Image Priors for Image Deblurring with Uncertain Blur

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We consider the problem of non-blind deconvolution of images corrupted by a blur that is not accurately known. A common choice for non-blind deconvolution algorithms is to use methods that rely on an exact blur estimate. However, small errors in the blur estimate result in visible artifacts in the restored image, which may not be removed by future iterations.

We propose a novel image prior to remove artifacts introduced by blur errors. To achieve this goal we use a dictionary-based prior learned only from the input blurred image and a database of images, and propose a method to prune ambiguities in the prior due to blur.

Consider an observed degraded image g

$$g = k * f + n \tag{1}$$

where $*$ is the convolution operator, k is a blur kernel, or point spread function (PSF), f is the noiseless and sharp image, and n is additive noise generated during the acquisition process. The aim of image deblurring is to estimate the noiseless image f given the noisy image g and the kernel k.

The imaging model in (1) can be written as a matrix-vector operation.

$$\vec{g} = K\vec{f} + \vec{n} \tag{2}$$

However, typically the linear system has infinite solutions due to the noise n being larger than the smallest singular values of the matrix K.

One way to obtain a unique solution is to introduce additional linear equations, which we call *image priors*, via a matrix A and a vector $\vec{b}$

$$A\vec{f} = \vec{b} \tag{3}$$

To enforce this regularity, we consider applying the same linear constraints to all pixels in patches of $L \times L$ pixels. For this purpose, we extract patches of $L \times L$ pixels centered at each pixel of the image f, rearrange the pixel intensities of each patch as a column vector and collect all such vectors into a matrix $F \in \mathbf{R}^{L^2 \times N}$, where N is the number of pixels in f. We can then write our prior as

$$F = DC, \tag{4}$$

We choose a dictionary made of a mix of both an external dictionary D_0 and the image itself ($D = [D_0\ F]$).

To learn C, we face two important challenges: The first challenge is that F is typically not available and the second is that we do not have enough equations to obtain a unique matrix C.

To deal with the first challenge, we extract noisy and blurred patches G_i from the image $g = k * f + n$. Let B be the matrix of patches extracted from the blurred noiseless image $b = k * f$. Since $B = KF = KDC$, we can express the blurred patches in B in terms of the blurred dictionary $E \triangleq KD$ using the *same* correspondence matrix C.

The second challenge, *i.e.*, the non uniqueness of the matrix C is due to the overcompleteness of the dictionary D. We introduce additional constraints on C by exploiting image self-similarities. We consider a patch at pixel i, B_i, found as a weighted average of similar patches D_j extracted from either the same image or from a dictionary of patches. Specifically, if we consider all the patches as vectors in $\mathbf{R}^{L^2}$, then we have

$$B_i = \sum_{j=1}^{M} D_j \frac{\phi(D_i, D_j)}{\sum_{\ell=1}^{M} \phi(D_i, D_\ell)}, \quad i = 1, \dots, N, \tag{5}$$

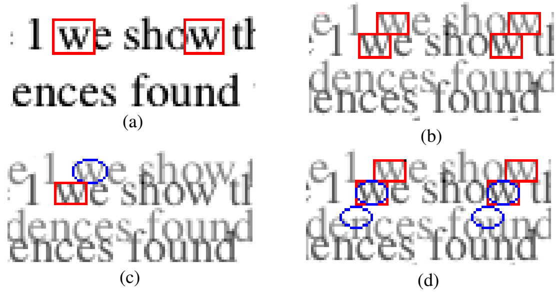

Figure 1: **Example of ambiguous correspondence correction.** In this toy example we show the correction performed by the correlation-based method. The PSF consists of only two peaks, which result in an overlap of two copies of the sharp image with two relative shifts. **(a)** Sharp image of text, and two correct correspondences (red squares) of the character 'w'. **(b)** Blurred version of the previous image which leads to additional incorrect correspondences. **(c)** Selected patch (red square) and the area corresponding to the second peak of the PSF (blue circle). **(d)** The correlated patches shown in (a) are obtained by overlapping the correspondence sets of the two patches (blue circles and red squares).

where ϕ is a positive semi-definite kernel that measures the similarity between two patches. In our work we use the following kernel,

$$\phi(D_i, D_j) = \begin{cases} 1 & \|D_i - D_j\|_2^2 \le \varepsilon^2 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where ε is proportional to the standard deviation of noise. It is easy to see that (5) can be written in matrix form as

$$B = DC^{nlm}, \tag{7}$$

where D is a matrix of patches ($D = [G\ E_0]$ in our case) and $\{C^{nlm}\}_{j,i} = \frac{\phi(D_i, D_j)}{\sum_{\ell=1}^{M} \phi(D_i, D_\ell)}$ is the correspondence matrix obtained from the procedure in eq. (5).

When we apply this procedure to a blurred image, incorrect correspondences may be generated. We distinguish two types: *false negatives*, *i.e.*, correspondences found in the sharp image, but not in the blurred one, and *false positives*, *i.e.*, correspondences present in the burred image but not in the sharp one. In Fig. 1 we provide a synthetic example to illustrate why blur generates false positives.

To reduce the false positives we suggest to use our (partial) knowledge of the blur. Formally, let $C_p = \{j \in \mathbb{Z} : ||B_p - B_{p+j}||^2 \le \varepsilon^2\}$ be the set of correspondences for the pixel p learned from the blurred image and let $\mathcal{K} = \{i \in \mathbb{Z} : |max(k) - k_i| \le \tau\}$ be the set of non-zero entries of the PSF k, where τ is a threshold based on blur noise. For each patch centered at p we enforce that its improved set of correspondences $\hat{C_p}$ be the intersection of all the correspondence sets of patches at pixels with relative displacement given by the main PSF peaks, *i.e.*, where the *consensus* is full: $\hat{C_p} = \bigcap_{i \in K} C_i$.

We finally pose the problem of recovering the sharp image f via the following convex optimization problem

$$\begin{aligned} \min_{f,n,e} \quad & \frac{1}{2}\|A\vec{f} - \vec{b}\|_2^2 + \beta\|\nabla f\|_2 + \frac{\lambda}{2}\|n\|_2^2 + \gamma\|e\|_1 \\ \text{subject to} \quad & g = h * f + n + e \end{aligned} \tag{8}$$

In the experimental validation our algorithm performance is overall better than the state-of-the-art methods when the blur kernel is noisy.