# A New Feature-preserving Nonlinear Anisotropic Diffusion Method for Image Denoising


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Image denoising is a long-studied subject with growing importance in the field of image processing and computer vision for various applications. Existing denoising methods [1] commonly use a first-order derivative to detect edges in order to achieve a good balance between noise removal and feature preserving. However, if edges are partly lost to a certain extent or contaminated severely by noise, these methods may not be able to recover them and thus fail to preserve other features that are bounded by edges. To overcome this problem, we propose a novel feature-preserving denoising method by combining the first- and second-order nonlocal derivatives to form a new feature detector in a nonlinear diffusion model. Experimental results demonstrate that our denoising method can achieve a higher Peak-signal-to-noise ratio (PSNR) and higher mean similarity index (MSSIM) than several commonly used algorithms when applied to natural images.

Let $\boldsymbol{I}(x) = \{I(x_i)\}_{i=1,2,\ldots,N}$ be a one-dimensional image containing edges and blobs, as shown in Figure 1. A simplest way to detect an edge

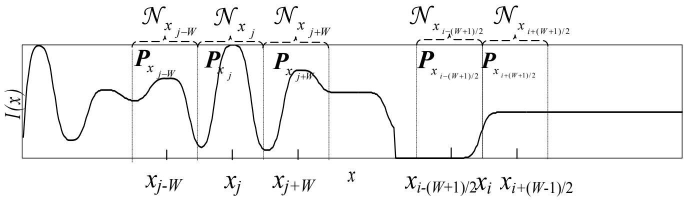



Figure 1: A schematic showing nonlocal difference in 1-D space

and blob is to apply the gradient and Laplacian operator. However, the presence of noise can lead to false detections because the measurement of pixel-level difference is prone to be corrupted by noise. Based on the concept of patch difference which was firstly introduced in the popular nonlocal means (NLM) filter [2], we define a first- (1st NLD) and second-order nonlocal derivative (2nd NLD) at pixel $x_i$ as:

$$\left|\nabla_{\mathrm{NL}} I(x_i)\right| = \left\| \boldsymbol{P}_{x_{i-(W+1)/2}} - \boldsymbol{P}_{x_{i+(W+1)/2}} \boldsymbol{Q} \right\|_{2,\sigma} = \sqrt{\sum_{k=-(W-1)/2}^{(W-1)/2} G_\sigma(k)\left(I(x_{i-(W+1)/2+k}) - I(x_{i+(W+1)/2-k})\right)^2} \quad ,(1)$$

and

$$\left|\nabla^2_{\mathrm{NL}} I(x_j)\right| = \left\| \boldsymbol{P}_{x_j} - \left(\boldsymbol{P}_{x_{j-W}} + \boldsymbol{P}_{x_{j+W}}\right)/2 \right\|_{2,\sigma} \quad , \qquad (2)$$

where $\boldsymbol{P}_{x_i} = \left[I(x_{i-(W-1)/2}), I(x_{i-(W-1)/2+1}), \ldots I(x), \ldots I(x_{i+(W-1)/2})\right]^{\mathrm{T}}$ is a patch at $x_i$, $W$ is the patch size, $G_\sigma$ is a normalized Gaussian kernel with Std $\sigma$ and $\boldsymbol{Q}$ is a matrix with ones on the secondary diagonal and zeros elsewhere. Eq. (1) and (2) are more reliable than the pixel-level first-order and second-order operator to measure edges and blobs under noise contamination, respectively.

By incorporating the combination of 1st and 2nd NLD into the diffusion model [1], we form a more sophisticated feature-preserving nonlinear anisotropic diffusion (FP-NAD),

$$\partial I(x_i,t)/\partial t = \mathrm{div}\left[c\left(\left|\nabla_{\mathrm{NL},\sigma} I(x_i,t), \nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|\right) \cdot \nabla I(x_i,t)\right] \quad , \qquad (3)$$

where the diffusion coefficient (DC) is defined as

$$c\left(\left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|\right) = \exp\left[-\left(\left(w_1 \cdot \left|\nabla_{\mathrm{NL},\sigma} I(x_i,t)\right| + w_2 \cdot \left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|\right)/h\right)^2\right], \quad (4)$$

$I(x_i, t = 0) = I_0(x_i)$ is the initial noisy image, $\nabla$ is the gradient operator and div is the divergence operator. The weights $w_1$ and $w_2$ should be appropriately chosen for balancing the contributions of 1st NLD and 2nd NLD. We define $w_1$ and $w_2$ as,

$$w_1 = \frac{\left|\nabla_{\mathrm{NL},\sigma} I(x_i,t)\right|}{\left|\nabla_{\mathrm{NL},\sigma} I(x_i,t)\right| + \left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|}, \quad w_2 = \frac{\left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|}{\left|\nabla_{\mathrm{NL},\sigma} I(x_i,t)\right| + \left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|}. \quad (5)$$

