Morphometric Shape Analysis with Measurement Covariance Estimates

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Abstract: We propose a shape analysis system based upon the description of landmarks with measurement covariance, which extends statistical linear modelling processes to 'pseudo landmarks' for scientific studies. We discuss the properties of our approach and how measurement covariances can be considered characteristic of the local shape. Our formulation includes corrections for parameter bias, induced by the degrees of freedom within the linear model. The method has been implemented and tested on measurements from fly wing, hand and face data. We use these data to explore possible advantages and disadvantages over the use of standard Procrustes/PCA analysis. In the process we show how appropriate weighting provides more efficient use of the available landmark data.

Motivation:

- Pseudo landmarks are in-appropriately weighted in morphometric analysis leading to statistical problems and the need to constrain landmark definitions.
- We seek a more general framework for the analysis of landmark points which allows the use of measurement covariances.
- We seek to improve the statistical power of data.
- We need to consider morphometric/biological requirements.
- The literature claims that measurement distributions are in-estimable. Here, we show the opposite.

Theory:

- According to the statistical theory of Likelihood, assumed distributions should match sample covariance (plus corrections).
- Conventional Procrustes alignment assumes an isotropic distribution about the mean shape. Also, PCA assumes isotropic measurement noise about the fitted hyper-plane.
- Biological variation and pseudo landmarks invalidate the statistical requirements by Procrustes and PCA.
- Persistence in isotropic assumptions for non-isotropic data will lead to poor statistical performance.

Methods:

- We generalise the least-squares cost function to a Mahalanobis distance using Likelihood.
- Similar to Procrustes, scale, rotation and translation are applied to the original data to get an aligned version of the data.
- A 2x2 covariance for each landmark is derived from the measurement process (all of which are composed into a single matrix).
- Ghost points are scaled projections relative to the mean shape (where uniform independent noise distribution is obtained).
- PCA is applied to the sum of correlations between ghost points.
- In case of aligning to the mean shape, one should use a covariance consistent with the distribution around the mean.
- The use of free parameters during model fitting reduces the sample covariance from residuals. To obtain a true unbiased estimate, we account for the amount of covariance due to rotation, translation and scale.
- To stabilise the alignment process, any variation about the mean which could have been described by an alignment parameter is removed from the correlation function prior to model construction.

Experiments:

- For the hand data, the largest variances correspond to the points which are not well constrained or to some extent unstable.
- For the face data, variations are similar on the points which are on symmetrical locations, and the orientation of error bars for points on the symmetry line are approximately vertical.
- For the fly wing data, error bars are oriented as expected based on the structures observed (using a simple taxonomy).
- Stability test gives an indication of the appropriateness of the assumed linear model (symmetry/reproducibility).
- The new method gives Fisher information (FI) values almost twice those obtained using Procrustes. As FI is proportional to the quantity of data, this demonstrates that the changes away from the isotropic assumption inherent to Procrustes/PCA has a significant effect on the efficacy of the model, equivalent to having defined only half as many landmarks from the outset.

Figure 1: Typical landmarks on sample images from the hand data (40 samples, 56 landmarks), face data (200 samples, 68 landmarks) and fly wing data (200 samples, 15 landmarks).

Figure 2: Aligned points when the new method is applied to the hand, face and fly wing data.

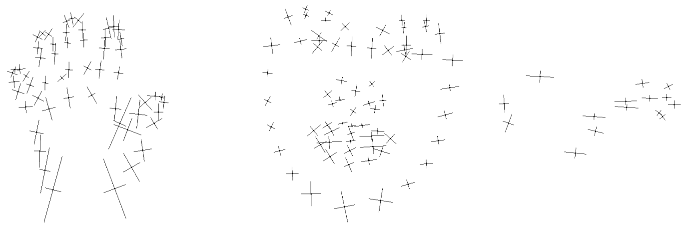

Figure 3: Magnified error bars from the covariance matrices estimated by the new method for the hand data (×20), face data (×20) and fly wing data (×30).

Conclusions: The expected advantages of the new method are:

- it will lead to a consistent way of incorporating all forms of landmark into our analysis;
- it removes the instabilities inherent in the analysis due to poorly determined points;
- it affords the application of an eigen-vector analysis statistical rigour;
- it offers the possibility of interpreting the linear modelling process as a statistical approximation, with consequent interpretations of the requirement for the number of linear model components;
- finally, generalisation of the approach would seem to be possible which would support the analysis of curves and surfaces.