Pose Estimation of Kinematic Chain Instances via Object Coordinate Regression

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Derivation for the Estimation of Articulation Parameters.

Here we provide the derivation for Eq. (2) and Eq. (3) in the paper used to find articulation parameters from two point correspondences.

Revolute Joints: We will first consider the case of revolute joints. We show the derivation for a rotation around the x-axis. The task is to derive the angle at the joint from two correspondences $(\mathbf{x}(i_1), \mathbf{y}_k(i_1))$ and $(\mathbf{x}(i_2), \mathbf{y}_{k+1}(i_2))$, with $\mathbf{y}_k(i) = (\mathsf{x}_k, \mathsf{y}_k, \mathsf{z}_k)^\top$. We abbreviate the squared distance between the two points in camera space as $d_\mathbf{x} = \|\mathbf{x}(i_1) - \mathbf{x}(i_2)\|^2$. We start with Eq. (1) in the paper and solve for θ.

$$d_{\mathbf{x}} = \|\mathbf{y}_k(i_1) - A_k(\theta_k)\mathbf{y}_{k+1}(i_2)\|^2, \text{ with } A_k(\theta_k) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_k & -\sin\theta_k & 0 \\ 0 & \sin\theta_k & \cos\theta_k & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$d_{\mathbf{x}} = (\mathsf{x}_k - \mathsf{x}_{k+1})^2 + (\mathsf{y}_k - (\cos(\theta_k)\mathsf{y}_{k+1} - \sin(\theta_k)\mathsf{z}_{k+1}))^2 + (\mathsf{z}_k - (\sin(\theta_k)\mathsf{y}_{k+1} + \cos(\theta_k)\mathsf{z}_{k+1}))^2 \tag{2}$$

$$\begin{aligned} d_{\mathbf{x}} = {} & (\mathsf{x}_k - \mathsf{x}_{k+1})^2 + \mathsf{y}_k^2 - 2(\mathsf{y}_k\cos(\theta_k)\mathsf{y}_{k+1} - \mathsf{y}_k\sin(\theta_k)\mathsf{z}_{k+1}) + (\cos(\theta_k)\mathsf{y}_{k+1} - \sin(\theta_k)\mathsf{z}_{k+1})^2 \\ & + \mathsf{z}_k^2 - 2(\mathsf{z}_k\sin(\theta_k)\mathsf{y}_{k+1} + \mathsf{z}_k\cos(\theta_k)\mathsf{z}_{k+1}) + (\sin(\theta_k)\mathsf{y}_{k+1} + \cos(\theta_k)\mathsf{z}_{k+1})^2 \end{aligned} \tag{3}$$

$$\begin{aligned} d_{\mathbf{x}} = {} & (\mathsf{x}_k - \mathsf{x}_{k+1})^2 + \mathsf{y}_k^2 + \mathsf{z}_k^2 \\ & + 2\sin(\theta_k)(\mathsf{y}_k\mathsf{z}_{k+1} - \mathsf{z}_k\mathsf{y}_{k+1}) + 2\cos(\theta_k)(-\mathsf{y}_k\mathsf{y}_{k+1} - \mathsf{z}_k\mathsf{z}_{k+1}) \\ & + 2\sin(\theta_k)\cos(\theta_k)(\mathsf{z}_{k+1}\mathsf{y}_{k+1} - \mathsf{z}_{k+1}\mathsf{y}_{k+1}) \\ & + \cos^2(\theta_k)(\mathsf{y}_{k+1}^2 + \mathsf{z}_{k+1}^2) + \sin^2(\theta_k)(\mathsf{y}_{k+1}^2 + \mathsf{z}_{k+1}^2) \end{aligned} \tag{4}$$

$$d_{\mathbf{x}} = (\mathsf{x}_k - \mathsf{x}_{k+1})^2 + \mathsf{y}_k^2 + \mathsf{z}_k^2 + \mathsf{y}_{k+1}^2 + \mathsf{z}_{k+1}^2 + a\sin(\theta_k) + b\cos(\theta_k), \tag{5}$$

where $a = 2(\mathsf{y}_k\mathsf{z}_{k+1} - \mathsf{z}_k\mathsf{y}_{k+1})$ and $b = -2(\mathsf{y}_k\mathsf{y}_{k+1} + \mathsf{z}_k\mathsf{z}_{k+1})$. It is known that

$$a\sin(\theta_k) + b\cos(\theta_k) = \sqrt{a^2+b^2}\sin(\theta_k + \operatorname{atan2}(b,a)). \tag{6}$$

