Solving Person Re-identification using Efficient Gibbs Sampling

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1 The Model

Given a set of observations $\mathcal{X} = \{\mathbf{x}_i\}_{i=1}^{N}$ across multiple camera views, where each observation corresponds to a complete trajectory within a camera FOV, person re-identification can be defined as the problem of identifying the set of indicator variables associated with the observations $\mathbf{z} = \{z_i\}_{i=1}^{N}$. To identify the label $z_i \in [1, \ldots, Z]$ associated with each trajectory, we utilise a combination of visual information, the appearance features, and the transitions between the cameras. To address the issues associated with appearance-based methods in our proposed person re-identification algorithm, we model each person's appearance using camera-specific illumination and camera gain. We identify the indicator labels by performing Bayesian inference using Gibbs sampling. Each observation $\mathbf{x}_i = \{\mathbf{a}_i, e_i, t_i, l_i\}$ consists of: $l_i \in [1, \ldots, L]$ the camera that records the observation; the time of entry e_i in a camera's FOV; the time of leaving the camera's FOV t_i; and the observed appearance features $\mathbf{a}_i$. We define the likelihood as

$$p(\{\mathbf{x}_i\}_{i=1}^{N} | \{z_i\}_{i=1}^{N}) = \prod_{j=1}^{N} p(\mathbf{a}_j | z_j, l_j)\, p(l_j | \{l_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j})\, p(e_j | \{t_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j}) \tag{1}$$

where $p(\{\mathbf{a}_j\}_{i=1}^{N} | z_j, l_j)$ is modelled as $\mathbf{a}_i = g_l(\mathbf{r}_z + \mathbf{w}_l)$, where g_l is the multiplicative gain constant of camera l, $\mathbf{r}_z$ is the RGB color model, averaged over the entire trajectory, $\mathbf{w}_l$ is the illumination noise associated with camera l, and the terms are distributed as:

$$g_l \sim \text{Gamma}(\alpha_l^g, \beta_l^g), \text{ which we approximate as } \mathcal{N}(\mu_l^g, (\Lambda_l^g)^{-1}) \tag{2}$$

$$\mathbf{r}_z \sim \mathcal{N}(\mu_z, (\Lambda_z)^{-1}) \tag{3}$$

$$\mathbf{w}_l \sim \mathcal{N}(\mu_l^w, (\Lambda_l^w)^{-1}) \tag{4}$$

The transitions between cameras are modelled as

$$l_j \mid \{l_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j-1} \sim \text{Mult}(l_j; \theta_{l_i}), i : z_i = z_j \wedge z_k \neq z_j, i < k < j \tag{5}$$

$$e_j \mid \{t_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j} \sim \mathcal{N}(e_j - t_i; \mu_{l_i, l_j}, \Lambda_{l_i, l_j}^{-1}), i : z_i = z_j \wedge z_k \neq z_j, i < k < j \tag{6}$$

It is clear from its structure (Fig 1a) that this model does not allow for efficient inference, since the Markov blanket of any observation is the complete set of observations and indicator variables preceding it. Yet if the latent indicator variables are known, the observations of a person become independent of all other persons, and the model becomes much simpler (Fig 1b)

2 Illumination per Person: Derivation of the Precision Conditional Distribution

The posterior distribution over the camera precision parameter for camera l in terms of the likelihood distribution and prior distribution is given as,

$$p(\Lambda_l^w | A_l, \alpha_l, \beta_l, \mu_l^w, \mu_z, l, z) = p(A_l | \alpha_l, \beta_l \mu_l^w, \Lambda_l^w, \mu_z, \Lambda_z, z, l) p(\Lambda_l^w) \tag{7}$$

