Hierarchical-Hybrid Shape Representation for Medical Shapes

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Recently, shape analysis has become of increasing interest in the medical community due to its potential in capturing the morphological variations across a population. The high quality 3D images captured can be used to extract 3D shape of the organs. 3D models of organs can also be used for training personnel, for visualization during image guided interventions and in simulations. A compact shape model that has implicit and explicit forms will aid in some of these medical use-cases.

We propose a compact hybrid shape model as a combination of Extended Superquadrics (ESQ) [1] and Radial basis interpolation function (RBF). The hybrid shape model in its parametric form is given as $(f(\theta,\phi) = h(\theta,\phi)+g(\theta,\phi))$. h is the extended superquadric function and g is radial basis interpolation function. The points on the surface of the shape are given by

$$(X,Y,Z) = f(\theta,\phi)$$

$$\begin{aligned} X &= a \times sign(cos\theta cos\phi)|cos\theta|^{\varepsilon_2}|cos\phi|^{\varepsilon_1} + r.cos\theta cos\phi \\ Y &= b \times sign(sin\theta cos\phi)|sin\theta|^{\varepsilon_2}|cos\phi|^{\varepsilon_1} + r.sin\theta cos\phi \\ Z &= c \times sign(sin\phi)|sin\phi|^{\varepsilon_1} + r.sin\phi \end{aligned} \tag{1}$$

$$-\pi/2 \leq \theta \leq \pi/2, -\pi \leq \phi \leq \pi.$$

Constants a, b and c give the extent of the ESQ. Exponents ε_1 and ε_2 are the shape parameters of the ESQ. The exponents are functions of θ(azimuth) and ϕ(elevation) respectively, modeled using cubic splines. Offset r is a function of both θ and ϕ, modeled by RBF. An ESQ at an arbitrary position also has translation and rotation parameters. The implicit representation of the shape model is given as

$$\begin{aligned} F(x,y,z)^{\frac{\varepsilon_1}{2}} + \sum_{j=1}^{N} w_j \lambda(d_j) = 1 \\ F(x,y,z) = \left[\left(\frac{|x|}{a}\right)^{2/\varepsilon_2} + \left(\frac{|y|}{b}\right)^{2/e2}\right]^{\varepsilon_2/\varepsilon_1} + \left(\frac{|z|}{c}\right)^{2/\varepsilon_1} \end{aligned} \tag{2}$$

λ is a Gaussian with compact support, d_j is the cosine distance from the j^{th} RBF center, w_j is a weight associated with j^{th} RBF center.

To handle concavities in the shape we propose to hierarchically divide the shape into approximately convex parts [2, 3]. The two step process involves first, finding pairs of points, called *mutex pairs*, which cannot belong to the same convex part. Then, finding cutting planes that divide the shape into parts that have no *mutex pairs*. Given a shape S, points $x \in S, y \in S$ are said to be mutually exclusive if there exists a plane p such that x, y lie on p and are disconnected in the contour map produced by the projection of shape S onto p. A weight is associated with each *mutex pair* which is a measure of concavity of the shape between the pair of points. For each *mutex pair* a candidate cutting plane is defined as a plane that bisects the pair and has its normal parallel to the pair. The following heuristics are defined to find the minimum number of cutting planes required to remove all *mutex pairs*: 1) The cut reduces the total non-convexity of the shape; 2) The cut results in parts where at least one is likely to contain no mutex-pairs.

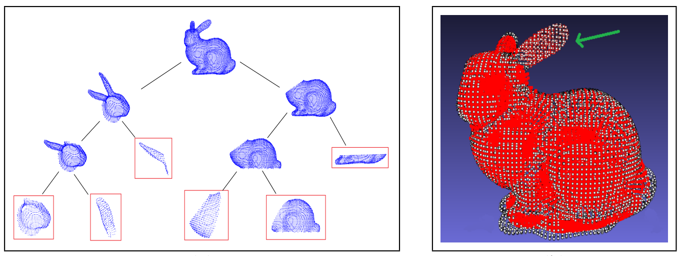
Given a shape as point cloud or a mesh, the method to build the hierarchical-hybrid shape representation is as follows. The input point cloud is hierarchically decomposed into approximately convex parts. The hybrid shape model is then fit to each of parts. LM method for non-linear least square fit is used to minimize the error of fit function derived from the implicit form of the shape model. The error of fit for a point P is defined as the distance from point P to a point Q on the ESQ surface such that $P-O = \beta(Q-O)$ where O is the center of the ESQ. The EoF is given as,

$$EOF = \sum_{i=1}^{Ndata} ||(x_i,y_i,z_i)|| \times |1 - F(x_i,y_i,z_i)^{\varepsilon_1/2}| + C_1 + C_2 + C_3 \tag{3}$$

Additional constraints C_1, C_2 and C_3 on the parameters of ESQ are also added to the EoF.

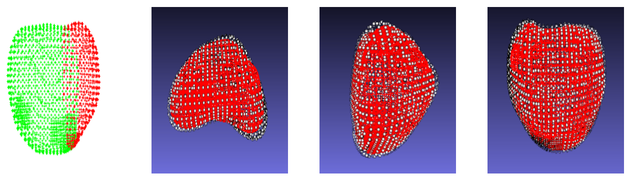
A binary tree is used for the shape representation. Each node in the tree represents a part of the shape. The intermediate nodes contain links to its constituent parts and holds the 2D shape of intersection between its constituent parts. The leaf nodes contain the hybrid shape representation of the part.

To reconstruct the shape, a bottom up approach is employed. The explicit form of the shape model is used to generate points/meshes for each part. The 2D intersection shape is then used to blend the parts together. Fig. 1 and Fig. 2 show fitting of the shape representation to data.



(a) (b)

Figure 1: Fitting our shape model to Stanford Bunny (8454 vertices): (a) Approximate convex decomposition, parts shown inside red box; (b) Shown in *Red* is HSSR fit, ground truth boundary points are shown in *White* and *Green* arrow points to the part that was retained as points due to large error of fit of the shape model.



(a) (b) (c) (d)

Figure 2: Shape representation for Prostate shape. (a) Prostate split into approximately convex parts; (b), (c) and (d) are axial, sagittal and coronal views of prostate. Red represents the mesh generated form HSSR. Shown in White are the ground truth organ boundary points

[1] Lin Zhou, and Chandra. Kambhamettu. "Extending superquadrics with exponent functions: modeling and reconstruction", IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 1999.

[2] Hairong Liu, Wenyu Liu, Latecki, L.J., "Convex shape decomposition," IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2010.

[3] Zhou Ren, Junsong Yuan, Chunyuan Li, Wenyu Liu, "Minimum near-convex decomposition for robust shape representation," IEEE International Conference on Computer Vision (ICCV), 2011.