Single-Image Super-Resolution via Adaptive Joint Kernel Regression

Chen Huang
yach23@gmail.com
Xiaoqing Ding
dingxq@tsinghua.edu.cn
Chi Fang
fangchi@tsinghua.edu.cn

State Key Laboratory of Intelligent Technology and Systems,
Tsinghua National Laboratory for Information Science and Technology,
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

Single image super-resolution (SR) methods can be broadly categorized into three classes: interpolation-based methods, reconstruction-based methods [7], and example-based methods [2, 3, 6]. The reconstruction-based methods often incorporate prior knowledge to regularize the ill-posed problem. For example, Zhang *et al*. [7] assembled the Steering Kernel Regression [5] (SKR)-based local prior and Nonlocal Means [1] (NLM)-based nonlocal prior. The example-based methods strongly rely on the chosen dictionary for satisfactory results. This paper focuses on learning good image priors and robust dictionaries for SR reconstruction. Among the extensively studied natural image priors, we choose to exploit the *local structural regularity* prior and *nonlocal self-similarity* prior in a coherent framework.

We propose in this paper an Adaptive Joint Kernel Regression (AJKR)-based prior to simultaneously exploit both image statistics. Our approach differs from others in several ways: 1) we combine a set of NLM-generalized local kernel regressors, which are more consistent with our nonlocal collaborative framework; 2) the proposed *regional redundancy* measure introduces higher-order statistics at the region level for each regression group, making the overall framework more adaptive (see Fig. 1(a)); and 3) an adaptive PCA-based dictionary learning scheme is adopted to bridge the gap of dictionaries learned online and offline by mixing them, and more importantly such a scheme together with its induced sparsity prior can adapt to the AJKR process in response to the regional redundancy measure (see the block diagram in Fig. 1(b)).

The imaging model for SR (assume $\mathbf{Y} \in \mathbb{R}^m$ is low resolution (LR) image, $\mathbf{X} \in \mathbb{R}^n$ is high resolution (HR) image) is usually expressed as

$$\mathbf{Y} = \mathbf{DHX} + \mathbf{V}, \tag{1}$$

where $\mathbf{D} \in \mathbb{R}^{m\times n}$ and $\mathbf{H} \in \mathbb{R}^{n\times n}$ are the downsampling matrix and blurring matrix respectively, and $\mathbf{V} \in \mathbb{R}^m$ is the Gaussian noise. To incorporate the AJKR prior, we first generalize NLM in the kernel regression framework

$$\hat{\mathbf{a}}_i = \arg\min_{\mathbf{a}} \|\mathbf{Y}_i - \Phi\mathbf{a}\|^2_{\mathbf{W}_i^N}, \tag{2}$$

where $\mathbf{Y}_i$ is a patch of neighboring pixels around location $\mathbf{x}_i$, and Φ and $\mathbf{a}$ are the regression bases and coefficients. $\mathbf{W}_i^N = diag\left[w_{i1}^N, w_{i2}^N, \ldots, w_{iL}^N\right]$ is the patch similarity-based kernel weight matrix with

$$w_{ij}^N = exp\left(-\frac{\|\mathbf{Y}_i - \mathbf{Y}_j\|^2_{\mathbf{W}_G}}{h_n^2}\right). \tag{3}$$

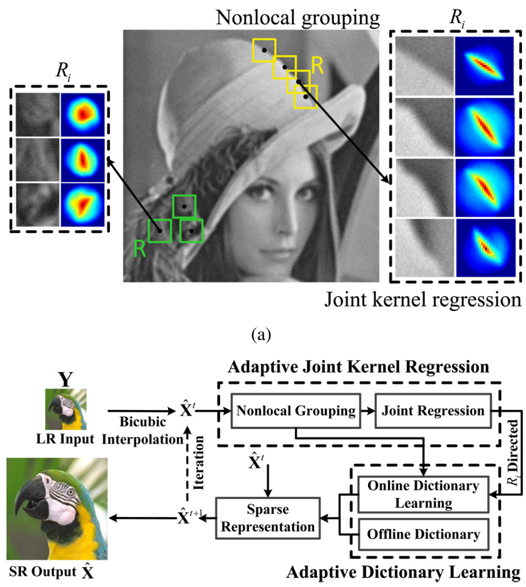
This procedure paves the way for our framework "harmonization" in a complete nonlocal sense. Then we combine all the local regressors over similar patches found in the nonlocal range $\mathcal{P}(\mathbf{x}_i)$ (sufficiently large image region), and weight them by their mutual similarities w_{ij}^N

$$\hat{\mathbf{a}}_i = \arg\min_{\mathbf{a}} \sum_{j\in\mathcal{P}(\mathbf{x}_i)} w_{ij}^N \|\mathbf{Y}_j - \Phi\mathbf{a}\|^2_{\mathbf{W}_j^N}. \tag{4}$$

