Solving Person Re-identification using Efficient Gibbs Sampling

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1 The Model

Given a set of observations $\mathcal{X} = \{\mathbf{x}_i\}_{i=1}^N$ across multiple camera views, where each observation corresponds to a complete trajectory within a camera FOV, person re-identification can be defined as the problem of identifying the set of indicator variables associated with the observations $\mathbf{z} = \{z_i\}_{i=1}^N$. To identify the label $z_i \in [1, \dots, Z]$ associated with each trajectory, we utilise a combination of visual information, the appearance features, and the transitions between the cameras. To address the issues associated with appearance-based methods in our proposed person re-identification algorithm, we model each person's appearance using camera-specific illumination and camera gain. We identify the indicator labels by performing Bayesian inference using Gibbs sampling. Each observation $\mathbf{x}_i = \{\mathbf{a}_i, e_i, t_i, l_i\}$ consists of: $l_i \in [1, \dots, L]$ the camera that records the observation; the time of entry e_i in a camera's FOV; the time of leaving the camera's FOV t_i; and the observed appearance features $\mathbf{a}_i$. We define the likelihood as

$$p(\{\mathbf{x}_i\}_{i=1}^N | \{z_i\}_{i=1}^N) = \prod_{j=1}^N p(\mathbf{a}_j | z_j, l_j)\, p(l_j | \{l_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^j)\, p(e_j | \{t_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^j) \tag{1}$$

where $p(\{\mathbf{a}_j\}_{i=1}^N | z_j, l_j)$ is modelled as $\mathbf{a}_i = g_l(\mathbf{r}_z + \mathbf{w}_l)$, where g_l is the multiplicative gain constant of camera l, $\mathbf{r}_z$ is the RGB color model, averaged over the entire trajectory, $\mathbf{w}_l$ is the illumination noise associated with camera l, and the terms are distributed as:

$$g_l \sim \text{Gamma}(\alpha_l^g, \beta_l^g), \text{ which we approximate as } \mathcal{N}(\mu_l^g, (\Lambda_l^g)^{-1}) \tag{2}$$

$$\mathbf{r}_z \sim \mathcal{N}(\mu_z, (\Lambda_z)^{-1}) \tag{3}$$

$$\mathbf{w}_l \sim \mathcal{N}(\mu_l^w, (\Lambda_l^w)^{-1}) \tag{4}$$

The transitions between cameras are modelled as

$$l_j \mid \{l_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j-1} \sim \text{Mult}(l_j; \theta_{l_i}), i : z_i = z_j \wedge z_k \neq z_j, i < k < j \tag{5}$$

$$e_j \mid \{t_i\}_{i=1}^{j-1}, \{z_i\}_{i=1}^{j} \sim \mathcal{N}(e_j - t_i; \mu_{l_i, l_j}, \Lambda_{l_i, l_j}^{-1}), i : z_i = z_j \wedge z_k \neq z_j, i < k < j \tag{6}$$

It is clear from its structure (Fig 1a) that this model does not allow for efficient inference, since the Markov blanket of any observation is the complete set of observations and indicator variables preceding it. Yet if the latent indicator variables are known, the observations of a person become independent of all other persons, and the model becomes much simpler (Fig 1b)

2 Illumination per Camera: Derivation of the Mean Conditional Distribution

The posterior distribution over the appearance mean parameter for camera l in terms of the likelihood distribution and prior distribution is given as,

$$p(\mu_l^w | A_l, \alpha_l, \beta_l, \mu_l, \Lambda_l^w, l, z) = p(A_l | \alpha_l, \beta_l \mu_l^w, \Lambda_l^w, \mu_z, \Lambda_z, z, l) p(\mu_l^w) \tag{7}$$