We can use supplementary Eq. 6 and continue

$$d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{z}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_{k+1}^2 = \sqrt{a^2+b^2}\sin(\theta_k + \operatorname{atan2}(b,a)) \tag{7}$$

$$\frac{d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{z}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_{k+1}^2}{\sqrt{a^2+b^2}} = \sin(\theta_k + \operatorname{atan2}(b,a)). \tag{8}$$

When we apply the asin function we have to consider the two possible results:

$$\operatorname{asin}\left(\frac{d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{z}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_{k+1}^2}{\sqrt{a^2+b^2}}\right) = \theta_k + \operatorname{atan2}(b,a) \tag{9}$$

and

$$\pi - \operatorname{asin}\left(\frac{d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{z}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_{k+1}^2}{\sqrt{a^2+b^2}}\right) = \theta_k + \operatorname{atan2}(b,a)), \tag{10}$$

which lead to the two solutions from Eq. (2) in the paper:

$$\theta_k^1 = \operatorname{asin}\left(\frac{d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_k^2 - \mathsf{z}_{k+1}^2}{\sqrt{a^2+b^2}}\right) - \operatorname{atan2}(b,a) \tag{11}$$

and

$$\theta_k^2 = \pi - \operatorname{asin}\left(\frac{d_{\mathbf{x}} - (\mathsf{x}_k - \mathsf{x}_{k+1})^2 - \mathsf{y}_k^2 - \mathsf{y}_{k+1}^2 - \mathsf{z}_k^2 - \mathsf{z}_{k+1}^2}{\sqrt{a^2+b^2}}\right) - \operatorname{atan2}(b,a). \quad (12)$$

Prismatic Joints: We will now derive Eq. (3) in the paper, which addresses prismatic joints. We show the derivation for a translation along the x-axis. We start again with Eq. (1) from the paper:

$$d_{\mathbf{x}} = \|\mathbf{y}_k(i_1) - A_k(\theta_k)\mathbf{y}_{k+1}(i_2)\|^2, \text{ with } A_k(\theta_k) = \begin{pmatrix} 1 & 0 & 0 & \theta_k \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

$$d_{\mathbf{x}} = (\mathsf{x}_k - (\mathsf{x}_{k+1} + \theta_k))^2 + (\mathsf{y}_k - \mathsf{y}_{k+1})^2 + (\mathsf{z}_k - \mathsf{z}_{k+1})^2 \quad (14)$$

$$d_{\mathbf{x}} = \mathsf{x}_k^2 - 2\mathsf{x}_k(\mathsf{x}_{k+1} + \theta) + (\mathsf{x}_{k+1} + \theta)^2 + (\mathsf{y}_k - \mathsf{y}_{k+1})^2 + (\mathsf{z}_k - \mathsf{z}_{k+1})^2 \quad (15)$$

$$d_{\mathbf{x}} = \mathsf{x}_k^2 - 2\mathsf{x}_k\mathsf{x}_{k+1} - 2\mathsf{x}_k\theta + \mathsf{x}_{k+1}^2 + 2\mathsf{x}_{k+1}\theta + \theta^2 + (\mathsf{y}_k - \mathsf{y}_{k+1})^2 + (\mathsf{z}_k - \mathsf{z}_{k+1})^2 \quad (16)$$

$$0 = \theta^2 + 2(\mathsf{x}_{k+1} - \mathsf{x}_k)\theta + \mathsf{x}_k^2 - 2\mathsf{x}_k\mathsf{x}_{k+1} + \mathsf{x}_{k+1}^2 + (\mathsf{y}_k - \mathsf{y}_{k+1})^2 + (\mathsf{z}_k - \mathsf{z}_{k+1})^2 - d_{\mathbf{x}}. \quad (17)$$

We can reformulate the equation as

$$0 = \theta^2 + p\theta + q, \quad (18)$$

with $p = 2(\mathsf{x}_{k+1} - \mathsf{x}_k)$ and $q = (\mathsf{x}_k - \mathsf{x}_{k+1})^2 + (\mathsf{y}_k - \mathsf{y}_{k+1})^2 + (\mathsf{z}_k - \mathsf{z}_{k+1})^2 - d_{\mathbf{x}}$. Solving supplemental Eq. 18 is a standard problem. The solutions are equivalent to Eq. (3) in the paper:

$$\theta_k^1 = -\frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 - q} \quad (19)$$

and

$$\theta_k^2 = -\frac{p}{2} - \sqrt{\left(\frac{p}{2}\right)^2 - q}. \quad (20)$$

References

[1] E. Brachmann, A. Krull, F. Michel, S. Gumhold, J. Shotton, and C. Rother. Learning 6d object pose estimation using 3d object coordinates. In *ECCV*, 2014.