, because it requires a lot of time for generation of rotated templates, SVD and 2D-FFT.

1.1 Overview of the proposed method

We propose *Fast Eigen Matching* which accelerate matching and learning process of the eigenspace method for RITM. Our contributions are as follows:

1. By focusing on the circularity of in-plane rotation and concentration of power spectrums to low frequency, we compute *fast-eigenimages* $\boldsymbol{H}$ by expanding a templates using Fourier basis, which leads to the use of FFT in a matching process (Fig.1 middle). Matching speed is further improved by a non-negligible amount even though RITM has been rigorously studied in terms of matching acceleration.

2. By utilizing the fact that Fourier expansion in polar coordinates is efficiently transformed to frequency domain using Hankel transform[11], our method computes 2D-Fourier transform of each *fast-eigenimages* $\tilde{\boldsymbol{H}}$ in polar coordinate (Fig.1 right). This computation is equivalent to existing learning method, i.e., time-consuming rotated template generation, numerical SVD and 2D-FFTs in Cartesian Coordinates, but substantially boosts the learning process by avoiding these time-consuming computation.

Our experiments revealed that the learning, matching, and total processes respectively becomes 120, 3, and 36 times faster, while keeping comparable matching performance compared to previous method. As a representative example, we show an application to global localization with a Particle Filter[14].

1.2 Related work

Variants of Cross-Correlation (CC), such as Zero mean Normalized CC (ZNCC)[7], Phase only Correlation (POC)[13], and Gradient Correlation (GC)[16] are widely used because of their robustness against noise, especially for textureless images. However, using these correlation-based methods for RITM are inefficient, as correlation for every rotated template should be computed. Many efforts have been paid to the accelerate matching process.

There have been attempts to improve efficiency by evaluating only important pixels. For example, Borgefors used edge pixels[2], Liang used discriminative pixels based on co-occurrence[8], and Ouyang used the projection to the Haar basis and skip evaluating pixels that had no possibilities to be the maximum[10]. These methods are intended to speedup the correlation with a single template rather than the whole template matrix. They are thus still inefficient for RITM.

Rotation Invariant Phase Only Correlation (RIPOC)[15, 4]utilize the fact that the power spectrums are invariant to translation and decouple the estimation of rotation and translation, thus each of them requires only one correlation. Thought RIPOC is highly efficient, it is prone to fail when there exists a power spectrum in the background of a query similar to the template, as it discards phase information in the process of estimating rotation.

In the *Eigen Template* method[6], most relevant to ours, they applied the idea of eigenspace methods to RITM and developed an efficient matching algorithm using pre-computed 2D-Fourier transformed *eigenimages*. However to the best of our knowledge there is no methods that focuses on the efficient computation of a 2D-Fourier transformed *eigenimages* in the literature of the eigenspace method.

2 Fast Eigen Matching

2.1 Problem Formulation

The similarity of RITM for query $f \in \mathbb{R}^{L1\times L2}$ and a template rotated by θ, $T_\theta \in \mathbb{R}^{K1\times K2}$, is defined using CC [1]such that

$$g(x,y,\theta) = \int\int_R f(x+x',y+y')T_\theta(x',y')dx'dy', \tag{1}$$

where, (x,y,θ) are translational and rotational parameters and R is the region where a template and the query overlaps. Eq.(1) relates how close the template matches the query for each parameter. For fixed translation, similarity is computed with the dot product of cropped query $f_{(x,y)} \in \mathbb{R}^{K1\times K2}$ (left top corner is (x,y)) and a template rotated by θ, namely

$$g(x,y,\theta) = \langle f_{(x,y)}, T_\theta \rangle. \tag{2}$$

It is obvious that to get finer rotational resolutions N, evaluating the above similarity takes a long time. Eigenspace methods are based on the decomposition of a template matrix $\boldsymbol{T} := [T_0, T_{(2\pi)/N,...,}T_{2\pi-(2\pi)/N}] \in \mathbb{R}^{K1\times K2\times N}$ with the product of two orthogonal matrices, *eigenimages* $\{H_1,...,H_M\} \in R^{K1\times K2}$ and *eigenfunctions* $\boldsymbol{\Psi} \in \mathbb{R}^{M\times N}$, i.e. $T_\theta \approx \sum_m H_m \psi_m(\theta)$, which yield approximation of Eq.(2) that can be expressed as

$$g(x,y,\theta) \approx \sum_m^M \langle f_{(x,y)}, H_m \rangle \psi_m(\theta) = \sum_m^M r_{m(x,y)} \psi_m(\theta) = \boldsymbol{r}_{(x,y)} \boldsymbol{\Psi}, \tag{3}$$

[1]We will concentrate our discussion to approximate CC of RITM for simplicity. It is straight forward to get approximations of NCC/ZNCC/POC/GC using the results of CC.