We define a Gaussian distribution as the prior for the illumination mean parameters μ_l^w and a Gaussian distribution for the likelihood. Additionally, for convenience, we approximate the gain parameters α_l, β_l in the above Eqn 7 terms of a Normal distribution $N(\mu_l^g, \lambda_l^g)$, where $\mu_l^g = \alpha_l \beta_l$ and $\lambda_l^g = \frac{1}{\alpha_l \beta_l^2}$. Thus, eqn 7 can now be re-wriiten as

$$p(\mu_l | A_l, \mu_l^g, \lambda_l^g, \mu_l, \Lambda_l, \mu_z, l, z) = p(A_l | \mu_l^g, \lambda_l^g, \mu_l^w, \Lambda_l^w, \mu_z, \Lambda_z, z, l) p(\mu_l^w) \tag{8}$$

where, μ_l^w, Λ_l^w represent the mean and precision of the Gaussian distribution defined over the l camera's offset value, $\text{A}_l = \{a_i\}_{i=1}^{N_l}$represents the set of observations observed by camera l, where N_l indicates the number of trajectories belonging to camera l. The Gaussian likelihood distribution is given as,

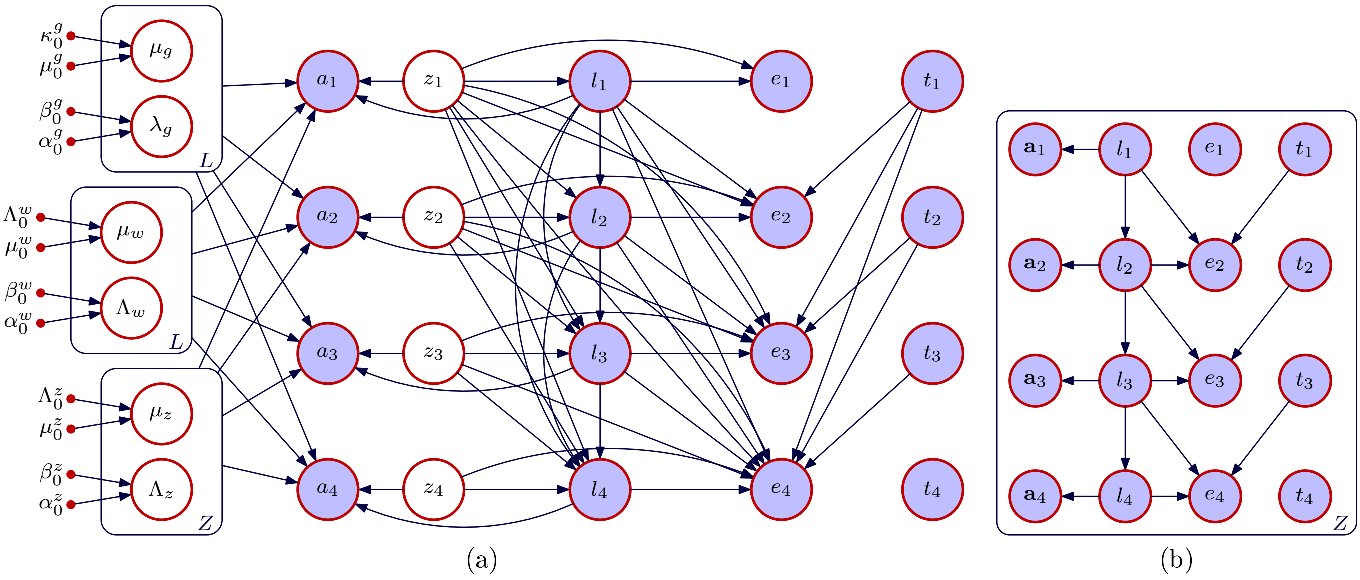

Figure 1: (a) Full graphical model of our probabilistic person re-identification algorithm and (b) Graphical model if the latent variables z_i are known.

$$p(A_l|\mu_l^g, \lambda_l^g, \mu_l^w, \Lambda_l^w, \mu_z, \Lambda_z, z, l) = \prod_{i=1}^{N_l} N(a_i|\mu_l^g(\mu_z + \mu_l^w), \lambda_l^g(\Lambda_z + \Lambda_l^w)) \tag{9}$$

