

Shape from Focus with Adaptive Focus Measure and High Order Derivatives

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Abstract

Shape From Focus (SFF) methods frequently use a single focus measure to obtain a depth map. Common focus measures are fixed and spatially invariant. In this paper we present a framework to create an adaptive focus measure based on ensemble of basis focus operators. Using the proposed framework we derive a new spatially variant focus measure obtained from linear combination of image derivatives. This approach effectively generalizes some of the existing measures. A new measure emerged from the proposed framework includes high order derivatives and presents a highly reliable focus measure. We rely on the focus curve standard deviation (CSTD) to determine the linear coefficients in our model. The emerged focus measure copes effectively with texture variation, strong intensity edges and depth discontinuities. Using CSTD we further suggest a new approach for aggregation in the focus volume succeeded by reconstruction based on the focus curve centroid. This different approach of aggregation and reconstruction yields improved depth maps, respecting shape smoothness and depth discontinuities for diversity of textured images. We assess the performance of our new approach by extensive experiments with highly realistic synthetic images and real images including two unique cases captured in the wild. In terms of focus measure, we significantly outperform the state-of-the-art, while presenting superior results comparing to two previously published alternatives.

1 Introduction

Three-dimensional (3D) shape reconstruction is a fundamental problem in machine vision applications. Among numerous methods Shape From Focus (SFF) presents a dense and passive optical method for 3D shape recovery using the degree of focus as a cue to estimate 3D shape. In SFF setting, a stack of images is captured while shifting the focus plane along the optical axis, by moving the camera, the object or changing the optical setting. The shape is then recovered by finding the plane at which each point in the image appears in focus (sharply). The SFF technique has been successfully utilized in many applications, such as micro-biology, PCB inspection, robot control, and even colonoscopy.

The basic step in SFF is to compute a sharpness quality for each pixel in the image stack, commonly referred to as *focus measure*. Numerous focus measures have been suggested

in the past including the popular Gray Level Variance (GLV), Modified Laplacian (ML) [16] and Tenenbaum-Gradient (TEN) [8] to name a few. More recent focus measures rely on 3D image space [9] high statistical moments [23] and discrete wavelet analysis [10]. However, image derivatives are still used as a strong cue for sharpness relying on intensity edges that typically exist in the scene texture map [8, 16]. Commonly, Shape From Focus (SFF) methods use a single focus measure [9, 8, 16, 23]. Many factors, including window size, noise level, illumination and shape characteristics affect the performance of a focus measure. A single focus measure is often unable to account for the diverse types of scenarios particularly for images captured in unconstrained conditions such as open nature [17, 20].

Focus measure is a local property, commonly evaluated in a small 2D window around the point of interest. Common focus measures are *spatially invariant* therefore incapable to adapt to variations in the texture and 3D shape of the scene. Several studies attempt to address this problem by changing the window size [9, 15] while most recent works show that the use of more than one focus measure provides a better 3D shape. These methods first extract multiple depth maps from separate focus measures. However, each one of the resulting depth maps typically includes certain errors. In order to obtain a reliable shape, the extracted depth maps are then merged based on their likelihood to be part of a coherent 3-D surface [9, 8, 10]. Yet, it is often difficult to correct the obtained errors after the depth extraction and one would better deal with the problem beforehand, in properly modeling the focus measure. Such approach will further be favorable computationally, since the full reconstruction pipeline is then performed once instead of separately for each focus measure. In this paper we first suggest a new adaptive focus measure that can directly result in improved depth maps. The new focus measure is embedded in the linear space spanned by image derivatives with different orders and directions. We determine the important coefficients in the linear space by a focus curve characteristic, specifically the *curve standard deviation* (CSTD). The proposed method exhibits a focus quality measure that is spatially variant and adaptive to image content and object shape. While focus measures often use image derivatives as high as second order, our framework allows employing considerably higher order derivatives found to be very effective. We refer to this new measure as *Adaptive High Order* (AHO) focus measure.