We define a Wishart distribution as the prior over the illumination precision parameter Λ_l^w and a Gaussian distribution for the likelihood. Note that for convenience, we approximate the gain parameters α_l, β_l in the above Eqn 7 terms of a Normal distribution $N(\mu_l^g, \lambda_l^g)$, where $\mu_l^g = \alpha_l \beta_l$ and $\lambda_l^g = \frac{1}{\alpha_l \beta_l^2}$. Thus, eqn 7 can now be re-wriiten as

$$p(\Lambda_l^w | A_l, \mu_l^g, \lambda_l^g, \mu_l^w, \mu_z, l, z) = p(A_l | \mu_l^g, \lambda_l^g, \mu_l^w, \Lambda_l^w, \mu_z, \Lambda_z, z, l) p(\Lambda_l^w) \tag{8}$$

where, μ_z, Λ_z represent the mean and precision of the Gaussian distribution defined over the z appearance, and $\mathrm{A}_l = \{a_i\}_{i=1}^{N_l}$represe the set of observations and N_l corresponds to the number of trajectories observed by camera l. The Gaussian likelihood distribution is given as,

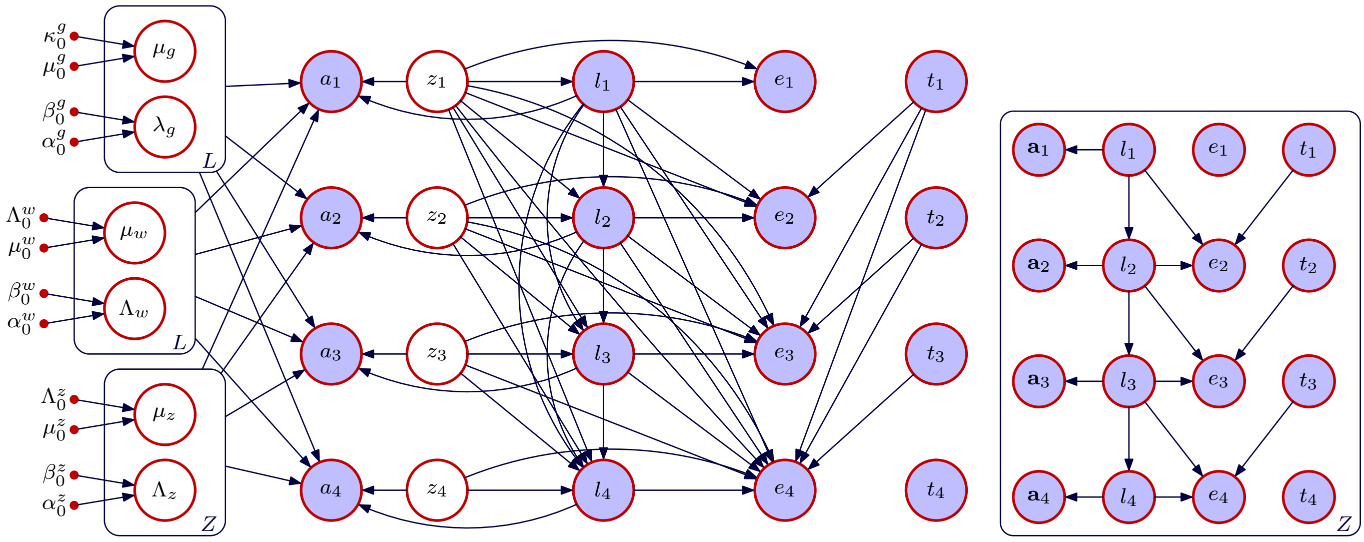

Figure 1: (a) Full graphical model of our probabilistic person re-identification algorithm and (b) Graphical model if the latent variables z_i are known.

$$p(A_l|\mu_l^g,\lambda_l^g,\mu_l^w,\Lambda_l^w,\mu_z,\Lambda_z,z,l)=\prod_{i=1}^{N_l}N(a_i|\mu_l^g(\mu_z+\mu_l^w),\lambda_l^g(\Lambda_z+\Lambda_l^w)) \tag{9}$$