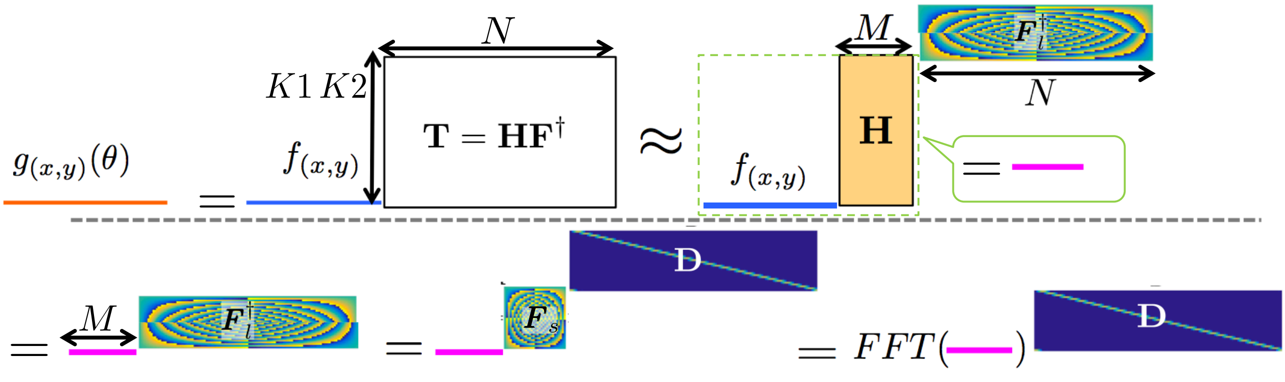

Fig. 2: Fast RITM using FFT

where M ($M < N$) is the number of *eigenimages* used to approximate Eq.(1), $\psi_m \in \mathbb{R}^N$ is a row vector of $\boldsymbol{\Psi}$, and $\boldsymbol{r}_{(x,y)} := [\langle f_{(x,y)}, H_1\rangle, \ldots, \langle f_{(x,y)}, H_M\rangle] \in \mathbb{R}^M$ is response. We observed that most of the computations in the matching process are consumed in computing similarity from response, i.e. computing product of response $\boldsymbol{r}_{(x,y)}$ and *eigenfunctions* $\boldsymbol{\Psi}$. In 2.2, we will show a novel algorithm that speeds up the matching process by making this computation efficient.

By convolving each *eigenimages* H_k with query f, we can get the similarity for all translations, $\boldsymbol{g}(\theta) \in \mathbb{R}^{L1 \times L2}$. These convolutions are usually done in the Fourier domain using 2D-Fourier transform of H_k and f for efficiency, [2]namely

$$\boldsymbol{g}(\theta) \approx \sum_m^M \{\mathscr{F}_{x,y}^{-1}(\mathscr{F}_{x,y}(f) \circ \tilde{H}_m)\} \cdot \psi_m(\theta), \tag{4}$$

where $\tilde{H}_m$ is 2D-Fourier transformed *eigenimage* in Cartesian coordinates which is zero padded to size of query, $L1 \times L2$. Thus it is necessary to speedup the computation of the 2D-Fourier transformed *eigenimages* $\tilde{H}_m$ when a template changes frame by frame. In 2.3, we will show a novel algorithm that calculates $\tilde{H}_m$ much more efficiently than computing it in Cartesian coordinates.

2.2 Fast Rotation Invariant Template Matching Using FFT

We approximate $\boldsymbol{T}$ with the product of the low frequency part of the Discrete Fourier Transform (DFT) matrix and orthogonal vectors which we call *fast-eigenimages*. This factorization enables the use of FFT to speedup the computation of Eq.(3) in matching process.