Shape from focus pipeline includes further steps. Computation of the focus measure per-pixel in the image stack, yields the *focus volume* (also known as data space). Then typically noise and systematic errors are filtered out from the focus volume by aggregation, prior to depth extraction. Linear filtering (e.g. convolution with a uniform kernel) is well known to blur depth discontinuities. Other methods therefore utilize non-linear filtering in the focus volume by adaptively varying the kernel size and weights e.g. anisotropic filtering [9, 8]. Finding the cue for window kernel adjustment is then the key problem. In this paper we further make use of the CSTD value to present a new adaptive filtering, that respects texture variations, intensity edges and depth discontinuities.

Finally, the reconstruction (depth extraction) can be performed by Winner Takes All, namely setting the depth values at each point according to the maximum focus level. Often a fitting function is used, modeling the focus peak by Gaussian [16], Lorentzian [14], or a polynomial [19] to reject outliers during reconstruction. However, these methods disregard the smoothness constraint between neighboring points in the spatial domain whereas the assumption of ideal optical model can further cause systematic errors. Shape can also be extracted by fitting a surface in the focus volume known as Focus Image Surface (FIS) [8, 13, 19]. In the FIS approach an optimization problem is solved where the resulting surface is forced to pass through the focus measure peaks while preserving shape smoothness.

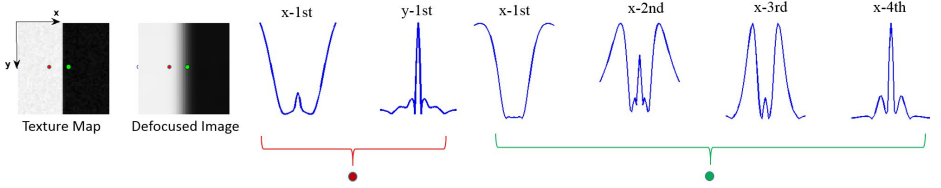


Figure 1: The importance of derivative direction and order. A toy example of a flat equifocal surface with subtle texture and strong intensity edge at the center. The texture map with a sample defocused image are shown left. Two focus curves are computed at the red dot, from the x (horizontal) and y (vertical) first derivatives. The derivatives up to order four for x are shown right for the green dot. Note that only the first order y and the fourth order x derivatives yield the correct localization of the maximum focus measure.

Yet, surface approximation methods hardly cope with sharp depth discontinuities due to the smoothness constraint. In this study, we use the focus curve *centroid* for depth extraction (after aggregation), an approach widely used for localization in microscopic imaging [21]. The focus curve centroid introduces a non-parametric cue for depth estimation, insensitive to outliers in the focus curve.

Experiments with synthetic and real images show the effectiveness of our new focus measure and SFF model in dealing with sharp depth discontinuities and fine structures. The suggested focus measure outperforms the state-of-the-art focus quality measures qualitatively and quantitatively. We further compare our SFF model to two previously published methods performing the whole pipeline of focus measure+aggregation+reconstruction and show superior results.

2 Focus Measure

Consider an image stack $I_z(x, y)$, consisting of Z images, each of size $X \times Y$ captured by an imaging system with a shallow depth of field. In the spatial domain, the focus volume is computed locally as $\phi : (X \times Y) \times Z \mapsto \mathbb{R}$ assigning a focus measure to each point in the image stack. The *focus curve* is the set of all focus measures associated to a certain point in the spatial domain ($x_0 \in X, y_0 \in Y$) and presents a curve in the focus volume determined by $\phi(x_0, y_0, z)$. Let us now define a linear space with the basis functions as the image derivatives, in different orders and directions:

$$\phi_{ij} \approx \left| \frac{\partial^i I}{\partial \mathbf{x}_j^i} \right| \quad (1)$$

where $i \in [1, 2, \dots, n]$ and $j \in [1, 2, \dots, m]$ index the derivative order and the discrete directions respectively (not to be confused with pixel location). For the standard image grid we use $m = 4$ directional derivatives enumerating 1, 2 as standard orthogonal x, y and 3, 4 as diagonal directions. The $\approx$ sign indicates the numerical approximation of the term calculated by central difference scheme [8]. Note that in this framework the celebrated Modified Laplacian (ML) focus measure is obtained as a particular case of $\phi_{21} + \phi_{22}$. This choice of basis focus measures is not arbitrary as the order and the directions of the derivatives play an important role in local adjustment of the focus measure. A toy example shown in Figure 1 demonstrates the importance of derivative order and directional operation. This example contains an equifocal surface with a subtle random noise texture and a strong horizontal gray