The Wishart prior distribution in 8 is given as,

$$p(\Lambda_l^w|\alpha_0^w,\beta_0^w)\propto|\Lambda_l^w|^{\frac{(\alpha_0^w-4)}{2}}exp\left(-\frac{1}{2}tr(\beta_0^w\Lambda_l^w)\right) \tag{10}$$

where, α_0^w,β_0^w represent the fixed hyper-parameters for the prior distribution. Since, the Wishart distribution is a conjugate prior for the Gaussian likelihood, the resultant posterior is also a Wishart distribution given as,

$$p(\Lambda_l^w|\alpha_n^w,\beta_n^w)\propto|\Lambda_l^w|^{\frac{(\alpha_n^w-4)}{2}}exp\left(-\frac{1}{2}tr(\beta_n^w\Lambda_l^w)\right) \tag{11}$$

where, α_n^w,β_n^w represent the hyper-parameters for the posterior distribution. To solve for the posterior distribution, we first combine the likelihood distribution (eqn 9) with the prior distribution (10). Thus the resultant posterior distribution is given as,

$$\propto\prod_{i=1}^{N_l}N(a_i|\mu_l^g(\mu_z+\mu_l^w),\lambda_l^g(\Lambda_z+\Lambda_l^w))|\Lambda_l^w|^{\frac{(\alpha_0^w-4)}{2}}exp\left(-\frac{1}{2}tr(\beta_0^w\Lambda_l^w)\right) \tag{12}$$

$$\propto\prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z+\Lambda_l^w)|^{n_l^z/2}\right]exp\left(-\frac{1}{2}\left\{\sum_{z=1}^{Z}\sum_{i=1}^{n_l^z}(a_i-\mu_l^g(\mu_z+\mu_l^w))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(a_i-\mu_l^g(\mu_z+\mu_l^w)\right\}\right)$$
$$\ldots|\Lambda_l^w|^{\frac{(\alpha_0^w-4)}{2}}exp\left(-\frac{1}{2}tr(\beta_0^w\Lambda_l^w)\right) \tag{13}$$

Note that in Eqn 13, we choose to factorize the likelihood distribution according to each person observed in the camera. In Eqn 13 n_l^z represents the number of trajectories belonging to person z observed by camera l. Note that $N_l=\sum_z n_l^z$. Before we proceed with our analytical derivation, we define certain terms. Firstly, we define the empirical mean and sum of square matrices, which can be given by,

$$\bar{a}=\frac{1}{n}\sum_i a_i$$
$$S^2=\frac{1}{n-1}\sum_{i=1}^{n}(a_i-\bar{a})^T(a_i-\bar{a})$$

Using the above definitions, we can expand the terms inside {} of the first exponential term in 13 as follows,

$$
\begin{aligned}
&= \sum_{z=1}^{Z}\sum_{i=1}^{n_l^z}\big((a_i-\overline{a})^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(a_i-\overline{a}) + (\overline{a}-\mu_l^g(\mu_z+\mu_l^w))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-\mu_l^g(\mu_z+\mu_l^w))\big)\\
&= \sum_{z=1}^{Z}\big(S_l^2(n_l^z-1)(\lambda_l^g(\Lambda_z+\Lambda_l^w)) + n_l^z(\overline{a}-\mu_l^g(\mu_z+\mu_l^w))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-\mu_l^g(\mu_z+\mu_l^w))\big)\\
&= \sum_z S_l^2(n_l^z-1)(\lambda_l^g(\Lambda_z+\Lambda_l^w))\\
&\quad \ldots + \sum_z n_l^z(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w))) \qquad (14)
\end{aligned}
$$