Such a scheme allows us to exploit both local/nonlocal image priors collaboratively, and can act as a regularization term for SR reconstruction. We further propose a regional redundancy measure $R_i = \sum_{j\in\mathcal{P}(\mathbf{x}_i)} \left(w_{ij}^N\right)^2$ to determine the confidence of these regression groups and thus control their relative effects of regularization using $\mathbf{R} = diag\left[R_1, R_2, \ldots, R_n\right]$. This leads to the matrix form of our SR optimization function

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} \|\mathbf{Y} - \mathbf{DHX}\|_2^2 + \lambda\,\|(\mathbf{I}-\mathbf{K})\mathbf{X}\|^2_{\mathbf{R}}, \tag{5}$$

where $\mathbf{K}$ packs the *equivalent regression kernels* derived from the solutions of Eq. (4). Although previous models [6, 7] also considered the unified priors, ours is much more coherent and adaptive with the redundancy measure that introduces higher-order statistics.



(b)

Figure 1: (a) Graphical illustration of the AJKR process; (b) Block diagram of our SR algorithm.

The AJKR framework can further benefit from dictionary-based methods. We follow the adaptive PCA strategies in [7] and [2] to learn the offline and online dictionaries $\mathbf{B}_0$ and $\mathbf{B}_1$, respectively. By combining the two dictionaries ($\mathbf{B} = [\mathbf{B}_0\ \mathbf{B}_1]$) and enforcing sparsity under the dictionary representation (α), we can reach to our final optimization function

$$\hat{\alpha} = \arg\min_{\alpha} \|\mathbf{Y} - \mathbf{DHB}\circ\alpha\|_2^2 + \lambda\,\|(\mathbf{I}-\mathbf{K})\mathbf{B}\circ\alpha\|^2_{\mathbf{R}} + \beta\,\|\alpha\|_1. \tag{6}$$

Compared with similar schemes for combined dictionaries (*e.g*. [4]), our combined dictionary together with its induced sparsity prior distinguishes itself by offering the ability to interact with the regression prior in response to the redundancy measure in a unified framework.

Details of SR implementation of this method is described in this paper. The large variety of experiments show that our method achieves state-of-the-art performance and is robust with minimum artifacts even under large amounts of noise and blur. The extension to image deblurring also produces outstanding results as compared with other methods.

[1] A. Buades, B. Coll, and J. M. Morel. A non-local algorithm for image denoising. In *Proc. CVPR*, pages 60–65, June 2005.

[2] P. Chatterjee and P. Milanfar. Clustering-based denoising with locally learned dictionaries. *IEEE TIP*, 18(7):1438–1451, July 2009.

[3] Weisheng Dong, Lei Zhang, Guangming Shi, and Xiaolin Wu. Image deblurring and super-resolution by adaptive sparse domain selection and adaptive regularization. *IEEE TIP*, 20(7):1838–1857, 2011.

[4] Daniele Perrone, Avinash Ravichandran, René Vidal, and Paolo Favaro. Image priors for image deblurring with uncertain blur. In *Proc. BMVC*, pages 114.1–114.11, 2012.

[5] H. Takeda, S. Farsiu, and P. Milanfar. Kernel regression for image processing and reconstruction. *IEEE TIP*, 16(2):349–366, 2007.

[6] Haichao Zhang, Jianchao Yang, Yanning Zhang, and Thomas S. Huang. Non-local kernel regression for image and video restoration. In *Proc. ECCV*, pages 566–579, 2010.

[7] Kaibing Zhang, Xinbo Gao, Dacheng Tao, and Xuelong Li. Single image super-resolution with non-local means and steering kernel regression. *IEEE TIP*, 21(11):4544–4556, 2012.