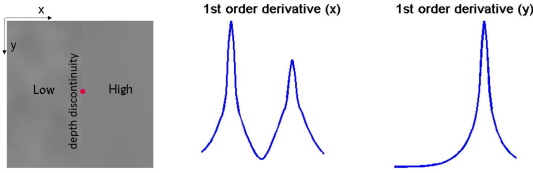


Figure 2: The effect of derivative direction at depth discontinuity. Two focus curves for a point near a depth discontinuity are depicted. Only the derivative along the depth edge (y) yields the desired focus curve.

level gradient at the center. The first order x -derivative exhibits a triple mode curve (see Fig. 1) caused by the gray level edge, smeared gradually due to defocus. As the gradient is diffused horizontally (see the defocused image in Figure 1), two peaks appear in the focus curve, corresponding to the near and far defocused images. The weak peak at the center is due to the subtle texture as it reaches the best focus. Furthermore, Figure 1 shows that among several image derivatives only the vertical first order and the horizontal fourth order are reliable focus curves. This simple demo reveals the weakness of popular focus measures such as the TEN [6] (sum of absolute first order derivatives). Averaging the erroneous x component in TEN with the correct y term is likely to shift the focus curve peak from its correct localization. In addition to the directional image derivatives our model further embeds high order derivatives (up to 10th) [9]. The benefit of such derivatives is in the narrow well-behaved focus curves that they exhibit. One may argue the accuracy of high order derivatives in a sampled data associated with amplification of high frequency noise. Yet, assuming sufficient sampling rate (spatial resolution) the defocus acts as a low pass filter, diminishing the high frequency noise. It should be mentioned that our interest in the focus curve is limited to a local neighborhood around the focus curve peak. A correct curve pattern near the peak is therefore sufficient for a reliable focus curve rather than absolute accuracy. Moreover, the linear combination of image derivatives allows a degree of freedom. Erroneous focus curves that may emerge from high order derivatives are suppressed in our framework by attenuation of their associated weights (cf. section 2.1).

Next, we demonstrate in Figure 2 a typical case of depth discontinuity. A toy example is again shown consisting of a step surface with two distinct depth levels and uniform texture. Figure 2 shows two focus curves for a point in the vicinity of the depth step edge. Note how the focus measure based on derivative across the step (x) fails to detect the correct depth level. On the other hand, the derivative along the edge yields the desired focus curve unimodal with correct localization of the maximum.

The above simplified tests show that in the vicinity of intensity edges and depth discontinuities there are mainly two types of derivative curves. The first type is a symmetric, unimodal and narrow curve, characterized by a low curve standard deviation (CSTD) while the second type exhibits a multimodal or wide peak curve with high CSTD. The common derivative based focus measures therefore sum reliable components characterized by low CSTD with multimodal or insignificant peak curves associated with high CSTD. This paper suggests a focus measure based on a combination of image derivatives represented by a vector in a linear space described by:

$$\psi(x, y, z) = \sum_{i=1}^n \sum_{j=1}^m \alpha_{ij}(x, y) \hat{\phi}_{ij}(x, y, z) \quad (2)$$

where, α_{ij} denote the coefficients and can be viewed as confidence measures. In order to set all focus curves into the same scale, we normalize each curve by its maximum level and denote it by $\hat{\phi}$. We call the emerged operator ψ as the Adaptive High Order (AHO) focus measure. This framework further allows different choices of the basis functions, namely a different ensemble of focus measures. The combination of different focus operators offers the benefit that a failure of a particular operator locally, can be compensated by other reliable terms.