Next, we replace Eqn 14 inside the {} of the first exponential term in Eqn 13 and re-write the equation, resulting in the posterior distribution

$$
\begin{aligned}
&\propto \prod_{z=1}^{Z}\Big[|\lambda_l^g(\Lambda_z+\Lambda_l^w)|^{n_l^z/2}\Big]\\
&\ldots exp\left(-\frac{1}{2}\left\{\sum_z S_l^2(n_l^z-1)(\lambda_l^g(\Lambda_z+\Lambda_l^w)) + \sum_z n_l^z(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))\right\}\right)\\
&\ldots |\Lambda_l^w|^{\frac{(\alpha_0^w-4)}{2}} exp\left(-\frac{1}{2}tr(\beta_0^w\Lambda_l^w)\right)\\
&\propto \prod_{z=1}^{Z}\Big[|\lambda_l^g(\Lambda_z+\Lambda_l^w)|^{n_l^z/2}\Big]\,|\Lambda_l^w|^{\frac{(\alpha_0^w-4)}{2}} exp\left(-\frac{1}{2}tr(\beta_0^w\Lambda_l^w)\right)\\
&\ldots exp(-1/2\{\sum_z S_l^2(n_l^z-1)(\lambda_l^g(\Lambda_z+\Lambda_l^w)) \qquad (15)\\
&\ldots + \sum_z n_l^z(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w))))\}) \qquad (16)
\end{aligned}
$$

Next, we expand the terms inside {} of the second exp. of eqn 16. Firstly, the second term i.e. $\sum_z n_l^z(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\overline{a}-(\mu_l^g(\mu_z+\mu_l^w)))$ expands as follows,

$$
\begin{aligned}
&= \sum_z n_l^z\overline{a}^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))\overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\mu_l^g(\mu_z+\mu_l^w)))\\
&\quad \ldots + \sum_z n_l^z(\mu_l^g(\mu_z+\mu_l^w)))^T(\lambda_l^g(\Lambda_z+\Lambda_l^w))(\mu_l^g(\mu_z+\mu_l^w)))\\
&= \sum_z n_l^z\overline{a}^T\lambda_l^g\Lambda_z\overline{a} + \sum_z n_l^z\overline{a}^T\lambda_l^g\Lambda_l^w\overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_z\\
&\quad - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_l^w - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g\mu_z - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g.\mu_l^w\ldots\\
&\quad \ldots + \sum_z n_l^z\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_z)\mu_z + \sum_z n_l^z\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_l^w)\mu_z\\
&\quad \ldots + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_l^w + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_l^w\\
&\quad \ldots + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_z)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_l^w)\mu_l^w \qquad (17)
\end{aligned}
$$

In Eqn 17, we omit terms not containing Λ_l^w, as multiplicative constants are absorbed into the normalising constant. Thus, Eqn 17 simplifies as,

$$
\begin{aligned}
&= \sum_z n_l^z \overline{a}^T \lambda_l^g \Lambda_l^w \overline{a} - \sum_z n_l^z 2\overline{a}^T (\lambda_l^g \Lambda_l^w) \mu_l^g \mu_z \dots \\
&- \sum_z n_l^z 2\overline{a}^T (\lambda_l^g \Lambda_l^w) \mu_l^g . \mu_l^w + \sum_z n_l^z \mu_l^{g2} \mu_z^T (\lambda_l^g \Lambda_l^w) \mu_z \\
&+ \sum_z n_l^z \mu_l^{g2} \mu_l^{wT} (\lambda_l^g \Lambda_l^w) \mu_l^w + \sum_z n_l^z 2\mu_l^{g2} \mu_z^T (\lambda_l^g \Lambda_l^w) \mu_l^w
\end{aligned} \tag{18}
$$

