Compressive Inverse Light Transport

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A forward light transport simulates global illumination in a scene given direct lighting or corresponding light source emission. It embodies the forward rendering process, a cornerstone of computer graphics, which aggregates the effect of light bouncing in a scene. An inverse light transport reverses the forward process; it enables undoing of interreflections and separation of light bounces in a real scene. In all current work, an inverse light transport matrix (*i*-LTM) is obtained by inverting a forward light transport matrix (*f*-LTM). For a projector-camera setup, a *f*-LTM can easily exceed the size of $10^5 \times 10^5$. Acquiring such a large *f*-LTM can take hours or days while inverting the *f*-LTM requires various forms of approximation which compromises on the accuracy of the inverse light transport. In this work, we propose a way of computing *i*-LTM directly from the measurements without the need for a prior and explicit reconstruction of the *f*-LTM. We show that within the framework of *compressive inverse theory*, *i*-LTM can be obtained by compressed sensing directly without additional computational cost and aggregated error from matrix inversion. This is done by computing the response of each pixel by projecting patterned illumination. We probe the light transport matrix by m illumination conditions $\mathbf{L} = [\mathbf{l}_0, \ldots, \mathbf{l}_m]$ to obtain their corresponding observations $\mathbf{C} = [\mathbf{c}_0, \ldots, \mathbf{c}_m]$, which is,

$$\mathbf{C} = \mathbf{T}\mathbf{L} \Leftrightarrow \mathbf{C}^T = \mathbf{L}^T\mathbf{T}^T \Leftrightarrow \mathbf{c}'_i = \mathbf{L}^T\mathbf{t}'_i, \tag{1}$$

where $\mathbf{t}'_i$ is a column in $\mathbf{T}^T$ that represents the reflectance function of the i-th pixel in the camera image and $\mathbf{c}'_i$ is a column of $\mathbf{C}^T$. Eq.1, relating illumination and observation by $\mathbf{T}$, can be rewritten as:

$$\mathbf{C}^T(\mathbf{T}^{-1})^T = \mathbf{L}^T \tag{2}$$

This formulation maps directly to a compressive sensing context where $\mathbf{C}^T$ fulfills the role of the encoding matrix, and the $\mathbf{L}^T$ represents the codes. To exploit sparsity in both rows and columns of $\mathbf{T}^{-1}$, $\mathbf{T}^{-1}$ can be further represented as $\mathbf{T}^{-1} = \mathbf{W}\widehat{\mathbf{T}^{-1}}\mathbf{W}^T$. The right transformation $\mathbf{W}^T$ operates on the rows of the transport matrix to exploit the coherency within the rows, while the left transformation $\mathbf{W}$ operates on the columns of the *i*-LTM.

$$\mathbf{C}^T\mathbf{W}(\widehat{\mathbf{T}^{-1}})^T = \mathbf{L}^T\mathbf{W}. \tag{3}$$

The optimization problem for reconstructing the *i*-LTM $(\mathbf{T}^{-1})^T$ becomes:

$$\text{(P3)} \qquad \min \|\mathbf{h}_i\|_0 \text{ s.t. } \mathbf{C}^T\mathbf{W}\mathbf{h}_i = \mathbf{L}_w(i), \quad i = 1, \ldots, N \tag{4}$$

where $\mathbf{h}_i$ denotes the i-th column of $(\widehat{\mathbf{T}^{-1}})^T$, *i.e.* $(\widehat{\mathbf{T}^{-1}})^T = [\mathbf{h}_0, \ldots, \mathbf{h}_N]$, and $\mathbf{L}_w(i)$ denotes the i-th column of $\mathbf{L}^T\mathbf{W}$. For orthonormal matrix $\mathbf{W}$, the following lemma shows a necessary and sufficient condition for this problem to have a unique solution.

Lemma 0.1. *Given a set $J \subseteq \{1, 2, \ldots N\}$, define the matrix $\mathbf{T}_J$ as the one formed from $\mathbf{T}$ by using columns from the set J. Let $\mathbf{h}_i \in \mathbb{S}_k$ be the ith column of $(\widehat{\mathbf{T}^{-1}})^T$. There exist a unique solution $\hat{\mathbf{h}}_i$ to the minimization procedure P(3) such that $\hat{\mathbf{h}}_i = \mathbf{h}_i$,*
if and only if

$$\ker(\mathbf{L}^T) \cap \mathbf{T}_J = \{0\}, \{\forall J | |J| \le 2k\}$$

It can be interpreted as follows, for the minimizer $\hat{\mathbf{h}}_i$ of (P3) to be the i-th column of $(\widehat{\mathbf{T}^{-1}})^T$, a necessary and sufficient condition is that, any of the K-subspaces spanned by $\mathbf{T}$ is not contained in the null space of $\mathbf{L}^T$.