2.1 Confidence Measure

In this section we present our model for setting the coefficients (weights) in (2) for the basis focus operators. To this end, we employ the CSTD¹ computed by:

$$\sigma_{ij}(x, y) = \frac{1}{\sqrt{N_{ij}}} \left[\int_{\bar{Z}} (z - \mu_{ij})^2 \hat{\phi}_{ij}(x, y, z) dz \right]^{\frac{1}{2}} \quad (3)$$

where $\mu_{ij}(x, y)$ is the *centroid* of the focus curve $\hat{\phi}_{ij}$ and N denotes the normalization factor, $N_{ij} = \int \hat{\phi}_{ij}(x, y, z) dz$. The integration domain $\bar{Z}$ corresponds to the range where focus values are over half of the maximum level. This restriction of the domain prevents the bias due to the focus curve long tail. Favoring narrow peaked curves (*i.e.* lower CSTD) leads to a monotonically decreasing weighting scheme with respect to CSTD. We use a *Lorentzian* function for this relation:

$$\alpha_{ij}(x, y) = \left[1 + \left(\frac{\sigma_{ij}(x, y)}{\rho} \right)^2 \right]^{-1}, \quad (4)$$

where ρ determines the rate of decay. Substituting the resulting weights from (4) in (2) yields the new AHO focus measure. Note that the this focus measure is spatially variant.

3 Aggregation

Once the focus volume in (2) is computed, we perform an aggregation in order to remove the residual inconsistencies in the focus volume, prior to depth extraction. A common concept behind the aggregation is averaging over a patch around the point of interest with 2D or 3D kernels. While 2D kernels disregard the relation between neighboring depth values, 3D kernels in the focus volume tend to smear the depth edges. However the z -axis provides additional information regarding the similarity of depth levels between two nearby points. While common similarity measures allow neighboring points to have close depth values (focus peak localization), we define a new similarity measure based on the CSTD. In this aggregation scheme, reliable curves, characterized by small discrepancy from a reference CSTD, are effectively averaged. The weighting in the aggregation window is then subjected to following scheme:

$$\omega(x, y) = \left[1 + \left(\frac{\sigma(x, y) - \bar{\sigma}}{\rho} \right)^2 \right]^{-1}, \quad (5)$$

¹In many physical applications this term is referred to Full Width Half Maximum

where we use the same parameter ρ as in (4). The new reference $\bar{\sigma}$ is set to the median CSTD emerged from all the focus curves in the stack and corresponds to the texture of the scene and the imaging optics. Adding this parameter improves the noise tolerance since noise patterns such as salt and pepper often exhibit low CSTD, mimicking the desired focus curve and deteriorating the aggregation outcome. Note that although $\bar{\sigma}$ is constant the weighting is space-variant due to the variation of CSTD (σ) per-pixel. We aggregate the focus measure separately for each image in the stack, denoting the aggregated measure after t iteration by $\tilde{\psi}_t$ ($\tilde{\psi}_0 \equiv \psi$). The aggregated measure is then given by

$$\tilde{\psi}_t(x, y, z) = \sum_{(x', y') \in W(x, y)} \hat{w}(x', y') \tilde{\psi}_{t-1}(x', y', z), \quad \forall z \in Z \quad (6)$$

where $W(x, y)$ denotes the window domain and $\hat{w}$ stands for the *normalized* weights given in (5). This approach resembles the successful non-local means in denoising [9], and is insensitive to the window size. In the proposed scheme, aggregation windows located near a depth discontinuity will initially average two shifted focus curves (ideally Gaussians), creating a multimodal curve with high CSTD. The iterative aggregation then suppresses averaging between distinct depth levels as the corresponding points associated with high CSTD are assigned lower weights in later iterations.

4 Reconstruction

A naive approach for extracting the depth map $d(x, y)$ from the focus volume is ‘Winner Takes All’ (WTA):

$$d(x, y) = \underset{z}{\operatorname{argmax}} \tilde{\psi}(x, y, z), \quad (7)$$

where the depth values are assigned according to maximum focus level [18] [19]. Previous methods have suggested fitting at the focus peak using various approximations such as Gaussian [19]. However a preset fitting model may become invalid after the aggregation, as the average of two non-identical Gaussians is not a Gaussian any more. The focus curve sharpness plays an important role in reliability of depth estimation, since relatively flat peaks are highly sensitive for localization of the maximum. In fact the focus curve width depends not only on the texture strength but also the imaging optics. A non-parametric cue for depth estimation is the focus curve *centroid* also robust to outliers that commonly appear in computation of the focus curves:

$$d(x, y) = \frac{1}{M} \int_{\tilde{Z}} z \tilde{\psi}(x, y, z) dz \quad (8)$$

where $M = \int_{\tilde{Z}} \tilde{\psi}(x, y, z) dz$ is a normalization factor and $\tilde{Z}$ denotes the region of the normalized focus curve (in the range $[0, 1]$) above a threshold $\tilde{\psi} > \tau_\psi$. Finally, a flowchart outlining the algorithm steps is provided in Fig. 3.

