Towards Efficient and Compact Phenomenological Representation of Arbitrary Bidirectional Surface Reflectance

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The visual appearance of real-world surfaces is the net result of surface reflectance characteristics when exposed to illumination. Appearance models can be constructed using phenomenological models which capture surface appearance through mathematical modeling of the reflection process. This yields an integral equation, known as *reflectance equation*, describing the surface radiance, which depends on the interaction between the incident light field and the surface *bidirectional reflectance distribution function* (BRDF). The theoretical formulation and modeling of the reflectance equation is of a great interest to numerous computer vision tasks, such as illumination modeling, surface reflectivity estimation/analysis, shape and albedo recovery, shape from shading, photometric stereo, object detection and recognition, to name a few.

The surface BRDF is a function defined on the cartesian product of two hemispheres corresponding to the incident and outgoing directions; the natural way to represent such a hemispherical function is to use hemispherical basis. However, due to their compactness in the frequency space, spherical harmonics (SH) have been extensively used for this purpose. In this paper, we address the geometrical compliance of hemispherical basis for representing surface BRDF. We propose a tensor product of the hemispherical harmonics (HSH) to provide a compact and efficient representation for arbitrary BRDFs, while satisfying the Helmholtz reciprocity property.

Under the assumption of no surface emittance, a surface point $\mathbf{x}$ is locally seeing the surrounding world through a unit hemisphere Ω oriented by the surface normal $\mathbf{n}(\mathbf{x})$ at this point, see Fig. 1. When it is illuminated by radiance $L(\mathbf{x},\mathbf{u}_i)$ coming in from a directional region of solid angle $d\omega_i$ at direction $\mathbf{u}_i = (\theta_i,\phi_i)$, it receives irradiance $L(\mathbf{x},\mathbf{u}_i)\cos\theta_i d\omega_i$, where $\cos\theta_i$ accounts for the foreshortening effect, $\theta_i,\theta_o \in [0,\pi/2]$ and $\phi_i,\phi_o \in [0,2\pi]$. The BRDF, denoted by $\rho_{bd}(\mathbf{x},\mathbf{u}_i,\mathbf{u}_o)$ is the ratio of the surface radiance in the outgoing direction $\mathbf{u}_o$ to the irradiance coming from the incident direction $\mathbf{u}_i$ at the same surface point. Assuming homogeneous surfaces, the *surface reflectance function* can be obtained by integrating irradiance-weighted BRDF over all incoming directions.

$$\mathscr{R}(\mathbf{x},\mathbf{u}_o) = \int_{\Omega_i} L(\mathbf{u}_i)\rho_{bd}(\mathbf{u}_i,\mathbf{u}_o)\cos\theta_i d\omega_i \tag{1}$$

Consider the product of two hemispherical harmonics basis functions to give a mapping $\Omega\times\Omega \to \mathbb{R}$ from the tensor product of two hemispheres to the real line, it is possible to define a combined basis function $H_n^m(\mathbf{u}_i)H_p^q(\mathbf{u}_o)$. However, in order to preserve the Helmholtz reciprocity property, we define what we call *Helmholtz HSH-based basis* by symmetrizing the combined basis with respect to the incident and outgoing directions.

$$\mathscr{H}_{np}^{mq}(\mathbf{u}_i,\mathbf{u}_o) = N_{np}^{mq}\left(H_n^m(\mathbf{u}_i)H_p^q(\mathbf{u}_o) + H_p^q(\mathbf{u}_i)H_n^m(\mathbf{u}_o)\right) \tag{2}$$

where N_{np}^{mq} is a normalization factor which guarantee the basis orthonormality. Thus, an arbitrary BRDF can be represented in terms of the Helmholtz HSH-based basis as follows,

$$\rho_{bd}(\mathbf{u}_i,\mathbf{u}_o) = \sum_{n=0}^{\infty}\sum_{p=0}^{\infty}\sum_{m=-n}^{n}\sum_{q=-p}^{p} a_{np}^{mq}\mathscr{H}_{np}^{mq}(\mathbf{u}_i,\mathbf{u}_o) \tag{3}$$