2.2.1 Approximation of similarity using low frequency part of DFT matrix

We will show that the template matrix $\boldsymbol{T}$ can be approximated using low the frequency part of the DFT matrix. It is well known that the covariance matrix of $\boldsymbol{T}$ (comprising from in-plane rotation of an image) is a circulant[3] and that a closed form solution for eigenvalue decomposition of a circulant matrix is computed using a DFT matrix $\boldsymbol{F}$ [3] , i.e. $\boldsymbol{T}^\top \boldsymbol{T} = \boldsymbol{F}\boldsymbol{\Lambda}\boldsymbol{F}^{\dagger}$[4]

[2] $\circ$: Hadamard product. $*$: convolution. $\mathscr{F}_{x,y}, \mathscr{F}_{x,y}^{-1}$: Forward and Inverse 2D-Fourier transform.
[3] Precisely, this discussion is true only for infinite-resolution images.
[4] † : conjugate transpose

where $\mathbf{\Lambda} \in \mathbb{R}^{N \times N}$ is a diagonal eigenvalue matrix. To summarize, $\boldsymbol{T}$ can be decomposed as

$$\boldsymbol{T} = \boldsymbol{U}\mathbf{\Lambda}^{1/2}\boldsymbol{F}^{\dagger} = \boldsymbol{H}\boldsymbol{F}^{\dagger},\ \boldsymbol{U}^{\top}\boldsymbol{U} = \boldsymbol{I}, \tag{5}$$

where $\boldsymbol{H} = [H_1, ..., H_N]$, we call this *fast-eigenimages*. We observed that the intensity in $\boldsymbol{T}$ changes slowly in the rotational direction for many natural images. This observation leads to the following approximation using the M-lowest frequency parts of the DFT matrix, $\boldsymbol{F}_l$

$$\boldsymbol{T} \approx [H_1, ..., H_M]\boldsymbol{F}_l^{\dagger},\ \boldsymbol{F}_l \in \mathbb{R}^{N \times M}. \tag{6}$$

Since H_m are perpendicular to each other, $\mathbf{\Psi}$ in Eq.(1) can be replaced with $\boldsymbol{F}_l$, i.e.

$$\boldsymbol{g}(x,y) \approx \boldsymbol{r}_{(x,y)}^{\top}\boldsymbol{F}_l^{\dagger},\ \boldsymbol{g}(x,y) \in \mathbb{R}^{N}, \tag{7}$$

it gives a similarities of each discretized rotations (Fig.2 top).

2.2.2 *Fast* calculation of similarity using M-dimensional FFT

By decomposing $\boldsymbol{F}_l$ in Eq.(7) using the small DFT matrix $\boldsymbol{F}_s \in \mathbb{R}^{M \times M}$, i.e. $\boldsymbol{F}_l = \boldsymbol{F}_s\boldsymbol{D}$, we get the approximation of Eq.(1) using M-dimensional FFT (Fig.2 bottom):

$$\boldsymbol{g}(x,y) \approx \mathrm{FFT}(\boldsymbol{r}_{(x,y)}^{\top})\boldsymbol{D},\ \mathbf{D} \in \mathbb{R}^{M \times N}. \tag{8}$$

Since most of the elements in $\mathbf{D}$ are approximately zero, the computational complexity of Eq.(8) is $\mathcal{O}(M\log_2 M)$ instead of MN in the existing method.

2.2.3 Pixel culling using upper bound of similarity

For some applications, users are interested in finding sets of parameters in which the similarity exceeds the predefined threshold. Since the norms of row vectors of $\boldsymbol{F}_l$ are the same, the L^1-norm of response vector, $\sum_m^M |r_{(x,y)}(m)|$ gives the upper bound of similarity. We can safely skip the calculation of similarity for which the upper bound is smaller than the threshold.

2.3 Efficient computation of 2D-Fourier transform of *fast-eigenimage*

In cases where templates changes frame by frame, the learning process should also be computed efficiently. We show a way to compute 2D-Fourier transform of *fast-eigenimages* using Hankel transform in polar coordinates, which avoids time-consuming rotated template generation, decomposition and 2D-FFT. This makes it possible to obtain extremely fast and memory-efficient learning algorithm while keeping the good property of FFT for fast matching. An overview of the proposed algorithm compared to the conventional one is shown in Fig.3.