5 Experimental Evaluation

Our test bed is composed of three sets of highly realistic synthetic images created by Autodesk Maya ray tracer and three sets of real images. In the synthetic stacks (consisting 40-90

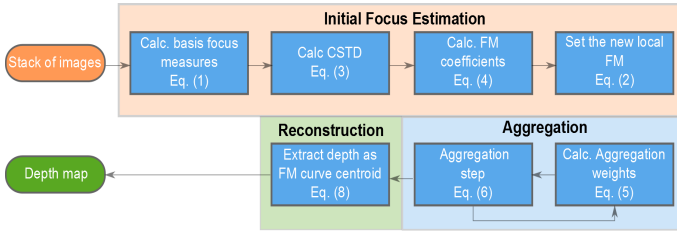


Figure 3: The Processing Flow Chart.

images), the texture maps were first created by Autodesk Maya, then the images were defocused according to the true shape. For defocus we used a Gaussian kernel in the ‘Cloth’ and ‘Synth-Cone’ and a hexagonal aperture blur, simulated by Adobe Photoshop in the checkerboard ‘Cube’. The synthetic sets were designed to capture particular challenges in SFF such as texture variation, strong image edges, depth discontinuities as well as non-standard defocus kernel. Our first real data set is composed of a stack of 95 images captured by a microscope [8]. Additionally, there are two newly introduced real data sets consisting of 34 images of an Antlion² and only 6 images of a Spider² captured in the wild, by a SLR camera. For more details on the imaging system of these two sets the reader is referred to [8].

In our experiments we use the derivative orders up to $n = 10$, considering $m = 4$ discrete directions. We fix the threshold parameter in (8) to be $\tau_\psi = 0.9$ for the synthetic images and $\tau_\psi = 0.7$ for the real sets. The decay factor in equations (4) and (5) was set to $\rho = 6$. In the aggregation stage we used 15×15 window size along 15 iterations. We separately assess the performance of the new focus measure and the complete SFF model in order to analyze the individual contributions.

5.1 Focus measure

In order to evaluate the performance of our focus measure we present a comparative analysis with the state-of-the-art focus measures as reported in [14]. The resulting depth maps are shown in Fig. 4. To allow a fair comparison, all depth maps were extracted in the same manner by Winner Takes All, without further processing (e.g. aggregation or fitting). The results show the significant improvement obtained by our AHO focus measure, well coping with different textures as well as intensity and depth edges. The ‘Cloth’ and the ‘Synth-Cone’ examples demonstrate our capability to handle high texture variations, reliably recovering the smooth depth map. The ‘Middlebury-Cone’ case presents the effectiveness of the new focus measure to handle depth discontinuities. The AHO results outperform the compared methods also in the semi-realistic ‘Cube’ test, where the checkerboard pattern exhibits a combination of high intensity edges and textureless patterns in addition to strong depth discontinuities at the object outline. Finally, our depth map for the ‘Real-Cone’ test with non-uniform illumination [15] is again the best outcome.

We further conduct a quantitative comparison in terms of depth map RMSE (Root Mean Square Error) and report the results in Table 1. Notably the AHO achieves results that are more than twice as accurate as the best focus measures in the literature. This high accuracy is partly due to the unprecedented usage of very high order derivatives in our focus measure.