Substituting Eqn 18 inside the {} in Eqn 16, we get

$$
\begin{aligned}
&\propto \prod_{z=1}^{Z} \left[|\lambda_l^g (\Lambda_z + \Lambda_l^w)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2}} exp\left(-\frac{1}{2} tr(\beta_0^w \Lambda_l^w) \right) \\
&\dots exp(-\frac{1}{2}\{ \sum_z S_l^2 (n_l^z - 1)(\lambda_l^g (\Lambda_l^w)) + \sum_z n_l^z \overline{a}^T \lambda_l^g \Lambda_l^w \overline{a} - \sum_z n_l^z 2\overline{a}^T (\lambda_l^g \Lambda_l^w) \mu_l^g \mu_z \\
&\dots - \sum_z n_l^z 2\overline{a}^T (\lambda_l^g \Lambda_l^w) \mu_l^g . \mu_l^w + \sum_z n_l^z \mu_l^{g2} \mu_z^T (\lambda_l^g \Lambda_l^w) \mu_z \\
&\dots + \sum_z n_l^z \mu_l^{g2} \mu_l^{wT} (\lambda_l^g \Lambda_l^w) \mu_l^w + \sum_z n_l^z 2\mu_l^{g2} \mu_z^T (\lambda_l^g \Lambda_l^w) \mu_l^w \})
\end{aligned} \tag{19}
$$

Solving for α_n^w To solve for the hyper-parameter term α_n^w, we compare the posterior distribution in Eqn 19 with the posterior distribution given in Eqn 11. On comparing the multiplicative constants and using the following identities

$$
\begin{aligned}
det(cA) &= c^n det(A) \\
(x^a)^b &= x^{ab} \\
det(A + B) &= det(A) det(I + BA^{-1})
\end{aligned}
$$

We can observe

$$
\begin{aligned}
|\Lambda_l^w|^{\frac{(\alpha_n^w - 4)}{2}} &= \prod_{z=1}^{Z} \left[|\lambda_l^g (\Lambda_z + \Lambda_l^w)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2}} \\
&= \prod_{z=1}^{Z} \left[\left(\lambda_l^{g3} \right)^{n_l^z/2} |(\Lambda_z + \Lambda_l^w)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2} + \frac{1}{2}} \\
&= \left(\lambda_l^{g3} \right)^{n_z^l/2} \prod_{z=1}^{Z} \left[|(\Lambda_z + \Lambda_l^w)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2} + \frac{1}{2}} \\
&= (\lambda_l^g)^{\frac{3n}{2}} \prod_{z=1}^{Z} \left[|\Lambda_l^w|^{n_l^z/2} |(\Lambda_z \Sigma_z + I)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2} + \frac{1}{2}} \\
&= (\lambda_l^g)^{\frac{3n}{2}} |\Lambda_l^w|^{n/2} \prod_{z=1}^{Z} \left[|(\Lambda_z \Sigma_z + I)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2} + \frac{1}{2}} \\
&= (\lambda_l^g)^{\frac{3n}{2}} \prod_{z=1}^{Z} \left[|(\Lambda_z \Sigma_z + I)|^{n_l^z/2} \right] |\Lambda_l^w|^{\frac{(\alpha_0^w - 4)}{2} + \frac{1}{2} + \frac{n}{2}}
\end{aligned} \tag{20}
$$

In the above derivation, comparing the exponents of the determinant

$$
\boxed{
\begin{aligned}
\frac{(\alpha_n^w - 4)}{2} &= \frac{(\alpha_0^w - 4)}{2} + \frac{1}{2} + \frac{n}{2} \\
\alpha_n^w - 4 &= \alpha_0^w - 4 + 1 + n \\
\alpha_n^w &= \alpha_0^w + n + 1
\end{aligned}
} \tag{21}
$$

Solving for β_n^w To solve for the final hyper-parameter term β_n^w, we compare the posterior distribution in Eqn 19 with the posterior distribution given in Eqn 11. On comparing the exp and using the following identities

$$
\begin{aligned}
x^T \Sigma x &= trace(x^T \Sigma x) \\
trace(A+B) &= trace(A) + trace(B) \\
trace(x^T \Sigma x) &= trace(x^T x \Sigma)
\end{aligned}
$$