2.3.1 Similarity in polar domain

The similarity of θ rotated template and query in Cartesian coordinates are given using polar transformed non-rotated image T_0 as,

$$g(x,y,\theta) = \int\int f_{(x,y)}(r,\tau)T_0(r,\tau-\theta)\rho d\rho d\tau, \tag{9}$$

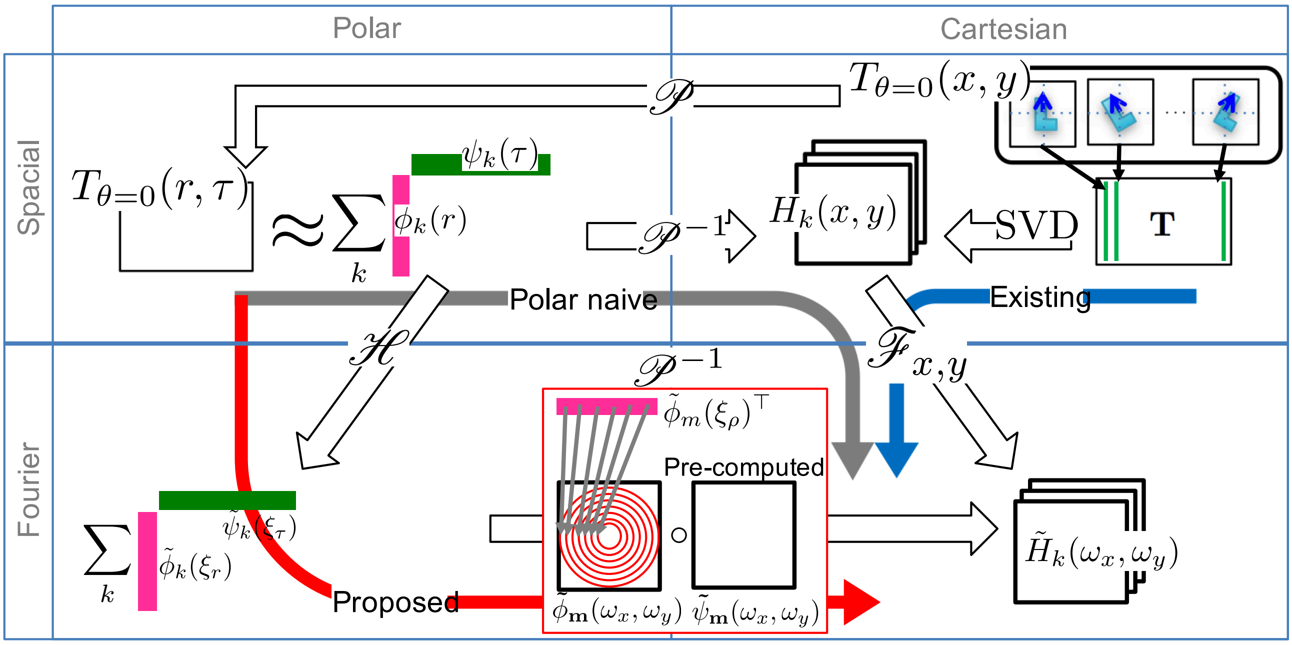

Fig. 3: Overview of proposed learning algorithm.

, where, ρ is radius and τ is angle from center of image. Eq.(9) is based on the relation between Cartesian and polar coordinates, $T_0(\rho,\tau-\theta) = T_\theta(x,y)$ where $x := \rho \cdot cos(\tau)$, $y := \rho \cdot sin(\tau)$. Using Fourier expansion, we can expand T in Eq.(9) to

$$T_0(\rho,\tau-\theta) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} \phi_m(\rho) exp(i(\tau-\theta)m), \tag{10}$$

where $\phi_m(r)$ is a Fourier coefficient and N is the number of discretization for rotational direction. This expansion can be computed efficiently directly from a Cartesian image using Fast Polar Fourier Transform[1].

2.3.2 Approximation with low frequency part of Fourier basis

Discussions similar to that in (2.2.1) leads to the approximation of Eq.(10) using the M-lowest frequency parts of the Fourier basis. Substituting this result into Eq.(9) leads to the same approximation as Eq.(7), where m-th element of response vector $\boldsymbol{r}_{(x,y)}$ is give as follows,

$$r_{m(x,y)} = \int\int f_{(x,y)}(\rho,\tau) H_m(\rho,\tau) d\rho d\tau,\ H_m(\rho,\tau) := \rho\,\phi_m(\rho)\,\psi_m(\tau), \tag{11}$$

where $H_m(\rho,\tau)$ is the *fast-eigenimage* in polar coordinates, and ψ_m is Fourier basis. This approximation corresponds to Eq.(7) in Cartesian coordinates.