² Images are courtesy of Ilia Lutsker

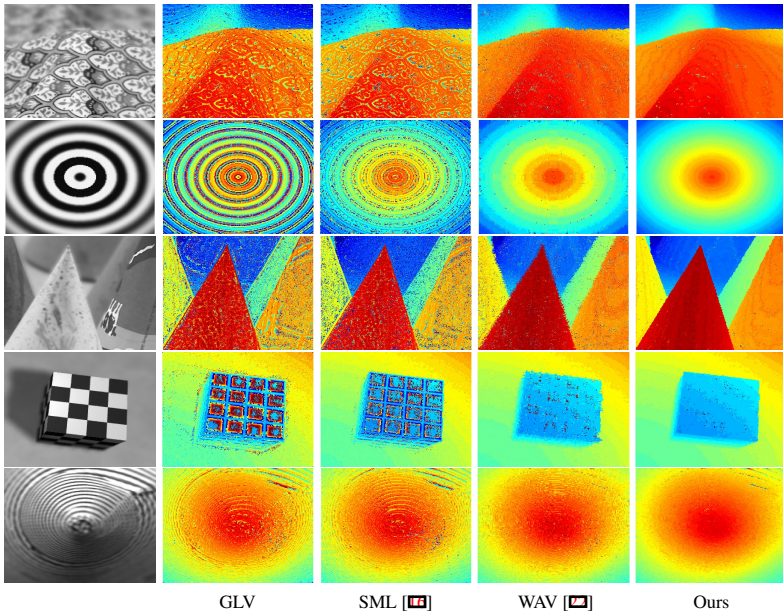


Figure 4: Comparison of focus measures. Color coded depth maps (from near red to far blue) extracted by applying WTA over different focus measures. From top to bottom: ‘Cloth’, ‘Synth-Cone’, ‘Middlebury-Cones’, ‘Cube’ and ‘Real-Cone’. Note the significant improvement obtained using the proposed focus measure.

Object	TEN [10]	GLV	ML [10]	WAV [10]	AHO
Cloth	2.22	2.6	1.42	1.78	0.76
Synthetic-Cone	7.75	7.05	3.73	1.05	0.33
Middlebury-Cones	3.58	3.42	1.56	2.49	1.17
Cube	7.02	5.18	3.63	1.67	0.76
Average	5.14	4.56	2.59	1.75	0.76

Table 1: Comparison of focus measure operators in terms of depth map RMSE, with respect to our AHO approach. Best results are in bold.

Figure 5 shows the effect of the highest derivative order considered in the model (n in (1)) on the depth map accuracy. As observed the accuracy is improved as higher order derivatives are added to the model. In the ‘Cube’ test case for instance, dominated by checkerboard texture, the improvement continues up to the 10th order. In cases where high order derivatives are not beneficial, they are likely to be attenuated by the weighting scheme in our framework.

5.2 Shape from Focus

In this section we present the performance of our complete pipeline for shape reconstruction using the proposed focus measure, aggregation and reconstruction. Due to lack of a benchmark in SFF and unavailable code for existing methods, we compare our results to two recently published methods that we implemented and optimized: the SFF approach in [10] based on *Bilateral* filtering of the focus volume and the *Anisotropic* focus aggregation of [10]. As the ‘Antlion’ and ‘Spider’ image stacks are captured outdoor without precise

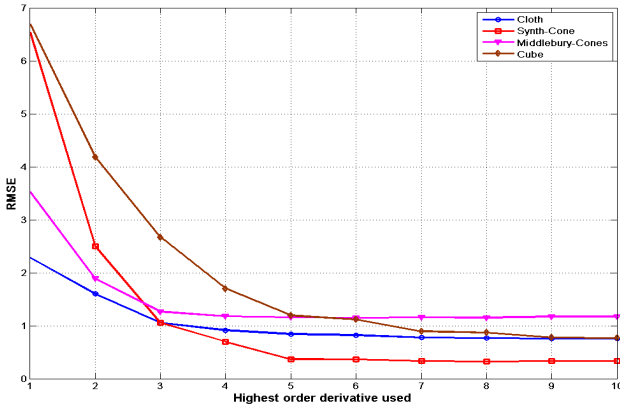


Figure 5: The effect of the highest derivative derivative order considered on the accuracy. The plots correspond to our four synthetic data sets with available ground-truth.

