Manifold-enhanced Segmentation through Random Walks on Linear Subspace Priors

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Segmentation of skeletal muscles in 3D Magnetic Resonance Imaging, which poses some very specific issues: simultaneous multi-object segmentation, non-discriminative appearance of the muscles, partial contours between them, large inter-subject variations, spurious contours due to fat infiltrations. For these reasons, it is necessary to impose knowledge-based shape priors into segmentation methods. In [2, 3, 4], segmentation is achieved with multi-object elastical deformable models, using either a reference atlas or a hierarchical statistical prior model. A discrete optimization procedure on a graph framework is used in [7], where a higher-order pose invariant shape prior is imposed on surface landmarks nodes. A pixel-wise region-based approach was proposed in [1] where prior knowledge is enforced by embeding the model into a statistical low-dimensional non-linear manifold through PCA in the Isometric Log-Ratio space. Random Walks for image segmentation, presented in [6], is a numerical method for computing globally large discrete regions-based segmentation methods, and is notoriously robust to partial contours. Prior knowledge on intensity within the RW formulation was introduced in [5].

In this paper, we propose a segmentation method based on RW, in which shape deformation is constrained to remain close to a PCA shape space built from training examples. Using the PCA allows us to model complex non-rigid shape variations relying on a few eigen-modes. Our method also benefits from the high performances of the RW optimization process.

The RW method amounts to computing the probability x_i^s that the node v_i is assigned to the label s . It was shown ([6]) that this probabilities minimize the functional:

$$E_{\mathrm{RW}}^{s}\left(x^{s}\right) = x^{sT} L x^{s} \tag{1}$$

where L is the combinatorial Laplacian matrix of the graph, defined as:

$$\forall\,(i,j) \in \mathcal{E},\; L_{ii} = \sum_{k} w_{ik},\; L_{ij} = -w_{ij}. \tag{2}$$

It is common practice to use as a Gaussian weighting function for w_{ij}. Once computed x^s for each label s, the segmentation is obtained by retaining the label of maximum probability: $\hat{s}_i = \arg\max_s x_i^s$.

Since minimizing (1) is an independent process for each label s, the whole RW process can be equivalently synthesized in one equation, via concatenation of the x^s and diagonal concatenation of L:

$$E_{\mathrm{RW}}\left(x\right) \quad = \quad x^{T} \bar{L} x. \tag{3}$$

The principle of a shape space is to design a low dimensional affine space approximating this implicit space. Assume we possess T co-registered segmented training volumes. We perform the PCA on vectors $\left\{\hat{x}^i\right\}_{i=1\ldots T}$, which are ground truth segmentations represented as probability vectors. Retaining the n eigen-modes of greatest variance, the projection of any segmentation in the shape space is represented as:

$$\tilde{x} = \bar{x} + \Gamma\gamma.$$

Thus, any segmentation x can be expressed as:

$$x = dx + \Gamma\gamma + \bar{x} \tag{4}$$

where $dx \in \mathbb{R}^{KN\times 1}$ is the deviation of x from the shape space.

In order to obtain a segmentation which remains close to the shape space, we want to minimize the objective function (3) with respect to both dx and γ, while keeping dx small. This leads us to the following functional:

$$E_1\left(dx,\gamma\right) = \left(dx + \Gamma\gamma + \bar{x}\right)^T \bar{L}\left(dx + \Gamma\gamma + \bar{x}\right) + \lambda\left\|dx\right\|^2. \tag{5}$$

The minimum of (5) verifies:

$$\left(A^T \bar{L} A + \lambda B\right) y = A^T \bar{L} \bar{x} \tag{6}$$

with:

$$y = \begin{bmatrix} dx \\ \gamma \end{bmatrix},\; A = \left[I_{KN}\;\Gamma\right],\; B = \begin{bmatrix} I_{KN} & 0 \\ 0 & 0 \end{bmatrix}. \tag{7}$$

The system of equations (6) can be solved with iterative methods like Bi-Conjugate Gradient. Computation time is approximately 15 min per segmentation on a 2.8 GHz Intel process with 4GB of RAM. We present results obtained with our method on a set of 3D MR volumes of muscles. Our data set is comprised of 30 volumes of the right thigh of healthy subjects, covering a wide range of ages and morphologies. On figure 1, we show the effect of the PCA shape prior on one example of our dataset.

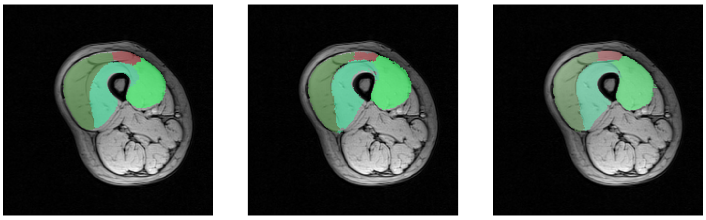

Figure 1: Effect of the PCA shape prior: (left) mean segmentation using $x = \bar{x}$, (middle) shape space segmentation using $x = \Gamma\gamma + \bar{x}$, (right) segmentation with shape prior using $x = dx + \Gamma\gamma + \bar{x}$. The shape space segmentation fits the boundaries better than the mean segmentation, but has fuzzy contours due to the approximation of projecting complex shapes into a linear subspace.

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