2.3.3 2D-Fourier transform of *fast-eigenimage* without 2D-FFT

Converting $H_m(\rho,\tau)$ in Eq.(11) to Cartesian coordinates using polar to Cartesian transform, $\mathscr{P}^{-1}$, yields *fast-eigenimage* similar to the one discussed in (2.2). However, transforming

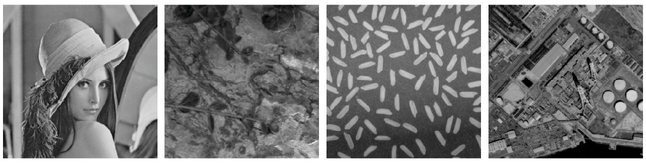

Fig. 4: Query images used

Table 1: Computation time[sec]

	Learning	Matching	Total
ZNCC	5.603	0.675	6.278
NA-GC	4.933	0.442	5.375
CE-GC	2.630	0.172	2.802
Proposed CF-GC	1.653	**0.055**	1.708
Proposed PF-GC	**0.022**	**0.055**	**0.077**

each $H_m(\rho,\tau)$ from polar to Cartesian coordinates and computing its 2D-FFT are computationally intensive (Fig.3 Polar naive). Instead, we compute 2D-Fourier transform in polar coordinates using Hankel transform $\mathscr{H}$, such that

$$\tilde{H}_m(\xi_\rho,\xi_\tau) = \mathscr{H}_m(\phi_m(\rho))\,\tilde{\psi}_m(\xi_\tau) = \tilde{\phi}_m(\xi_\rho)\,\tilde{\psi}_m(\xi_\tau), \tag{12}$$

where $\mathscr{H}_m(\phi_m(\rho)) = \int \rho\phi_m(\rho)J_m(\xi_\rho\rho)d\rho$ and J_m is the Bessel function of the first kind[11]. This transformation corresponds to 2D-FFT in Cartesian coordinates but is much faster. This efficient Fourier transformation using Hankel transform is on the fact that $f_{(x,y)}(\rho,\tau)$ is cyclic in rotation direction[11].

The result of each direct product in Eq.(12) (Fig.3 left bottom) can be transformed to Cartesian coordinates, but this transformations are computationally intensive. Instead, we compute polar to Cartesian transformation separately for $\tilde{\phi}_m(\xi_\rho)$ and $\tilde{\psi}_m(\xi_\rho)$. Result of Hankel transform, $\tilde{\phi}_m(\xi_\rho)$, which are functions that depend only on ρ, are efficiently converted to Cartesian-Fourier domain $\tilde{\phi}_m(\omega_x,\omega_y)$ using lookup table. Clearly, $\tilde{\psi}_m(\xi_\tau)$ are independent from template, thus we can pre-compute $\tilde{\psi}_m(\omega_x,\omega_y)$. Finally, 2D-Fourier transform of *fast-eigenimages* $\tilde{H}_m(\omega_x,\omega_y)$ are computed (Fig.3 Proposed) as,

$$\tilde{H}_m(\omega_x,\omega_y) = \tilde{\phi}_m(\omega_x,\omega_y)\,\tilde{\psi}_m(\omega_x,\omega_y). \tag{13}$$

If the templates are available in polar coordinates, $\tilde{H}_m(\omega_x,\omega_y)$ can be computed more efficiently. We will demonstrate this in chapter 4.

3 Evaluation

To demonstrate that our method achieves superior efficiency while keeping comparable matching performance with the existing method, we evaluated the computational time and detection rate using four representative images of size 512×512[pix], shown in Fig.4. For each image, we randomly rotated and cropped to generate a circular template of radius 60[pix], and added Gaussian noise of variance up to 0.1 to the query. Similarities between the query and the templates were calculated with each algorithm and parameter with peak are used for evaluation. We evaluated the matching performance with a "success" rate, defining "success" as achieving estimated translational and rotational errors of less than 2[pix] and 2[deg], respectively. The results are the average of 200 evaluations for each noise level. We have compared our method with ZNCC[7], GC[16] (NA-GC) and Edge Eigen[6] (CE-GC), Edge Eigen is an improved version of Eigen Template method and it is an approximation of GC. We also approximated GC, using decomposition in Cartesian coordinates (CF-GC

