Multi-scale Graph-based Guided Filter for De-noising Cryo-Electron Tomographic Data

Shadi Albarqouni[1,2]
http://campar.in.tum.de/Main/ShadiAlbarqouni

Maximilian Baust[1]
http://campar.in.tum.de/Main/MaximilianBaust

Sailesh Conjeti[1]
http://campar.in.tum.de/Main/SaileshConjeti

Ashraf Al-Amoudi[2]
https://www.dzne.de/en/sites/bonn/research-groups/al-amoudi.html

Nassir Navab[1,3]
http://campar.in.tum.de/Main/NassirNavab

[1] Chair for Computer Aided Medical Procedure (CAMP),
Technische Universität München (TUM)
Munich, Germany

[2] Deutsches Zentrum für Neurodegenerative Erkrankungen
(DZNE),
Bonn, Germany

[3] Chair for Computer Aided Medical Procedure (CAMP),
Johns Hopkins University
Baltimore, MD, USA.

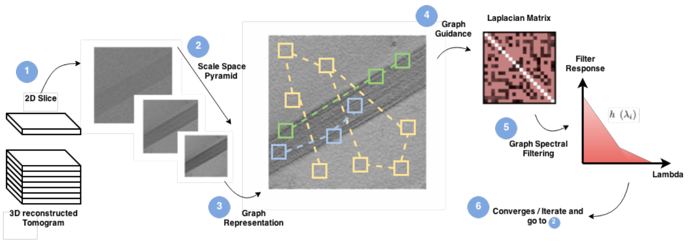


Figure 1: **MG^{2}F Framework:** A noisy image slice from the 3D reconstructed tomogram is fed to the algorithm, where the graph is built on a selected scale space image (i.e. coarse grid) acting as a guidance for the regularized graph spectral filter.

1 Introduction

Cryo-electron tomography (CET) is a powerful imaging technique in biological sciences which bridges the gap between the molecular and the cellular structural biology [5], giving a better understanding of protein interactions and thus better drug delivery strategies. In principle, similar to Computed Tomography (CT) in Medical Imaging, the acquired projections at limited angles are reconstructed back to create the 3D object, however, these projections are extremely noisy and have a low contrast. Therefore, many conventional filters failed in smoothing the background while preserving edges and interesting objects, which makes developing a denoising algorithm is very desirable for better interpretation.

We show in this paper how our methodology meets the hypothesis: a) By using a multi-scale pyramid for guidance we are able to detect meaningful scales and use them for guidance without oversmoothing fine scale structures. b) Using a patch-based approach, we can take advantage of redundant structures in the whole image rather than using a pre-defined spatial window for averaging similar pixels or patches. This way, we can preserve the local and global consistencies. c) By deriving explicit solution formulas for computing the intermediate filtering results we obtain an efficient algorithm.

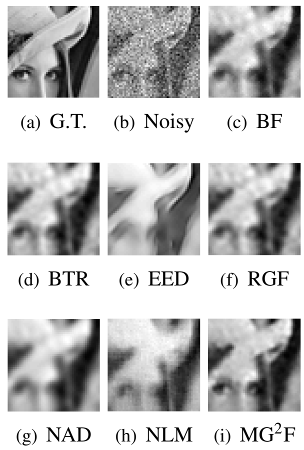
2 Methodology

Given a noisy image I_η, we collect N overlapping patches, which can be seen as data points $v = \{v_1, v_2, ..., v_N\} \in \mathbb{R}^{n\times N}$ lying on a manifold $\mathcal{M}$ embedded in $\mathbb{R}^n$ space such that $v = EI_\eta$, where E is an operator collecting patches and vectorize it, cf. Figure1. The relation between the data points can be represented by a k-NN connected, undirected, and weighted graph $G = \{v, \varepsilon, \omega\}$, where ε is the set of edges, and ω is the set of edge weights.

These weights are assigned using a heat kernel, however, the distance between these patches is computed on a certain structure scale σ_s where the noise manifest itself and can be used as a guidance for the graph spectral filter $h(\lambda_i)$, which is computed based on the eigenvalue decomposition of the normalized Laplacian matrix $\tilde{L_{\sigma_s}} := U\Lambda U^T$. This way, we can formulate the denoising problem as follows:

$$\hat{I}_f = \arg\min_{I_f} \left\{ \frac{1}{2}\|I_f - I_\eta\|_2^2 + \alpha S_{\sigma_s}(I_f) \right\}, \qquad (1)$$

where $\alpha > 0$ is the regularization parameter and $S_{\sigma_s}(I_f) = \frac{1}{2}Tr\left(vL_{\sigma_s}v^T\right)$ is the graph guidance regularization term.



Algorithm (Parameters)	PSNR (dB)
Bilateral (BF) [7] (σ_i=0.5, σ_r=1.5, W=10)	17.49
Beltrami (BTR) [2] (δ=0.1, *iter*=10)	17.37
EED [6] (ρ=4, *iter*=30)	11.27
NAD [4] (*iter*=10, κ=0.3)	16.50
NLM [1] (P=7, W=21, σ_s=4σ_n)	12.11
RGF [3] (σ_i=0.5, σ_r=1.5, *iter*=10)	17.49
MG^{2}F (α=0.8, *iter*=4, σ_h=0.1)	**17.78**

Figure 2: **Photographic Image:** Results of different algorithms on Lena image(128X128, SNR=7) along with a tabulated comparison to the proposed MG^{2}F filter.

The closed form solution can be written as

$$\hat{I}_f = E^T\left(\sum_{i=1}^{N} \frac{1}{(1+\alpha\lambda_i)} u_i \hat{v}_i\right) = E^T\left(\frac{1}{I+\alpha\tilde{L}_{\sigma_s}}\right) EI_\eta, \qquad (2)$$

where E^T denotes the reshaping process of the previously vectorised patches, and the spectral response of the filter $h(\lambda_i) = 1/(1+\alpha\lambda_i)$ controls the frequency decay and thus the degree of smoothness. A connection to classical filters and the sensitivity analysis are discussed in details.

3 Results

To give a good illustrative example, we run the algorithm on Lena image, which corrupted by an (i.i.d) Gaussian noise resulting in SNR of 7. Different algorithms are applied on this image, results are shown in Figure 2 for the cropped images. It is clear that our method gives an outperforming PSNR indicating for better contrast. A simulated and real CET data experiments in 2D and 3D are presented in the paper. Using the gold-standard metrics, we show that our denoising algorithm significantly outperforms the state-of-the-art methods such as NAD, NLM and RGF in terms of noise removal and structure preservation.

4 References

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