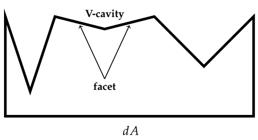
Shape from Shading for Rough Surfaces: Analysis of the Oren-Nayar Model

Yong Chul Ju
y.ju@mmci.uni-saarland.de
Vision and Image Processing
Saarland University, Germany

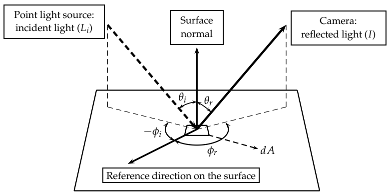
Michael Breuß
breuss@tu-cottbus.de
Institute for Applied Mathematics and Scientific Computing
BTU Cottbus, Germany

Andrés Bruhn
bruhn@vis.uni-stuttgart.de
Institute for Visualization and Interactive Systems
University of Stuttgar, Germany

Silvano Galliani
galliani@mia.uni-saarland.de
Mathematical Image Analysis Group
Saarland University, Germany



(a) Facet model for surface patch dA with many symmetric Lmabertian V-shape cavities.



(b) Illustration of diffuse reflectance parameters for SfS with Oren-Nayar reflectance.

Figure 1: Sketch of the Oren-Nayar surface reflection model.

Since almost five decades *Shape from Shading (SfS)* is one of the fundamental problems in computer vision. Having many interesting applications such as astronomy, terrain reconstruction, endoscopy or dentistry, the goal of SfS is to recover the surface of an object from a single input image under the assumption that a reflectance model and the light information are available. While, for a long time, research in SfS was mainly dominated by approaches based on relatively simple model assumptions such as an orthographic camera setup and a Lambertian surface model, recently more realistic concepts such as *perspective cameras* [4] and *non-Lambertian reflectance models* [5] found their way into research and led to considerable progress in the field.

One of these non-Lambertian reflectance models that seems particularly appealing is the *Oren-Nayar* reflectance model [2]. It allows to model rough materials such as concrete, plaster, clay or cloth realistically whose surface properties are considerably different from those of Lambertian surfaces. However, since the corresponding SfS models are rather involved, no theoretical analysis of such models with respect to model convexity or critical points has been performed so far. Moreover, it has not yet been investigated for which model parameters the popular and efficient fast marching (FM) method [6] can be applied safely to solve the resulting PDE of Hamilton-Jacobi type. This issue is particularly important, since the FM method has already shown impressive speed ups of up to factor 100 when applied on a pure experimental basis [7].

In our paper we perform such an in-depth analysis of the Oren-Nayar SfS model based on *Osher's criterion* [3]. This criterion allows to decide, if the FM method can be applied for solving a certain Hamilton-Jacobi equation, even if the underlying model is non-convex. By investigating this criterion for the Oren-Nayar model, we do not only succeed in providing concrete bounds for the model parameters that allow a safe application of the FM method, but we also put the findings of previous authors on a solid theoretical basis.

Figure 1 (a) illustrates the Oren-Nayar surface reflectance model that models rough surfaces by aggregating many Lambertian surface patches. In this context, the roughness is characterised by a Gaussian probability distribution of the patch slopes with standard deviation (roughness parameter) $\sigma \in [0, \frac{\pi}{2}]$. In our paper we investigate the SfS model by Ahmed and Farag [1] that additionally makes use of a light attenuation factor, that assumes the light source to be located in the centre of the camera and that models a perspective projection of the camera. This model is given by

$$H_{2\text{D}} = \mathtt{f}^2 I \frac{M+1}{A\sqrt{M+1}+BM} - e^{-2v} = 0 \tag{1}$$

with

$$M = \left[\mathtt{f}^2 \left|\nabla v(\mathbf{x})\right|^2 + \left(\nabla v(\mathbf{x}) \cdot \mathbf{x}\right)^2\right] \left(\frac{|\mathbf{x}|^2 + \mathtt{f}^2}{\mathtt{f}^2}\right) , \tag{2}$$

where the patch statistics σ and the different angles depicted in Figure 1 (b) enter the model via the local brighhtness I and the factors A and B [1]. To simplify the computations, the so-called Hamiltonian $H_{2\text{D}}$ is expressed here in terms of the *logarithm* of the sought depth $v = \ln u$.

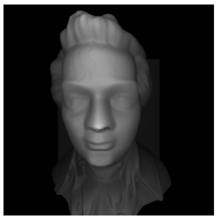
On the one hand we perform a *general* investigation of the model using Osher's criterion. In this context, we can prove that in 1-D for

$$0 \le \sigma < \sqrt{0.3869067207} \approx 0.622$$

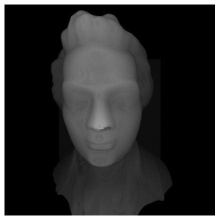
the FM method works by construction for the Oren-Nayar model. Moreover, we also conduct a *parameter dependent* analysis showing that the FM method can even be applied safely for a much wider range of settings that are particularly relevant for practical applications. Only strong discontinuites and very flat regions pose problems to the FM method. Based on the 1-D case we extend our theoretical findings also to the 2-D case. For the first time in the literature it becomes thus possible to *theoretically justify* the use of the FM method as solver for the SfS Oren-Nayar model which had been applied so far on a purely empirical basis only.

Numerical experiments demonstrate the validity of our theoretical analysis. They demonstrate a stable behaviour of the FM method for the predicted range of model parameters. As Figure 2 (d) shows, the FM method works reasonably well when the roughness parameter σ is within the admitted range. In constrast, in case σ is outside the range, the reconstruction shows significant problems, since the FM method fails.

Figure 2: Mozart Experiment.



(a) Input image, $\sigma = 0.5$. Fulfills theoretical bounds.



(b) Input image, $\sigma = \frac{\pi}{2}$. Exceeds theoretical bounds.

(c) Ground truth, $\mathtt{f} = 128$.

(d) Reconstruction of (a). Observation: FM works.

(e) Reconstruction of (b). Observation: FM fails.

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