In the vicinity of an edge, $\left|\nabla_{\mathrm{NL}} I(x_i,t)\right| > \left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|$, so $w_1 > w_2$ and the 1st NLD contributes more to the DC; in the vicinity of a blob, $\left|\nabla_{\mathrm{NL}} I(x_i,t)\right| < \left|\nabla^2_{\mathrm{NL},\sigma} I(x_i,t)\right|$, therefore $w_1 < w_2$ and the 2nd NLD contributes more. Since the DCs are small in the vicinity of both features and high in other regions, the diffusion (smoothing) process will be discouraged considerably in feature regions and encouraged in background regions, leading to a feature-preserving nonlinear diffusion method that preserves features and removes noise in the backgrounds.

We perform several experiments on natural images. As shown in a table below, our method achieves highest PSNR and MSSIM when denoising the image *Barbara*, compared to Perona and Malik (PM)[1], structure adaptive (SAFIR)[3] and block matching (BM3D)[4]. The

| $\sigma_n$ | PSNR/MSSIM values | | | |
|---|---|---|---|---|
| | Our method | PM | SAFIR | BM3D |
| 25 | **31.22/0.901** | 24.47/0.710 | 27.78/0.790 | 30.73/0.887 |
| 30 | **30.37/0.892** | 24.03/0.635 | 26.39/0.748 | 29.76/0.864 |
| 40 | **28.85/0.843** | 22.16/0.514 | 24.30/0.674 | 28.07/0.824 |

latter is a state-of-the-art algorithm. A visual comparison of *Barbara*'s face and knee is given in Figure 2. As seen, the nose, eyes on the face and textures on the knee are better preserved by our method, compared to the BM3D. Figure 3 shows results on a fragment of image *Parrot* [5] under severe noise contamination, indicating the validity of our method in dealing with extremely low-PSNR images.

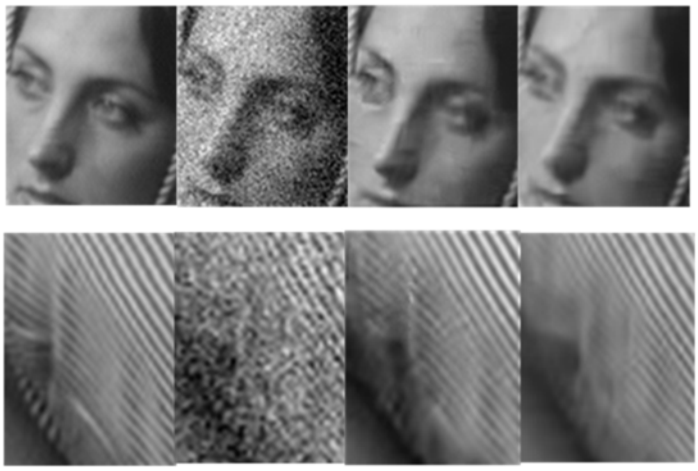
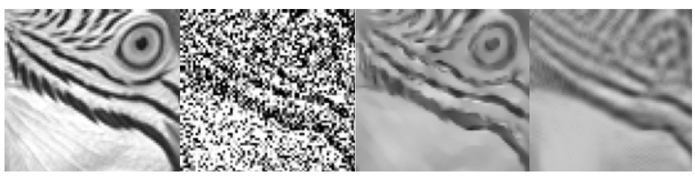


Figure 2: From left to right: Noisy-free and noisy images with AWGN of a Std $\sigma_n$= 25, results by our method and BM3D.



(a) +∞/1 (b) 8.40/0.223 (c) 15.29/0.622 (d) 14.46/0.484

Figure 3: (a) – (b) Noise-free and Noisy image ($\sigma_n$ = 120); (c) - (d) Denoised results by our method and BM3D, respectively. Two numbers under each image are the corresponding PSNR and MSSIM.


[1] P. Perona and J. Malik. Scale–space and edge detection using anisotropic diffusion. *IEEE TPAMI*, 12(7): 629-639, 1990.

[2] A. Buades *et al*.. A review of image denoising algorithms, with a new one. *Multi. Model. Simu.*, 4(2): 490– 530, 2005.

[3] C. Kervrann and J. Boulanger. Optimal spatial adaptation for patch-based image denoising. *IEEE TIP*, 15(10): 2866-2878, 2006.

[4] K. Dabov *et al*. Image denoising by sparse 3-D transform-domain collaborative filtering. *IEEE TIP*, 16(8): 2080-2095, 2007

[5] http://www.r0k.us/graphics/kodak/kodim23.html.