We can observe that

$$
\begin{aligned}
exp\left(-\frac{1}{2}tr(\beta_n^w \Lambda_l^w)\right) &= exp\left(-\frac{1}{2}tr(\beta_0^w \Lambda_l^w)\right) \\
&\dots exp(-\frac{1}{2}\{\sum_z S_l^2(n_l^z-1)(\lambda_l^g(\Lambda_l^w)) + \sum_z n_l^z \overline{a}^T \lambda_l^g \Lambda_l^w \overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g \Lambda_l^w)\mu_l^g \mu_z \\
&\dots - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g \Lambda_l^w)\mu_l^g.\mu_l^w + \sum_z n_l^z \mu_l^{g2} \mu_z^T(\lambda_l^g \Lambda_l^w)\mu_z \\
&\dots + \sum_z n_l^z \mu_l^{g2} \mu_l^{wT}(\lambda_l^g \Lambda_l^w)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g \Lambda_l^w)\mu_l^w\})
\end{aligned} \tag{22}
$$

In the above Eqn, the RHS can be expanded as following

$$
\begin{aligned}
&= exp(-\frac{1}{2}\{tr(\beta_0^w \Lambda_l^w) \\
&\dots + \sum_z \sum_{i=1}^{n_z^l}(a_i - \overline{a})^T(\lambda_l^g \Lambda_l^w)(a_i - \overline{a}) + \sum_z n_l^z \overline{a}^T \lambda_l^g \Lambda_l^w \overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g \Lambda_l^w)\mu_l^g \mu_z \\
&\dots - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g \Lambda_l^w)\mu_l^g.\mu_l^w + \sum_z n_l^z \mu_l^{g2}\mu_z^T(\lambda_l^g \Lambda_l^w)\mu_z \\
&\dots + \sum_z n_l^z \mu_l^{g2}\mu_l^{wT}(\lambda_l^g \Lambda_l^w)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g \Lambda_l^w)\mu_l^w\}) \\
&= exp(-\frac{1}{2}\{tr(\beta_0^w \Lambda_l^w) + tr(\sum_z \sum_{i=1}^{n_z^l}(a_i - \overline{a})^T(\lambda_l^g)(a_i - \overline{a})\Lambda_l^w) \\
&\dots + tr(\sum_z n_l^z \overline{a}^T \lambda_l^g \overline{a} \Lambda_l^w) - tr(\sum_z n_l^z 2\overline{a}^T(\lambda_l^g)\mu_l^g \mu_z \Lambda_l^w) \\
&\dots - tr(\sum_z n_l^z 2\overline{a}^T(\lambda_l^g)\mu_l^g.\mu_l^w \Lambda_l^w) + tr(\sum_z n_l^z \mu_l^{g2}\mu_z^T(\lambda_l^g)\mu_z \Lambda_l^w) \\
&\dots + tr(\sum_z n_l^z \mu_l^{g2}\mu_l^{wT}(\lambda_l^g)\mu_l^w \Lambda_l^w) + tr(\sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g)\mu_l^w \Lambda_l^w)\})
\end{aligned} \tag{23}
$$

Comparing the terms inside the exponential, after adding the traces and factoring out Λ_l^w, we get

$$
\boxed{
\begin{aligned}
\beta_n^w = \beta_0^w &+ \sum_z \sum_{i=1}^{n_z^l}(a_i - \overline{a})^T(\lambda_l^g)(a_i - \overline{a}) + \sum_z n_l^z \overline{a}^T \lambda_l^g \overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g)\mu_l^g \mu_z \\
\dots &- \sum_z n_l^z 2\overline{a}^T(\lambda_l^g)\mu_l^g.\mu_l^w + \sum_z n_l^z \mu_l^{g2}\mu_z^T(\lambda_l^g)\mu_z + \sum_z n_l^z \mu_l^{g2}\mu_l^{wT}(\lambda_l^g)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g)\mu_l^w
\end{aligned}
} \tag{24}
$$