The Gaussian prior distribution in 8 is given as,

$$p(\mu_l^w|\mu_0^w, \Lambda_0^w) \propto |\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T \Lambda_0^w (\mu_l^w - \mu_0^w)\right) \tag{10}$$

where, μ_0^w, Λ_0^w represent the fixed hyper-parameters for the prior distribution. Since, the Gaussian distribution is a conjugate prior for the Gaussian likelihood, the resultant posterior is also a Wishart distribution given as,

$$p(\mu_l^w|\mu_n^w, \Lambda_n^w) \propto |\Lambda_n|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_n^w)^T \Lambda_n (\mu_l^w - \mu_n^w)\right) \tag{11}$$

where, μ_n^w, Λ_n^w represent the hyper-parameters for the posterior distribution. To solve for the posterior distribution, we first combine the likelihood distribution (eqn 9) with the prior distribution (10). Thus the resultant posterior distribution $p(\mu_l^w|\mu_n^w, \Lambda_n^w)$ is given as,

$$\propto \prod_{i=1}^{N_l} N(a_i|\mu_l^g(\mu_z + \mu_l^w), \lambda_l^g(\Lambda_z + \Lambda_l^w))|\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T \Lambda_0^w (\mu_l^w - \mu_0^w)\right) \tag{12}$$

$$\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^o)|^{n_l^z/2}\right] exp\left(-\frac{1}{2}\left\{\sum_{z=1}^{Z}\sum_{i=1}^{n_l^z}(a_i - \mu_l^g(\mu_z + \mu_l^w))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(a_i - \mu_l^g(\mu_z + \mu_l^w)\right\}\right)$$
$$\ldots|\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T \Lambda_0^w (\mu_l^w - \mu_0^w)\right) \tag{13}$$

Note that in Eqn 13, we choose to factorize the likelihood distribution according to each person. In Eqn 13 n_l^z represents the number of trajectories belonging to person z observed by camera l. Note that $N_l = \sum_z n_l^z$. Before we proceed with our analytical derivation, we define certain terms. Firstly, we define the empirical mean and sum of square matrices, which can be given by,

$$\bar{a} = \frac{1}{n}\sum_i a_i$$
$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(a_i - \bar{a})^T(a_i - \bar{a})$$

Using the above definitions, we can expand the terms inside {} of the first exponential term in 13 as follows,

$$
\begin{aligned}
&= \sum_{z=1}^{Z}\sum_{i=1}^{n_l^z} \big((a_i - \overline{a})^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(a_i - \overline{a}) + (\overline{a} - \mu_l^g(\mu_z + \mu_l^w))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - \mu_l^g(\mu_z + \mu_l^w))\big) \\
&= \sum_{z=1}^{Z} \big(S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + n_l^z(\overline{a} - \mu_l^g(\mu_z + \mu_l^w))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - \mu_l^g(\mu_z + \mu_l^w))\big) \\
&= \sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) \\
&\quad \ldots + \sum_z n_l^z(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))
\end{aligned} \tag{14}
$$

Next, we replace Eqn 14 inside the {} of the first exponential term in Eqn 13 and re-write the equation, resulting in the posterior distribution $p(\mu_l^w|\mu_n^w, \Lambda_n^w)$

$$
\begin{aligned}
&\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^w)|^{n_l^z/2}\right] \\
&\ldots exp\left(-\frac{1}{2}\left\{\sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + \sum_z n_l^z(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))\right\}\right) \\
&\ldots |\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T\Lambda_0^w(\mu_l^w - \mu_0^w)\right) \\
&\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^w)|^{n_l^z/2}\right]|\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T\Lambda_0^w(\mu_l^w - \mu_0^w)\right) \\
&\ldots exp\left(-\frac{1}{2}\left\{\sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + \sum_z n_l^z(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))\right\}\right)
\end{aligned} \tag{15}
$$