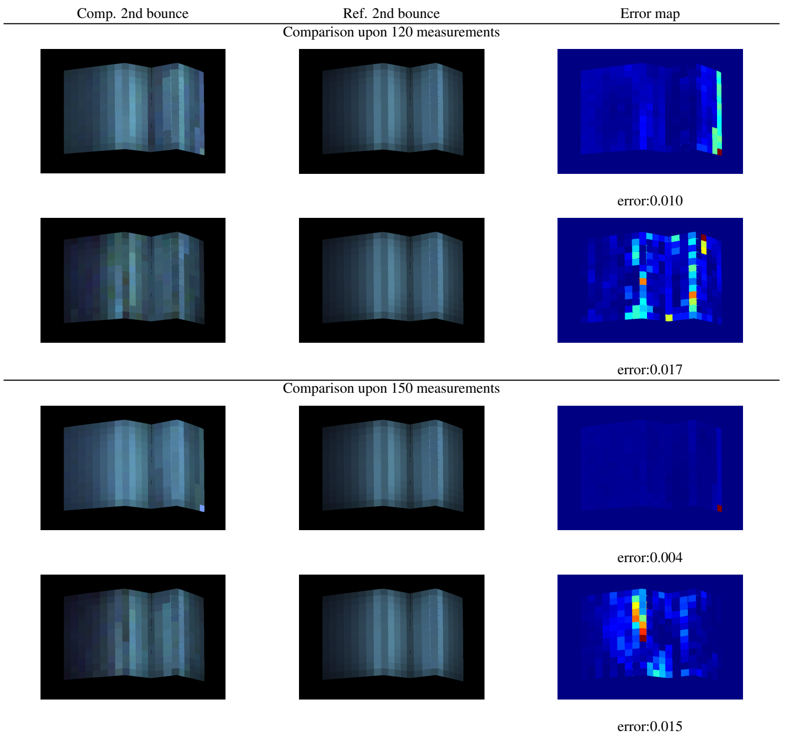

Figure 1: Quantitative comparison of the 2nd-bounces separated from the *i*-LTM acquired by our direct method and the two-phase method. The first row and the third row show the results of our direct method, while the second row and fourth row show the reconstruction results of the two stage method.

Now we develop a sufficient condition under which problem (P4) in Eq. 5 guarantees to have the same solution as problem (P3) thereby enabling efficient algorithms.

$$\text{(P4)} \qquad \min \|\mathbf{h}_i\|_1 \text{ s.t. } \mathbf{C}^T\mathbf{W}\mathbf{h}_i = \mathbf{L}_w(i), \quad i = 1, \ldots, N. \tag{5}$$

Our theorem below is based on a careful examination of submatrices consisting of an arbitrary collection of k columns. Please refer to our paper for explanation of notations.

Theorem 0.1. *Let $\mathbf{L}^T$ be any matrix of size $m \times N$ with D-RIP ratio $\gamma_{\mathbf{L}^T}(2k)$. Let $\kappa_k(\mathbf{T}) = \sigma^{(k)}_{max}(\mathbf{T})/\sigma^{(k)}_{min}(\mathbf{T})$, if*

$$\kappa^2_{2k}(\mathbf{T}) \cdot \gamma_{\mathbf{L}^T}(2k) < \sqrt{2} + 1 \tag{6}$$

Then, ℓ_1-minimization will exactly recover $\mathbf{h} \in \mathbb{S}_k$.

This theorem states that, provided that the submatrices of $\mathbf{T}$ is reasonably well-conditioned, we can still ensure an exact recovery of a sparse $\mathbf{h}$. When $\mathbf{L}^T$ has a smaller isometry constant δ_{2k}, *i.e.*, $\mathbf{L}^T$ contains more random patterns, then there is larger room for $\kappa^2_{2k}(\mathbf{T})$ to vary while still ensures reconstruction. On the contrary, if the submatrices of $\mathbf{T}$ are well-conditioned, *i.e.*, $\kappa^2_{2k}(\mathbf{T}) \to 1$, we can afford an illumination pattern $\mathbf{L}^T$ with less rows (fewer measurements). Fig.1 shows a quantitative comparison of the 2nd-bounces from the *i*-LTM acquired by our direct method and the two-phase method.