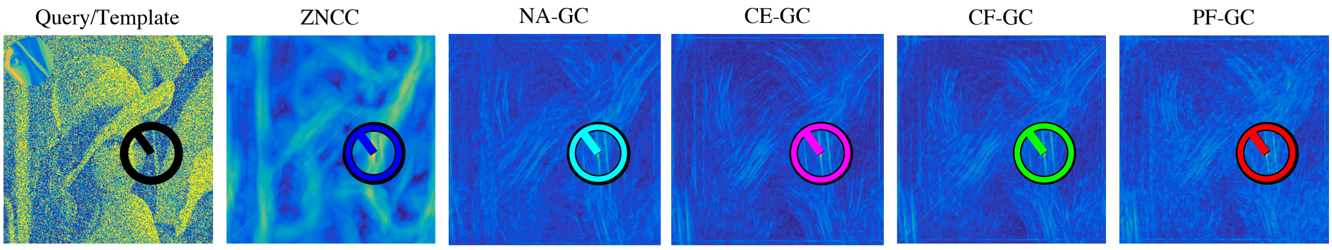

Fig. 5: Example detection result for Lena with Gaussian noise of variance 0.1. Left-most image shows the query and the template(Left top) and the others shows similarities, $max_\theta g(x,y,\theta)$, of each algorithm. CF-GC and PF-GC are proposed method.

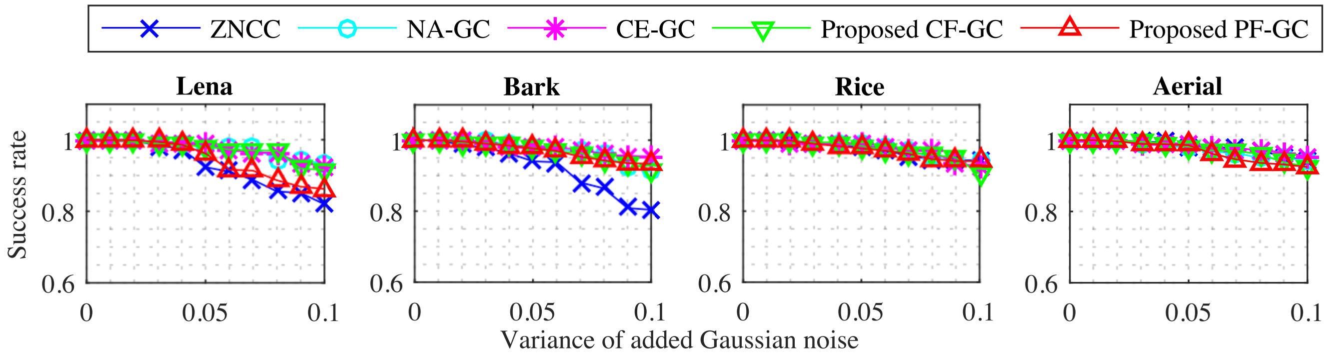

Fig. 6: Detection rate of each algorithm for different data set.

Eq.(7)) and polar coordinates (PF-GC Eq.(11)). The similarity of GC g_{GC} is defined as

$$g_{GC}(x,y,\theta) = \int\int_R f_x(x+x',y+y')T_{x,\theta}(x,y)dx'dy' + \int\int_R f_y(x+x',y+y')T_{y,\theta}(x,y)dx'dy', \tag{14}$$

where, $f_x, f_y, T_{x,\theta}$ and $,T_{y,\theta}$ are derivatives of f and T_θ. For proposed methods, CF-GC and PF-GC, each derivative was integrated to single complex image as $\bar{f} = f_x + if_y$. This is because treating two derivatives as single complex image is more efficient than computing separately for f_x and f_y, since our *eigenfunctions* (DFT) are complex. All algorithms were implemented in Matlab 2016a for Mac and evaluated using iMac Late 2012 with Intel Core i7 3.4GHz CPU and Nvidia Quadro K6000 GPU. Correlation was done in Fourier domain for all methods. We used $N = 512$ for ZNCC and N-GC, and for E-GC and F-GC we extract $M = 64$ *(fast-)eigenimages* from $N = 512$ discretized rotation. Bilinear interpolation are used for image rotation (for ZNCC, N-GC, CE-GC and CF-GC) and Cartesian to polar conversion (for PF-GC). Computation time of each method are shown in Table 1. Learning for ZNCC and NA-GC include image rotation and 2D-FFT of it. Examples of similarly (maxed for θ direction) are shown in Fig.5, and detection rate are shown in Fig.6