lab equipment we first register the image stack prior to SFF computation. Due to limited range of z samples we remove the background from the resulting depth maps by creating a mask from image segmentation. Figure 6 shows the recovered depth maps compared to the outcome of the two previous methods of [9] and [12]. Apparently our results are the closest to the ground truth. While in the ‘Cloth’ and ‘Antlion’ test cases, the AHO based SFF and the Anisotropic approach show comparable results, in the other four examples our approach outperforms both methods. Close observation shows our capability to recover a smooth surface such as the ‘Cloth’, preserving sharp depth discontinuities at ‘Cloth’ and ‘Cube’ boundaries and coping with the challenging textures such as the stripes on the ‘Synth-Cone’ or the checkerboard on the ‘Cube’. Note the total failure of the Anisotropic approach in the ‘Cube’ case caused by propagation of errors from the strong edges toward the textureless regions, in the diffusion process. In the Antlion case (fifth column from left) the Bilateral method yields an inferior result while the Anisotropic approach totally fails in the Spider case due the small number of image samples in the stack (only 6 images). A close view on the ‘Spider’ depth map (forth column-first row) reveals the shortcomings of the Bilateral result presenting a piecewise-constant depth map and a false edge on the flower petal, where our approach provides a reliable piecewise-smooth surface.

We also perform a quantitative comparison for the synthetic tests based on depth map RMSE, reporting the results in Table 2. The proposed method presents superior results for

Object	Bilateral [9]	Anisotropic [12]	AHO+Aggreg+Recons
Cloth	0.67	0.35	0.41
Synth-Cone	0.5	1.79	0.27
Middlebury-Cones	0.84	0.65	0.64
Cube	0.6	3.63	0.28
Average	0.66	1.6	0.4

Table 2: RMSE comparison using the Bilateral filtering method of [9] and the Anisotropic approach of [12]. Best results are in bold.

all the test cases comparing the Bilateral [9] method. The Anisotropic approach [12] however

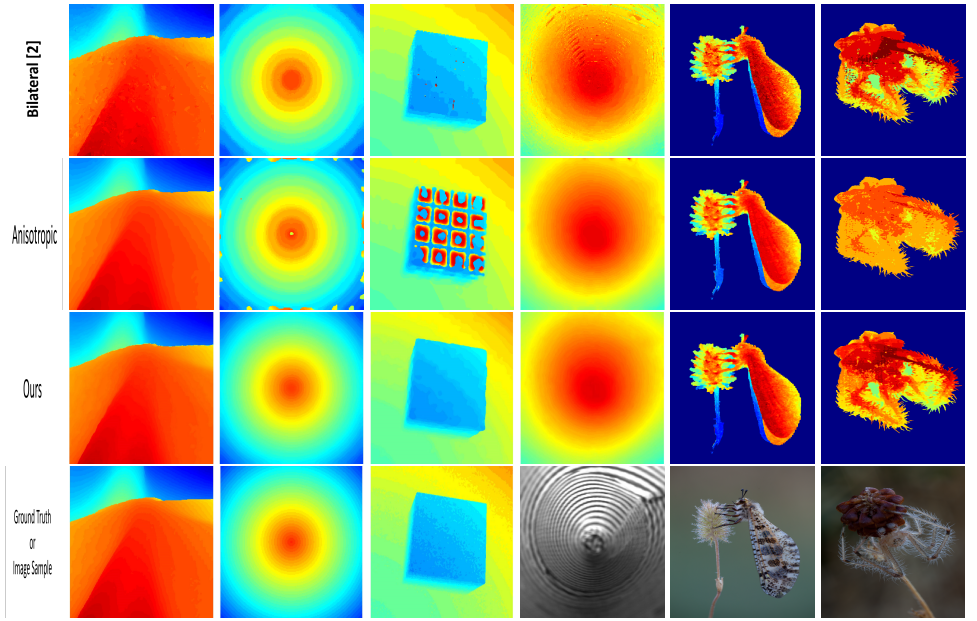


Figure 6: Comparison with previous methods. Color coded depth maps ranging from close (red) to far (blue). Note that except the Antlion case performing similarly to Anisotropic approach the proposed method shows the most reliable results. Zoom in for better visibility.

shows a lower RMSE only in the ‘Cloth’ test, characterized by a smooth shape. Our method further exhibits the lowest average RMSE.

6 Summary

We introduce a new framework for Shape from Focus based on the three known stages of initial focus measure estimation, focus volume aggregation and depth extraction. Our focus measure is composed of an ensemble of image derivatives in various orders and directions. The suggested multi-directional and high order derivatives along with a novel weighting function yields an adaptive focus quality measure that copes with the challenges in SFF such as high intensity edges, texture variation and depth discontinuities. A comparative evaluation shows that the new focus measure significantly outperforms the state-of-the-art. Considering the SFF pipeline we present superior results over two previously published methods. Another aspect of our method involves parallel computing. As the basis focus measures can be computed independently, the suggested scheme is ready for parallel computing.

