

# Accurate Closed-form Estimation of Local Affine Transformations Consistent with the Epipolar Geometry

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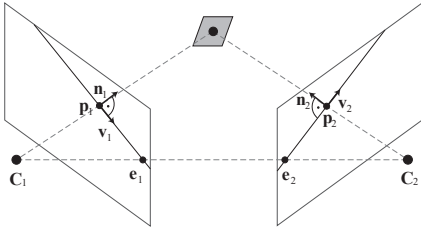
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A novel method is proposed for accurate estimation of local affine transformations for a pair of images satisfying the epipolar constraint. The method returns the closest, in least squares sense, affine transformation to an initial estimate consistent with the fundamental matrix.

**The contributions** of the paper: (i) the introduction of two novel constraints for a local affine transformation making it consistent with the fundamental matrix, and (ii) a method estimating an EG- $L_2$ -Optimal affinity – transformation which is consistent with the epipolar geometry (EG) –, by enforcing the proposed constraints.

An affine correspondence consists of a point pair  $\mathbf{p}_1, \mathbf{p}_2$  and a local affine transformation  $\mathbf{A}$  mapping the neighborhood of the points.



**The constraints** state that the  $2 \times 2$  matrix  $\mathbf{A}$  transforms the normal  $\mathbf{n}_1$  of the epipolar line related to point  $\mathbf{p}_1$  into  $\beta \mathbf{n}_2$ , where  $\mathbf{n}_2$  is the normal of the epipolar line related to point  $\mathbf{p}_2$  and  $\beta \in \mathbb{R}$  is a scalar. This statement is equivalent to  $\mathbf{n}_1 \mathbf{A}^{-T} = \beta \mathbf{n}_2$ . It is proven as well that  $\beta$  is determined by the epipolar geometry.

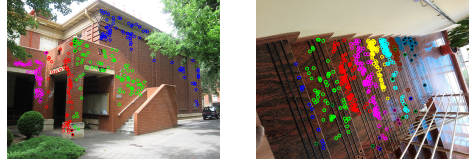
**The method** requires an affine correspondence  $\mathbf{p}_1, \mathbf{p}_2, \mathbf{A}'$ , i.e. estimated by an affine-covariant detector. The points  $\mathbf{p}_1$  and  $\mathbf{p}_2$  are optimally be corrected w.r.t. the epipolar geometry, in least squares sense, by the method of [4]. The proposed technique corrects  $\mathbf{A}'$  by simultaneously minimizing  $\|\mathbf{A} - \mathbf{A}'\|_F^2$  and enforcing the introduced constraints using a closed-form approach. It is proven that  $\|\mathbf{A} - \mathbf{A}'\|_F^2$  has both geometric and algebraic interpretations.

**The processing time** of the method is  $\approx 0.04$  ms in C++.

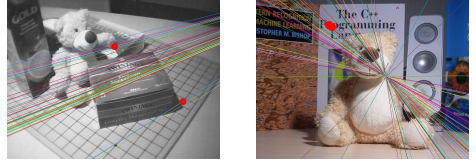
**Evaluation.** The method is validated on synthetic data and publicly available benchmarks. The corrected affinities are always more accurate than the output of the affine-covariant detector. As a side-effect, the detectors are compared – the most accurate is the Hessian-Affine augmented by view-synthesis a la ASIFT.

**Conclusions.** The algorithm has negligible time demand and always makes the input affinities more accurate. In problems involving local affine transformations in rigid scenes, the proposed method should always be used.

**Application 1.** Using the proposed results the detection and segmentation of multiple planes becomes more accurate [1].



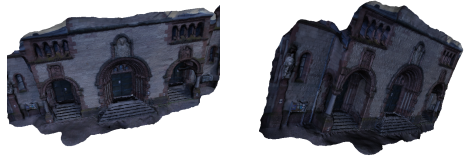
**Application 2.** Using equation  $\mathbf{n}_1 \mathbf{A}^{-T} = \beta \mathbf{n}_2$  the fundamental matrix is estimable from two affine correspondences.



**Application 3.** Surface normal estimation benefiting from precise affine correspondences [2].



**Application 4.** Precise affine correspondences significantly improve camera calibration as well as 3D reconstruction [3].



**Application 5.** In the paper, we use the method to compare the geometric precision of affine-covariant feature detectors.

- [1] D. Barath, J. Matas, and L. Hajder. Multi-H: Efficient recovery of tangent planes in stereo images. In *BMVC*, 2016.
- [2] D. Barath, J. Molnar, and L. Hajder. Novel methods for estimating surface normals from affine transformations. In *VISIGRAPP Selected Papers*, 2016.
- [3] I. Eichhardt and L. Hajder. Improvement of camera calibration using surface normals. In *ICPR*, 2016.
- [4] R. I. Hartley and P. Sturm. Triangulation. *CVIU*, 1997.