As demonstrated in Table 1, Fig.5, and 6, our polar decomposition methods (PF-GC) are much faster than the existing methods , and at the same time achieve comparable detection rate with them. Comparison of CE-GC (numerical SVD in Cartesian coordinates) and CF-GC (DFT in Cartesian coordinates) shows almost the same detection rate, it suggests that decomposing template matrix using DFT instead of numerical SVD boosts RITM without affecting the detection performance. Comparison of CF-GC (DFT in Cartesian coordinates) and PF-GC (DFT in Polar coordinates) shows slight deterioration at the expense of significant speedup.

Our hypothesis for this deterioration is that the error induce by Cartesian to polar trans-

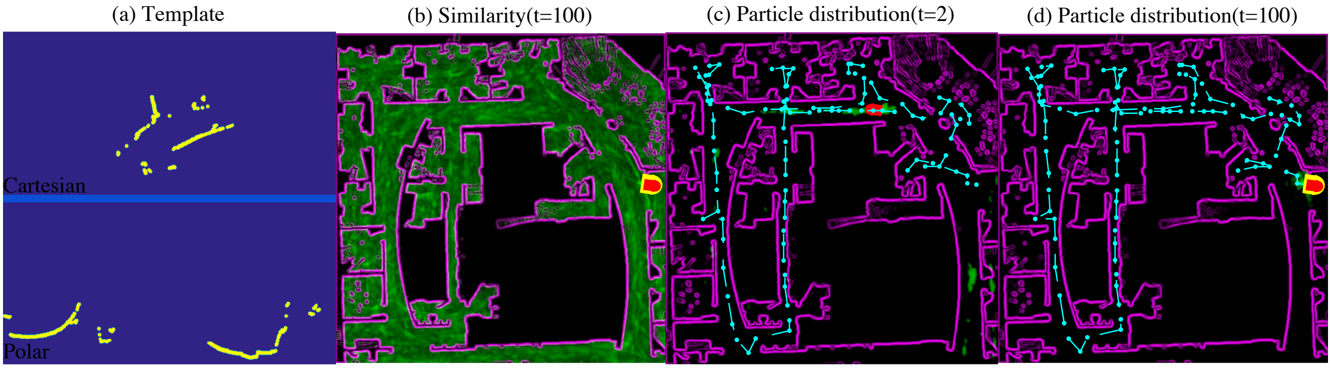

Fig. 7: Snapshot of localization process. (a)Template (b)Computed similarity (green) is super-imposed on query (Magenta). We can observe many local peaks. (c)(d) Particle distribution (green) is super-imposed on query for $t = 2$ and $t = 100$. Estimated robot pose are shown in yellow arrow, and the true pose and the path are shown in red arrow and cyan line and t in the figure indicate frame number. See also the supplemental video.

formation reduce the quality of *fast-eigenimage*s. Actually, as seen in next chapter, when data is available in polar coordinates we observed no noticeable difference for each of the decomposition method.

4 Application to global localization

Our algorithm is most useful when a template changes frame by frame and similarity for all parameters are necessary. We applied our method to the global localization task for a mobile robot equipped with a Laser Range Finder (LRF). Typically, features in a template are not unique in global map and thus it is difficult to localize uniquely through a single observation. To address this issue, we build a Particle Filter based Monte Carlo localization framework similar to [14]. The main difference with [14] is that likelihood of observed range data z, namely $p(z|\boldsymbol{x})$, were calculated for all discretized parameters in our implementation, instead of only where particle exist. Therefore our framework is easily recoverable from robot kidnapping, or global localization failure, by assuming that the robot might get kidnapped with a small probability.

We simulated odometry with a variance of 2[pix]/frame for translation and 1[rad]/frame for rotation. Range data were simulated by ray-tracing with Gaussian noise of variance 5[pix] in the range direction. For our PF-GC, range data converted to polar image of size 64×512[pix] (template) were matched with global map of size 512×512[pix](query), reconstructed from real range data[5]. For other methods range data was converted to circular image of radius 60[pix] as same as chapter 3. The results are shown in Fig.7, and Table 2. The computation time was almost same as that shown in Table 1, except that the learning time of CF-GC was further reduced. This is because we can skip Cartesian to polar transformation in this example.