References

- [1] Y. An, G. Kang, Il Jung Kim, H. S. Chung, and J. Park. Shape from focus through laplacian using 3D window. *International Conference on Future Generation Communication and Networking*, 2:46–50, 2008.
- [2] T. Aydin and Y.S. Akgul. A New Adaptive Focus Measure for Shape From Focus. In *BMVC*, 2008.
- [3] R. Ben-Ari. A unified approach for registration and depth in depth from defocus. *IEEE TPAMI*, 36(6):1041–1055, 2014.
- [4] A. Buades, B. Coll, and J. Morel. A non-local algorithm for image denoising. *CVPR*, 2:60 – 65, 2005.
- [5] T.S. Choi and M. Asif. Three-dimensional shape recovery from focused image surface. *ICASSP*, 6:3269–3272, 1999.
- [6] B. Fornberg. Generation of finite difference formulas on arbitrary spaced grids. *Mathematics of Computation*, 51(184):699–706, 1988.
- [7] R. Hariharan and a. N. Rajagopalan. Shape-from-focus by tensor voting. *IEEE Transactions on Image Processing*, 21(7):3323–3328, 2012.
- [8] E. Krotkov. Focusing. *International Journal of Computer Vision*, 1(3):223–237, 1988.
- [9] T. S. Choi M. T. Mahmood, M. Abdul. Optimal depth estimation by combining focus measures using genetic programming. *Information Sciences*, 181(7):1249–1263, 2011.
- [10] M. T. Mahmood. Shape from focus by total variation. *IVMSP*, pages 3–6, 2013.
- [11] M. T. Mahmood, S. O. Shim, and T. S. Choi. Shape from focus using principal component analysis in discrete wavelet transform. *Optical Engineering*, 48(5), 2009.
- [12] M. T. Mahmood, S. O. Shim, and T. S. Choi. Depth and image focus enhancement for digital cameras. *International Symposium on Consumer Electronics*, pages 50–53, 2011.
- [13] M. T. Mahmood, T. S. Choi, K. Young, and S. O. Shim. Estimating shape from focus by Gaussian process regression. *IEEE International Conference on Systems, Man and Cybernetics*, 1:1345–1350, 2012.
- [14] M. T. Mahmood, S. Mannan, and T. S. Choi. Sampling for shape from focus in optical microscopy. *IEEE TPAMI*, 34(3):564–73, 2012.
- [15] A. S. Malik and T. S. Choi. *Consideration of various noise types and illumination effects for 3D shape recovery*. 2008.
- [16] S. K. Nayar and Y. Nakagawa. Shape from focus. *TPAMI*, 16(8):824–831, 1994.
- [17] S. Pertuz, D. Puig, and M. A. Garcia. Analysis of focus measure operators for shape-from-focus. *Pattern Recognition*, 46(5):1415–1432, 2013.

- [18] H. Shoji, K. Shirai, and M. Ikehara. Shape from focus using color segmentation and bilateral filter. *IEEE 12th Digital Signal Processing Workshop*, pages 566–571, 2006.
- [19] M. Subbarao. Accurate Recovery of Three-Dimensional Shape from Image Focus. *TPAMI*, 1995.
- [20] M. Subbarao. Selecting the optimal focus measure for autofocusing and depth-from-focus. *TPAMI*, 20(8):864 – 870, 1998.
- [21] R. E. Thompson, D. R. Larson, and W. W. Webb. Precise nanometer localization analysis for individual fluorescent probes. *Biophysical Journal*, 82:2775–2783, 2002.
- [22] G. Yang and B. J. Nelson. Wavelet-based autofocusing and unsupervised segmentation of microscopic images. In *Int. Conference on Intelligent Robots and Systems*, volume 3, pages 2143–2148, 2003.
- [23] Y. Zhang, Y. Zhang, and C. Wen. New focus measure method using moments. *Image and Vision Computing*, 18(12):959–965, 2000.