A Phase Field Method for Tomographic Reconstruction from Limited Data

Russell J. Hewett
rhewett@mit.edu

Imaging and Computing Group
Department of Mathematics
Massachusetts Institute of Technology

Ian Jermyn
i.h.jermyn@durham.ac.uk

Department of Mathematical Sciences
Durham University

Michael T. Heath
heath@illinois.edu

Department of Computer Science
University of Illinois at Urbana-Champaign

Farzad Kamalabadi
farzadk@illinois.edu

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign

Solar phenomena such as active regions, flares, coronal mass ejections (CMEs), and solar wind, all of which contribute to geoeffective events, collectively referred to as *space weather*, are not well understood [1], yet are of critical importance due to modern society's reliance on technologies that can be disrupted by these events. Understanding such activity requires knowledge of the electron density of the solar corona (or solar atmosphere), but such knowledge is hard to come by for local, short-lived events such as CMEs.

Direct imaging of CME electron density via tomographic reconstruction is difficult because a maximum of three unique observations of any given event are available. Such a regime requires a reconstruction algorithm that is robust to sparse data. Mumford-Shah type models have previously been proposed for CME reconstruction [2] but have not been successful due, in part, to the complex topology of CME structures.

We present a method for reconstruction from sparse data that, similarly, uses an auxiliary segmentation to constrain the density, but we represent the segmentation using the phase field level set framework, thereby eliminating important topological limitations and allowing for smooth enforcement of two different regularization regimes, while retaining robustness. We use a fast variational algorithm to compute MAP estimates, and compare our results to classical regularized tomography for synthetic CME-like images.

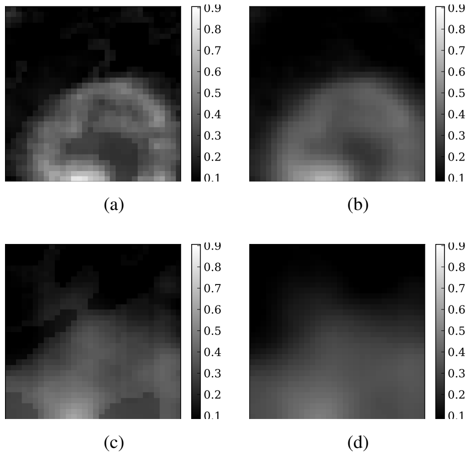

Figure 1: Comparison of joint segmentation-reconstruction with Tikhonov regularized reconstructions for CME image. (a) Joint segmentation-reconstruction and (b) Tikhonov regularized reconstruction with $\lambda = \lambda_-$ for 32 equispaced observation angles and 32 projections per angle. (c) and (d) same, for 3 equispaced observation angles.

Model We seek to infer the electron density f in a local section of the solar corona Ω and a segmentation, represented by a phase field ϕ classifying a subset $R \subset \Omega$ as part of a CME, given a set of coronagraphs Y and prior knowledge K, (*e.g.* parameter values). We compute the MAP estimate $(\hat{f}, \hat{\phi})$ from $-\ln P(f,\phi|Y,K) = E(Y|f,K) + E(f|\phi,K) + E(\phi|K)$:

$$(\hat{f},\hat{\phi}) = \arg\min_{(f,\phi)} \sum_j \frac{1}{2\sigma^2} \int_\Gamma \left(y_j - h_j(f)\right)^2 + \int_\Omega \Big\{ \frac{1}{2}(\lambda_+\phi_+ + \lambda_-\phi_-)(\nabla f \cdot \nabla f) - c_4 \nabla\phi \cdot \nabla f + c_1 \frac{1}{2} \nabla\phi\cdot\nabla\phi + c_2(\frac{1}{4}\phi^4 - \frac{1}{2}\phi^2) + c_3(\phi - \frac{1}{3}\phi^3) \Big\} . \quad (1)$$

The first line of (1) defines $E(Y|f,K)$, representing noisy tomographic measurements.

The third line defines the segmentation energy, $E(\phi|K)$. The phase field ϕ represents the region $R = \{x \in \Omega : \phi(x) > 0\}$; c_1, c_2, and c_3 are free parameters. The last two terms are a double well potential: for $|c_3| < c_2$, local minima occur at $\phi = \pm 1$. Coupled with the smoothing effect of the first term, the potential ensures that, away from the region boundary and for fixed R, ϕ takes the values 1 in R and -1 in $\Omega \setminus R$. Near the boundary, there is a smooth transition across an interface zone of width $4\sqrt{c_1/c_2}$. The effective energy controlling R is then a linear combination of the length (area) of the boundary and the area (volume) of the interior of R for 2D (3D) regions [3].

The second line defines $E(f|\phi,K)$, which couples the density and the phase field. In the first component, $\phi_\pm = (1 \pm \phi)/2$ act as pseudo-indicator functions for the CME and background regions, thus defining distinct Tikhonov regularization parameters, $\lambda_\pm$, for the interior and exterior of R. The second term favours large inward pointing ∇f on the boundary, because CMEs generally have sharply higher densities than the background. Thus, like [2], we model the background as smoother than the CME, and with a very different density, but unlike [2], the phase field defines a smooth change in the regularization parameter over the interface.

Algorithm Traditionally, energies such as (1) are optimized by split-step gradient descent methods relying on explicit finite differencing to solve the associated PDEs. However, the small time step required for stability leads to slow convergence and implicit methods allowing larger time steps are impractical due to increased computational complexity from the nonlinearity in the phase field potential. We resolve these issues by minimizing in both f and ϕ simultaneously using finite element discretization and a trust-region-based variation on Newton's method, the Levenberg-Marquardt method. In this approach, the length of the descent step is dependent upon the minimization algorithm and the local shape of the objective function, and is not explicitly constrained by the discretization.

Results We compare our results to those obtained using Tikhonov regularization (Fig. 1). We see that, even for the limited angle reconstructions, the joint segmentation-tomographic reconstructions have definition in the CME region that is not present in either of the Tikhonov regularized reconstructions. Our experiments show that our method is significantly more effective than Tikhonov regularized tomography alone, and resolves issues with CME topology and continuity of the density that affected previous work. Our model and optimization method easily extend to the full 3D CME reconstruction problem, though further work on parameter estimation is necessary to render the method automatic.

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[3] M. Rochery, I. Jermyn, and J. Zerubia. Phase field models and higher-order active contours. In *ICCV '05: Proceedings of the Tenth IEEE International Conference on Computer Vision*, pages 970–976, 2005.