One-sided Radial Fundamental Matrix Estimation

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For modern consumer cameras, often approximate calibration data is available, making applications such as 3D reconstruction or photo registration easier as compared to the pure uncalibrated setting. In this paper we adress the setting with calibrated-uncalibrated image pairs: for one image intrinsic parameters are assumed to be known, whereas the second view has unknown distortion and calibration. This situation arises e.g. when one would like to register archive imagery to recent photos. Very few existing solutions apply to the calibrated-uncalibrated setting. We propose a simple and numerically stable two-step scheme to first estimate radial distortion parameters and subsequently the focal length using novel solvers.

By using the distortion model proposed in [1], $p_u \propto \left(x_d y_d 1 + \lambda r_d^2\right)^T$ is the undistorted version of an observed image point $p_d = (x_d, y_d, 1)^T$ in an image with unknown radial distortion, $r_d^2 = (x_d - u)^2 + (y_d - v)^2$ for a known distortion center $(u,v)^T$, assumed to be at the image center, and λ is an unknown distortion parameter. The epipolar constraint becomes

$$q^T F p_u = q^T F \begin{pmatrix} x_d \\ y_d \\ 1 + \lambda r_d^2 \end{pmatrix} = q^T \underbrace{\left[F \mid \lambda F_3\right]}_{=:\hat{F}} \begin{pmatrix} x_d \\ y_d \\ 1 \\ r_d^2 \end{pmatrix}, \tag{1}$$

where we introduced the 3×4-matrix $\hat{F}$. F_3 denotes the 3rd column of F. By using 9 correspondences, the nullspace of $\hat{F}$ is three-dimensional, i.e.

$$\hat{F} = x\hat{X} + y\hat{Y} + z\hat{Z}. \tag{2}$$

We can fix z to 1 due to the scale ambiguity of $\hat{F}$. The constraints $\hat{F}_4 \propto \hat{F}_3$, i.e. $\lambda \hat{F}_4 = \hat{F}_3$, now read as

$$x\hat{X}_{i4} + y\hat{Y}_{i4} + \hat{Z}_{i4} = \lambda\left(x\hat{X}_{i3} + y\hat{Y}_{i3} + \hat{Z}_{i3}\right) \tag{3}$$

for $i = 1,2,3$. First, we can eliminate λ by taking ratios, leading to 3 polynomial equations in x and y only,

$$\begin{aligned} p_{ij}(x,y) &\stackrel{\text{def}}{=} \left(x\hat{X}_{i4} + y\hat{Y}_{i4} + \hat{Z}_{i4}\right)\left(x\hat{X}_{j3} + y\hat{Y}_{j3} + \hat{Z}_{j3}\right) \\ &\quad - \left(x\hat{X}_{j4} + y\hat{Y}_{j4} + \hat{Z}_{j4}\right)\left(x\hat{X}_{i3} + y\hat{Y}_{i3} + \hat{Z}_{i3}\right) \stackrel{!}{=} 0 \end{aligned} \tag{4}$$

for $(i,j) \in \{(1,2),(1,3),(2,3)\}$. We then compute two resultants (e.g. combining p_{12} with p_{13}, and p_{12} with p_{23}, respectively) leading to two degree 4 polynomials in x,

$$q_1(x) \stackrel{\text{def}}{=} a_1x^4 + b_1x^3 + c_1x^2 + d_1x + e_1 \stackrel{!}{=} 0 \tag{5}$$

$$q_2(x) \stackrel{\text{def}}{=} a_2x^4 + b_2x^3 + c_2x^2 + d_2x + e_2 \stackrel{!}{=} 0 \tag{6}$$

The leading monomial x^4 can now be eliminated by one step of Gaussian elimination leading to a final cubic polynomial,

$$r(x) \stackrel{\text{def}}{=} a_2q_1(x) - a_1q_2(x) \stackrel{!}{=} 0. \tag{7}$$