Since the set of associated Legendre polynomials is distinguished by the property that it contains a polynomial for every combination of order and degree [1], compared to Zernike polynomials which are restricted to even differences between polynomial order and degree, we provide an analytical analysis and experimental justification that for a given approximation order, our proposed Helmhholtz HSH-based basis provide better approximation accuracy when compared to the basis proposed by Koenderink *et al*. [3] [2], while avoiding the high computational complexity inherited from Zernike polynomials. We believe that our BRDF representation can be used in place of simple Lambertian models in algorithms such as shape-from-shading and photometric stereo. We validate our basis functions on Oren-Nayar and Cook-Torrance BRDF physical models. Scattered BRDF measurements which might violate the Helmholtz reciprocity property can be filtered out through the process of projecting them on the subspace spanned by our HSH-based basis, where the reciprocity property is preserved in the least-squares sense.

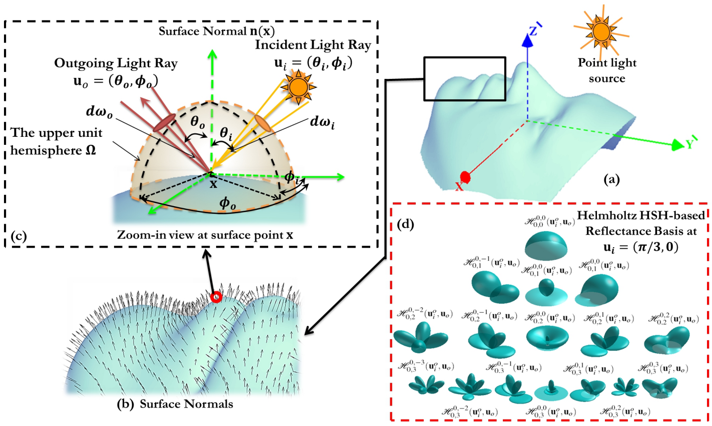

Figure 1: (a) An object's surface is illuminated by a distant point light source. We use the mean shape from USF HumanID 3D face database [4] for illustration. (b) An in-depth view of a surface patch showing surface normals at each surface point. (c) A zoom-in view at a surface point $\mathbf{x}$ seeing its surrounding world through a unit hemisphere Ω centered at the point and oriented by the surface normal $\mathbf{n}(\mathbf{x})$ at that point. (d) Visualization of up-to 3rd order of our proposed Helmholtz surface reflectance basis at an incident direction $\mathbf{u}_i = (\pi/3, 0)$.

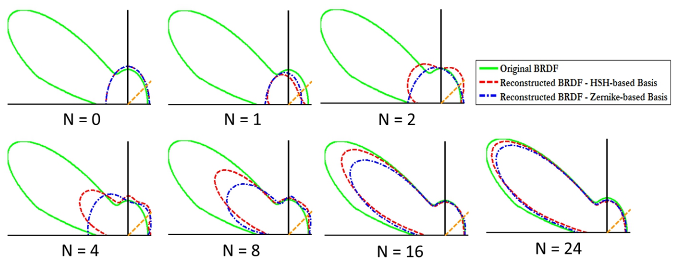

Figure 2: Example of fitting HSH-based basis (dashed red) versus the Zernike-based ones (dashed blue) on a Cook-Torrance BRDF (solid green) with high surface specular reflectivity, where $\rho_s = 0.458, \mathrm{m} = 0.125, \theta_i = \pi/3, \phi_i = 0$. Notice how HSH-based basis provide better fitting at lower orders when compared to the Zernike-based basis. For visualization purposes, the incident plane is plotted and dashed orange line represents the light incident direction.

[1] A. B. Bhatia and E. Wolf. On the circle polynomials of zernike and related orthogonal sets. In *Mathematical Proceedings of the Cambridge Philosophical Society*, number 50.

[2] J. Koenderink and A. van Doorn. Phenomenlogical description of bidirectional surface reflection. *Journal of the Optical Society of America*, 15(11):2903–2912, 1998.

[3] J. Koenderink, A. Van Doorn, and M. Stavridi. Bidirectional reflection distribution function expressed in terms of surface scattering modes. In *ECCV*, pages 28–39, 1996.

[4] S. Sarkar. Usf humanid 3d face database. University of South Florida, Tampa, FL, USA.