Primal-Dual convex optimization in large deformation diffeomorphic registration with robust regularizers

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Non-rigid image registration is a highly ill-posed problem. This means that a number of qualitatively different transformations can achieve the same image similarity after registration. This justifies the vast literature on image registration methods with differences on transformation characterization, regularizers, image similarity metrics, optimization methods, and additional constraints [3].

In the last decade, diffeomorphic registration has arisen as a powerful paradigm for non-rigid image registration, with application to Computational Anatomy. Large Deformation Diffeomorphic Metric Mapping (LDDMM) [1] and Diffeomorphic Demons [4] are among the most widespread methods for diffeomorphic registration. In both methods, transformations are characterized to belong to an infinite dimensional Riemannian manifold of diffeomorphisms, parameterized by flows of smooth vector fields in the tangent space. The invertibility of the transformations is numerically guaranteed by the use of sufficiently smooth regularizers. Customarily, these methods regularize the problem with the L^2-norm of some physically meaningful differential expression of the vector fields, or with the Gaussian smoothing of the vector fields.

Simultaneously to the development of the diffeomorphic registration paradigm, the computer vision community has shown a growing interest in robust regularizers based on the Total Variation (TV) norm. The popularity of these regularizers has increased thanks to the availability of optimization methods for solving this challenging problem. The ability of TV based regularizers to preserve discontinuities has led these methods to occupy top positions in optical flow benchmark studies and non-rigid image registration evaluations.

The purpose of this article is to propose a method for primal-dual optimization of convexified LDDMM problems, formulated with robust regularizers and image similarity norms related to the TV norm. The method is based on Chambolle and Pock algorithm with diagonal preconditioning [2].

Let $Diff(\Omega)$ be the manifold of diffeomorphisms. Let V be the corresponding tangent space at the identity. Let $\mathcal{L} = Id - \gamma\Delta$ be the autoadjoint Laplacian operator associated to the scalar product in V, providing the Riemannian metric in $Diff(\Omega)$. The LDDMM variational problem is given by the minimization of the energy functional

$$E(v) = E_{\text{reg}}(v) + \alpha E_{\text{img}}(v) = \langle \mathcal{L}v, \mathcal{L}v \rangle_{L^2} + \alpha \| I_0 \circ \phi^v_{1,0} - I_1 \|^2_{L^2}. \quad (1)$$

As a result of the composition of the image with the diffeomorphism $\phi^v_{1,0}$, this energy functional is non-convex. In order to apply Fenchel-Duality principles in the primal-dual optimization of the problem, the functional needs to be transformed into a convex energy. This is done using the stationary parameterization of diffeomorphisms and approximating the Gateaux derivative $\partial_v \phi^v_{1,0}$ by $-D\phi^v_{1,0} \cdot v$ using the normal coordinate representation, yielding

$$E_{conv}(v) = E_{\text{reg}}(v) + \alpha E_{\text{img}}(I_0 \circ \phi^{v_0}_{1,0} - I_1 + \nabla(I_0 \circ \phi^{v_0}_{1,0})^T v_0 - \nabla(I_0 \circ \phi^{v_0}_{1,0})^T v). \quad (2)$$

In this work, we study three robust regularizers liable to provide acceptable results in diffeomorphic registration: Huber, V-Huber and Total Generalized Variation (TGV). The general algorithm that solves the different variational problems is given by

Data: $v^0 \in V$, $\overline{v}_0 \in V$, $p^0 \in P$, $q^0 \in Q$,
Σ_p, Σ_q, T preconditioning matrices, $\theta \in [0,1]$
Result: $v \in V$, $p \in P$, $q \in Q$ solutions of the primal-dual problem
for $n \leftarrow 0$ **to** *maxits* **do**

$$p^{n+1} = (Id + \Sigma_p \partial F^*)^{-1}(p^n + \Sigma_p K \overline{v}^n)$$
$$q^{n+1} = (Id + \Sigma_q \partial G^*)^{-1}(q^n + \Sigma_q A \overline{v}^n)$$
$$v^{n+1} = v^n - TK^* p^{n+1} - TA^* q^{n+1}$$
$$\overline{v}^{n+1} = v^{n+1} + \theta(v^{n+1} - v^n)$$

end

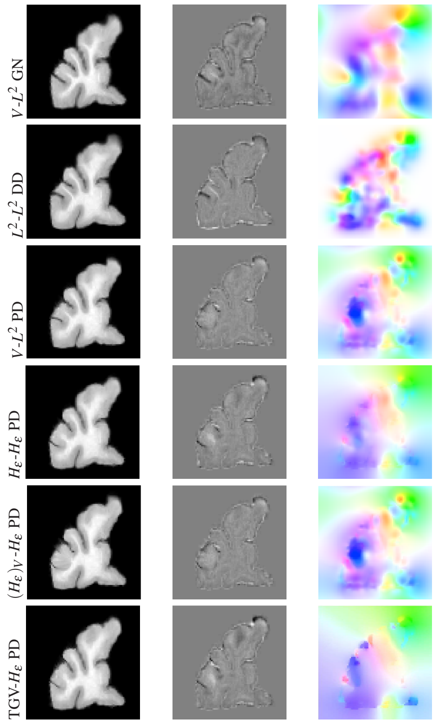

Figure 1: **2D MRI experiment.** Image registration results with the state of the art methods, and the proposed method and the considered regularizers. From left to right, warped sources ($I_0 \circ \phi^v_{1,0}$), residuals ($I_0 \circ \phi^v_{1,0} - I_1$) and velocity fields for the methods considered in the comparison.

The method is compared in a complex geometry 2D MRI data set with state of the art optical flow and diffeomorphic registration methods. In addition, the 3D version of the method is evaluated with the manual segmentations of the Non Rigid Image Registration Evaluation Project (NIREP) database.

Results in the 2D MRI data set have demonstrated that diffeomorphic solutions can be obtained for Huber, V-Huber and TGV regularizers, despite the preservation of discontinuities favored by the robust regularizers. The method has shown to be able to perform similarly to state of the art diffeomorphic registration methods in terms of the image similarity after registration.

The evaluation in the NIREP database has shown a comparable performance for V-Huber regularizer with respect to the original V-L^2 variational formulation and L^2-L^2 log-domain diffeomorphic Demons. For each region, the results obtained by V-Huber regularizer were with the best scoring methods. The performance of TGV regularizer was usually located above the worst performing methods and slightly below the best performing methods. Moreover, primal-dual optimization methods (with the exception of the TGV regularized) were more efficient than diffeomorphic Demons, widely used because of its computational efficiency.

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