This can be solved in closed form leading to one or three real solutions. For each possible value of x, a corresponding y can be extracted by a similar procedure. Two of the p_{ij} polynomials (which are quadratic) yield a linear equation in y after one Gaussian elimination step. The extended fundamental matrix is given by $\hat{F} = x\hat{X} + y\hat{Y} + \hat{Z}$, and λ can be obtained as the ratio $\lambda = \hat{F}_{14}/\hat{F}_{13} = \hat{F}_{24}/\hat{F}_{23} = \hat{F}_{34}/\hat{F}_{33}$. By construction all those ratios are equal. Since we dropped the rank constraint, the estimated fundamental matrix (i.e. the 3×3 submatrix $\hat{F}[1:3,1:3]$) will generally be of full rank. We enforce rank-2 using the SVD as in the 8-point algorithm.

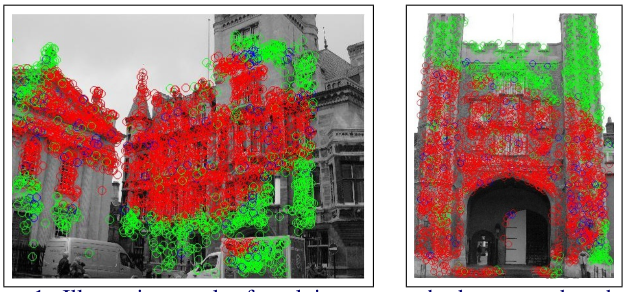

Figure 1: Illustrative result of applying our method compared to the results of the standard 8-point algorithm; red are the inliers found by both methods; green are the extra inliers found by our method; blue are inliers found by the standard 8-point not found by our method.

The focal length can be extracted in partially calibrated settings and we propose a different approach than [2]. Let F be a fundamental matrix, and K and K' camera intrinsics such that $E = (K')^T F K$ is an essential matrix. K is assumed to be known and K' is of the shape $\operatorname{diag}(f,f,1)$ for an unknown focal length f, hence we can incorporate K into F, yielding $E = \operatorname{diag}(f,f,1)F$. Plugging this expression into the trace constraint for essential matrices, $2EE^TE - \operatorname{tr}(EE^T)E = 0$, leads to a corresponding matrix constraint in terms of f,

$$\begin{aligned} G(f) &\stackrel{\text{def}}{=} 2\operatorname{diag}(f,f,1)FF^T\operatorname{diag}(f^2,f^2,1)F \\ &\quad - \operatorname{tr}\left(\operatorname{diag}(f,f,1)FF^T\operatorname{diag}(f,f,1)\right)\operatorname{diag}(f,f,1)F \stackrel{!}{=} 0. \end{aligned} \tag{8}$$

We determine f by minimizing the algebraic error, $\|G(f)\|_F^2$. First order optimality conditions, $d\|G(f)\|_F^2/df = 0$, yields a polynomial in f^5, f^3 and f. Since we can exclude the degenerate solution $f = 0$, a double quadratic polynomial in f^4 and f^2 can be obtained, which is trivial to solve after substituting $w = f^2$. Since f has to be strictly positive, up to two possible values for f need to be checked for optimality. Our algorithm has a complexity comparable to that of the standard 8-point algorithm for classical fundamental matrix, since the main step is finding the null space created by the nine correspondences (rather than the eight correspondences in the 8-point algorithm).

To test our the method on real images, we matched a set of uncalibrated/distorted images to an image with known intrinsics using different datasets. We ran our and the standard 8-point methods in a RANSAC framework.. Results show that our method uses a higher number of inliers with an equal or lower average epipolar error. In Fig. 1 we can see two typical situations were the standard 8-point method would use only the correspondences not heavily affected by radial distortion, whereas our method would use correspondences where radial distortion is severe.

[1] A.W. Fitzgibbon. Simultaneous linear estimation of multiple view geometry and lens distortion. In *Computer Vision and Pattern Recognition (CVPR)*, volume 1, pages 125–132, 2001.

[2] Magdalena Urbanek, Radu Horaud, and Peter Sturm. Combining off- and online calibration of a digital camera. In *International Conference on 3D Digital Imaging and Modeling (3DIM)*, pages 99–106, 2001.