5 Conclusion

We propose *Fast Eigen Matching* that speeds up the learning and matching processes of the eigenspace method for rotation invariant template matching(RITM). By experiments,

Table 2: Translational/rotational error of 50 frame used for evaluation and computation time

	Translational error [pix]		Rotational error [deg]		Computation time[sec]		
	mean	variance	mean	variance	Learning	Matching	Total
ZNCC	0.011	0.285	0.8652	0.0060	5.605	0.675	6.280
NA-GC	2.070	1.649	0.9568	0.0055	4.931	0.441	5.372
CE-GC	1.905	2.7870	0.8079	0.0122	2.630	0.170	2.800
Proposed CF-GC	1.826	1.171	0.9626	0.0154	1.663	0.055	1.718
Proposed PF-GC	2.145	0.916	0.6303	0.0061	**0.007**	**0.055**	**0.062**

we demonstrated that our methods achieved superior efficiency while keeping comparable matching performance. Our methods are successfully applied to global localization of mobile robot equipped with LRF where online learning is required. The algorithm can also be applicable to image data [12, 9].

References

[1] A. Averbuch, R.R. Coifman, D.L. Donoho, M. Elad, and M. Israeli. Fast and accurate polar fourier transform. *Applied and Computational Harmonic Analysis*, 21(2):145 – 167, 2006.

[2] Gunilla Borgefors. Hierarchical chamfer matching: A parametric edge matching algorithm. *IEEE Trans. Pattern Anal. Mach. Intell.*, 10(6):849–865, November 1988.

[3] Chu Yin Chang, Anthony a. Maciejewski, and Venkataramanan Balakrishnan. Fast eigenspace decomposition of correlated images. *IEEE Transactions on Image Processing*, 9(11):1937–1949, 2000.

[4] Paul Checchin, Franck Gérossier, Christophe Blanc, Roland Chapuis, and Laurent Trassoudaine. Radar Scan Matching SLAM using the Fourier-Mellin Transform. *Field and Service Robotic*, pages 1–10, 2010.

[5] Andrew Howard and Nicholas Roy. The robotics data set repository (radish).

[6] Gou Koutaki and Keiichi Uchimura. Occlusion Robust Pattern Matching Using Shape Based Eigen Templates. *IEEJ Transactions on Electronics, Information and Systems*, 133(1):134–141, 2013.

[7] John P Lewis. Fast template matching. In *Vision interface*, volume 95, pages 15–19, 1995.

[8] Dong Liang, Shunichi Kaneko, Manabu Hashimoto, Kenji Iwata, Xinyue Zhao, and Yutaka Satoh. Robust object detection in severe imaging conditions using co-occurrence background model. *International Journal of Optomechatronics*, 8(1):14–29, 2014.

[9] Masafumi Noda, Tomokazu Takahashi, Daisuke Deguchi, Ichiro Ide, Hiroshi Murase, Yoshiko Kojima, and Takashi Naito. Vehicle ego-localization by matching in-vehicle camera images to an aerial image. In *Computer Vision–ACCV 2010 Workshops*, pages 163–173. Springer, 2010.

[10] Wanli Ouyang, Renqi Zhang, and Wai-Kuen Cham. Fast pattern matching using orthogonal haar transform. In *Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on*, pages 3050–3057. IEEE, 2010.

[11] Robert Piessens. The hankel transform. 2000.

[12] Turgay Senlet and Ahmed Elgammal. A framework for global vehicle localization using stereo images and satellite and road maps. *2011 IEEE International Conference on Computer Vision Workshops (ICCV Workshops)*, pages 2034–2041, 2011.

[13] H.S. Stone, M.T. Orchard, E.-C. Chang, and S.A. Martucci. A fast direct fourier-based algorithm for subpixel registration of images. *Geoscience and Remote Sensing, IEEE Transactions on*, 39(10):2235–2243, Oct 2001.

[14] Sebastian Thrun, Wolfram Burgard, and Dieter Fox. *Probabilistic Robotics*. 2005.

[15] G. Tzimiropoulos, V. Argyriou, S. Zafeiriou, and T. Stathaki. Robust fft-based scale-invariant image registration with image gradients. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 32(10):1899–1906, Oct 2010.

[16] Georgios Tzimiropoulos, Vasileios Argyriou, and Tania Stathaki. A Frequency Domain Approach to Roto-translation Estimation using Gradient Cross-Correlation. *Bmvc08*, 2008.