Next, we expanding the terms inside {} of the second exp. of eqn 15. Firstly, the second term i.e. $\sum_l n_z^l(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\overline{a} - (\mu_l^g(\mu_z + \mu_l^w)))$ expands as follows,

$$
\begin{aligned}
&= \sum_z n_l^z\overline{a}^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))\overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\mu_l^g(\mu_z + \mu_l^w))) \\
&\quad \ldots + \sum_z n_l^z(\mu_l^g(\mu_z + \mu_l^w)))^T(\lambda_l^g(\Lambda_z + \Lambda_l^w))(\mu_l^g(\mu_z + \mu_l^w))) \\
&= \sum_z n_l^z\overline{a}^T\lambda_l^g\Lambda_z\overline{a} + \sum_z n_l^z\overline{a}^T\lambda_l^g\Lambda_l^w\overline{a} - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_z \\
&\quad - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_l^w - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g\mu_z - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g.\mu_l^w \ldots \\
&\quad \ldots + \sum_z n_l^z\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_z)\mu_z + \sum_z n_l^z\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_l^w)\mu_z \\
&\quad \ldots + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_l^w + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_l^w \\
&\quad \ldots + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_z)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_z^T(\lambda_l^g\Lambda_l^w)\mu_l^w
\end{aligned} \tag{16}
$$

In Eqn 16, we omit terms not containing μ_l^w, as multiplicative constants are absorbed into the normalising constant. Thus, Eqn 16 simplifies as,

$$
\begin{aligned}
&= -\sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_l^w - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g\mu_l^w + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_l^w \\
&\quad + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_l^w + \sum_z n_l^z 2\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_z + \sum_z n_l^z 2\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_z
\end{aligned} \tag{17}
$$

Next, we expand the terms inside the first exponential term in Eqn 15 $exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T \Lambda_0^w (\mu_l^w - \mu_0^w)\right)$ as,

$$exp\left(-\frac{1}{2}(\mu_l^w - \mu_0^w)^T \Lambda_0^w (\mu_l^w - \mu_0^w)\right) = exp\left(-\frac{1}{2}\left(\mu_l^{wT}\Lambda_0^w\mu_l^w - 2\mu_l^{wT}\Lambda_0^w\mu_0^w + \mu_0^{wT}\Lambda_0^w\mu_0^w\right)\right) \tag{18}$$

Substituting Eqn 17 inside the {} of the second exponential term in Eqn 15, and Eqn 18 into the first exponential term, and combining the exponential terms, we get

$$\begin{aligned}
&\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^w)|^{n_z^l/2}\right] |\Lambda_0^w|^{\frac{1}{2}} exp(-\frac{1}{2}\{\sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_z)\mu_l^g\mu_l^w \\
&\ldots - \sum_z n_l^z 2\overline{a}^T(\lambda_l^g\Lambda_l^w)\mu_l^g\mu_l^w + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_l^w + \sum_z n_l^z\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_l^w \\
&\ldots + \sum_z n_l^z 2\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_z)\mu_z + \sum_z n_l^z 2\mu_l^{g2}\mu_l^{wT}(\lambda_l^g\Lambda_l^w)\mu_z + \mu_l^{wT}\Lambda_0^w\mu_l^w - 2\mu_l^{wT}\Lambda_0^w\mu_0^w + \mu_0^{wT}\Lambda_0^w\mu_0^w\})
\end{aligned} \tag{19}$$

Solving for μ_n^w and Λ_n^w To solve for the hyper-parameter terms, we consider the terms inside the {} of the exponential term the posterior distribution in Eqn 19 and arrange them according to the terms $\mu_l^{wT}\mu_l^w$ and μ_l^w, resulting in,

$$\begin{aligned}
&= \mu_l^{wT}(\sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_z + \sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_l^w + \Lambda_0^w)\mu_l^w + \\
&\ldots \mu_l^{wT}\left(-\sum_z n_l^z 2\lambda_l^g\mu_l^g\Lambda_z\overline{a} - \sum_z n_l^z 2\mu_l^g\lambda_l^g\Lambda_l^w\overline{a} + \sum_z n_l^z 2\mu_l^{g2}\lambda_l^g\Lambda_z\mu_z + \sum_z n_l^z 2\mu_l^{g2}\lambda_l^g\Lambda_l^w\mu_z - 2\Lambda_0^w\mu_0^w\right) \\
&\ldots \sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + \mu_0^{wT}\Lambda_0^w\mu_0^w
\end{aligned} \tag{20}$$

