Combination of paths for interactive segmentation

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Boundary-based interactive segmentation methods aim to build a closed contour, very often using minimum cost paths linking user-provided ordered landmark points. While discrete methods rely on a graph modeling of the image, continous models address the problem by means of curve optimization. Among them, the minimal path method [1] find curves of minimal length defined from an edge-based cost function $P = g + w$, where g is a decreasing function of the image gradient magnitude defined over image domain $\mathcal{D}$, and w is a regularizing constant. Given two points a and b, the *geodesic path* between them,

$$\gamma_{a,b} = \underset{\mathcal{C}\subset\mathcal{D}}{\operatorname{argmin}} \left\{ L[\mathcal{C}] = \int_0^1 P(\mathcal{C}(u)) \left\| \mathcal{C}'(u) \right\| \mathrm{d}u \right\} \text{ s.t. } \begin{cases} \mathcal{C}(0) = a \\ \mathcal{C}(1) = b \end{cases} \quad (1)$$

can be obtained by considering the geodesic distance map, or *minimal action map*, $U_a : \mathcal{D} \to \mathbb{R}^+$, efficiently computable by the Fast Marching (FM) method [3]. The minimal path between a and b can be extracted by a gradient descent on U_a, starting from b until a is reached. This can fail to extract the desired curve, for instance when P is too noisy or when the length of the target curve is too important. To address this issue, several approaches build a *piecewise-geodesic curve*, i.e. a concatenation of geodesic curves connecting pairs of successive landmark points or *vertices*. Among them, the geodesically linked active contour (GLAC) model [2] is generated by joining end-to-end geodesic paths built from a set of vertices $\mathcal{V} = \{\mathbf{v}_i\}_{1\leq i\leq n}$. The GLAC approach consists in finding the sequence of n vertices which generates a piecewise-geodesic curve minimizing a weighted sum of an edge-fitting term and a region homogeneity term. While this model allows to blend the benefits of minimal paths and region-based terms and is relatively robust to local minima, it turns out to have a significant drawback, as its initial state is not necessarily a simple closed curve. This can occur for instance when the initial vertices are unevenly distributed around the target boundary. In this case, geodesics gather on particular sides of the target boundary, overlapping each other.

Hence, we propose a more relevant contour construction preserving the advantages of piecewise-geodesic curves. Assuming that a set of several possible relevant paths is available for each pair of successive vertices, we may select a single path from each set and combine these paths in order to build the best boundary curve. Let $\mathcal{A}_i$ be a set of K_i *admissible paths* linking the two vertices,

$$\mathcal{A}_i = \{\gamma_{i,j}\}_{1\leq j\leq K_i},$$

which we refer to as *admissible set*. Curve $\gamma_{i,1}$ is the geodesic path between $\mathbf{v}_i$ and $\mathbf{v}_{i+1}$. To generate supplemental paths $\gamma_{i,j}, 2 \leq j \leq K_i$, we propose an approach based on the extraction of *saddle points*. When propagation is performed from two source points a and b, yielding the *combined action map* $U_{a,b} = \min(U_a, U_b)$, the two propagation fronts meet at the **saddle points**, which are the mountain passes on the different roads travelling from a to b. For each pair of successive vertices $(\mathbf{v}_i, \mathbf{v}_{i+1})$, the combined action map is propagated simultaneously from $\mathbf{v}_i$ and $\mathbf{v}_{i+1}$, generating two monotonically advancing fronts. Among the locations where the two fronts collide, those which are also local minima of $U_{\mathbf{v}_i,\mathbf{v}_{i+1}}$ in the direction orthogonal to the propagation direction are kept as the K_i saddle points. Two gradient descents are performed both sides apart from each saddle point and the resulting assembled paths make up the admissible set $\mathcal{A}_i$. The combined action map, the detected saddle points and corresponding paths are shown in Fig. 1 (top row, left).

The computation of an admissible closed contour consists in picking up one path per admissible set $\mathcal{A}_i$ such that the resulting concatenation of selected paths minimizes energy E, designed to penalize contours that are not simple, poorly fitting to image edges or enclosing regions with

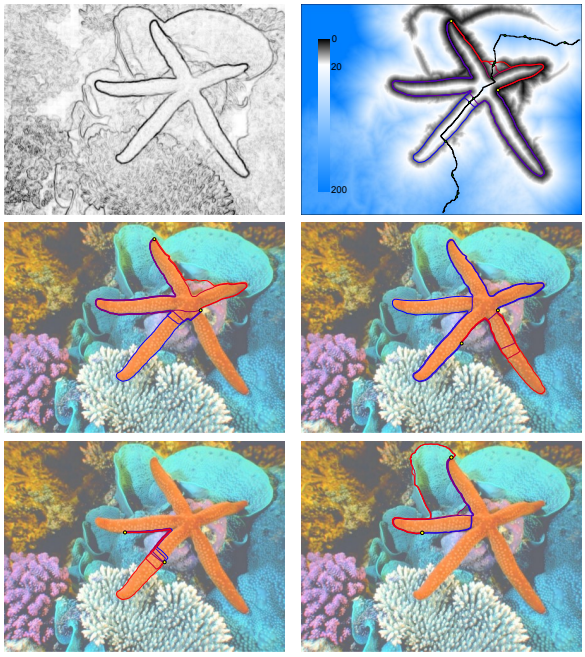

Figure 1: Top row: potential P. Medial curve (black) and saddle points (green) with corresponding paths drawn over combined action map. Middle and bottom row: Admissible set for each pair of successive vertices with $n = 4$. Paths are sorted according to their exteriority

overlapping color distributions:

$$E[\Gamma] = E_{\text{simplicity}}[\Gamma] + \omega_{\text{edge}} E_{\text{edge}}[\Gamma] + \omega_{\text{region}} E_{\text{region}}[\Gamma].$$

Basically, $E_{\text{simplicity}}$ is a novel term involving measures of self-overlapping and self-intersection. The edge term sums up a decreasing function of the gradient magnitude along the contour, whereas the region term is based on the Bhattacharya coefficient between color distributions of inner and outer regions. To determine the best sequence of labels $\{x_1, \ldots, x_n\}$, instead of an exhaustive search, we propose a greedy search in $O(n^2 K_{\max})$, where $K_{\max}$ is the maximum number of admissible paths over all sets $\mathcal{A}_i$. In each admissible set $\mathcal{A}_i$, paths are sorted according to increasing *exteriority* $\mathcal{X}$, i.e. the signed area, calculated with Green's theorem, formed by a given path $\mathcal{C}$ and the line segment from $\mathcal{C}(1)$ returning to $\mathcal{C}(0)$. The vertices being located clockwise, admissible paths are sorted from the innermost to the outermost (see Fig. 1, middle and bottom rows). Starting from an initial labelling corresponding to the most interior configuration, labels are changed according to a local search, by iteratively testing candidate labellings. At each iteration, candidate sequences are tested that differ from a single label from the current labelling, by increasing labels only. The candidate sequence leading to the smallest energy is kept along the iterations.

[1] L. Cohen and R. Kimmel. Global minimum for active contour models: a minimal path approach. *IJCV*, 24(1):57–78, 1997.

[2] J. Mille and L. Cohen. Geodesically linked active contours: evolution strategy based on minimal paths. SSVM, volume 5567 of *LNCS*, pages 163–174, Voss, Norway, 2009. Springer.

[3] J.A. Sethian. A fast marching level set method for monotonically advancing fronts. *PNAS*, 93(4):1591–1595, 1996.