Next, we use completing the square formula to derive μ_n^w. First, the completing the square formula for a generic framework is given as follows,

$$\begin{aligned}
x^TAx + x^TB + C &= (x - H)^TA(x - H) + K \\
K = C - \frac{1}{4}B^TA^{-1}B; &\qquad H = -\frac{1}{2}A^{-1}B
\end{aligned} \tag{21}$$

Comparing Eqn 20 and LHS of Eqn 21, we can see that,

$$\begin{aligned}
A &= \sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_z + \sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_l^w + \Lambda_0^w \\
B &= -\sum_z n_l^z 2\lambda_l^g\mu_l^g\Lambda_z\overline{a} - \sum_z n_l^z 2\mu_l^g\lambda_l^g\Lambda_l^w\overline{a} + \sum_z n_l^z 2\mu_l^{g2}\lambda_l^g\Lambda_z\mu_z + \sum_z n_l^z 2\mu_l^{g2}\lambda_l^g\Lambda_l^w\mu_z - 2\Lambda_0^w\mu_0^w \\
C &= \sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + \mu_0^{wT}\Lambda_0^w\mu_0^w
\end{aligned}$$

Writing Eqn 20 in the form of Eqn 21, we get

$$\mu_l^{wT}A\mu_l^w + \mu_l^{wT}B + C = (\mu_l^w - H)^TA(\mu_l^w - H) + K \tag{22}$$

$$H = \frac{\sum_z n_l^z\lambda_l^g\mu_l^g\Lambda_z\overline{a} + \sum_z n_l^z\mu_l^g\lambda_l^g\Lambda_l^w\overline{a} - \sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_z\mu_z - \sum_z n_l^z 2\mu\lambda_l^g\Lambda_l^w\mu_z + \Lambda_0^w\mu_0^w}{\sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_z + \sum_z n_l^z\mu_l^{g2}\lambda_l^g\Lambda_l^w + \Lambda_0^w} \tag{23}$$

$$K = \sum_z S_l^2(n_l^z - 1)(\lambda_l^g(\Lambda_z + \Lambda_l^w)) + \mu_0^{wT}\Lambda_0^w\mu_0^w - \frac{1}{4}B^TA^{-1}B \tag{24}$$

Using Eqn 22 inside the exponential term in Eqn 19, we get

$$\begin{aligned}
&\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^w)|^{n_l^z/2}\right] |\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}\left\{(\mu_l^w - H)^TA(\mu_l^w - H) + K\right\}\right) \\
&\propto \prod_{z=1}^{Z}\left[|\lambda_l^g(\Lambda_z + \Lambda_l^w)|^{n_l^z/2}\right] |\Lambda_0^w|^{\frac{1}{2}} exp\left(-\frac{1}{2}(\mu_l^w - H)^TA(\mu_l^w - H)\right) exp\left(-\frac{K}{2}\right)
\end{aligned} \tag{25}$$

Comparing the first exponential term in Eqn 25 with posterior distribution in Eqn 11, we can see that

$$\boxed{\begin{aligned}
\mu_n^w &= \frac{\sum_z n_l^z \lambda_l^g \mu_l^g \Lambda_z \overline{a} + \sum_z n_l^z \mu_l^g \lambda_l^g \Lambda_l^w \overline{a} - \sum_z n_l^z \mu_l^{g2} \lambda_l^g \Lambda_z \mu_z - \sum_z n_l^z 2\mu \lambda_l^g \Lambda_l^w \mu_z + \Lambda_0^w \mu_0^w}{\sum_z n_l^z \mu_l^{g2} \lambda_l^g \Lambda_z + \sum_z n_l^z \mu_l^{g2} \lambda_l^g \Lambda_l^w + \Lambda_0^w} \\
\Lambda_n^w &= \sum_z n_l^z \mu_l^{g2} \lambda_l^g \Lambda_z + \sum_z n_l^z \mu_l^{g2} \lambda_l^g \Lambda_l^w + \Lambda_0^w
\end{aligned}}$$