Skeleton Extraction via Anisotropic Heat Flow

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Skeleton is a thin version of the shape, which is a useful feature for shape description in image processing and computer vision. In skeleton extraction, it is an important task to compute a smooth and high quality medial function that can provide skeleton features easily and without noise. One of the main features for skeleton point detection is curvature maxima along the level-sets of the medial function. We present the proposed anisotropic heat diffusion as a novel medial function generation technique, which can remove noise in the image and can preserve the prominent curvatures of the object shape along the level-sets. The proposed anisotropic heat diffusion equation is obtained with the following considerations.

Let η denote the direction normal to the feature boundary (the gradient direction) through a given point in an image I, and let τ denote the tangent direction. These directions can be written in terms of the first derivatives of the image, I_x and I_y, as

$$\eta = \frac{(I_x, I_y)}{\sqrt{I_x^2 + I_y^2}}, \qquad \tau = \frac{(-I_y, I_x)}{\sqrt{I_x^2 + I_y^2}} \tag{1}$$

Since η and τ constitute orthogonal directions, the rotationally invariant Laplacian operator can be expressed as the sum of the second order spatial derivatives, $I_{\eta\eta}$ and $I_{\tau\tau}$, in these directions and the linear heat conduction equation can be written as follows,

$$\frac{\partial I}{\partial t} = \nabla^2 I = (I_{\eta\eta} + I_{\tau\tau}) \tag{2}$$

Omitting the normal diffusion, while keeping the tangential diffusion yields the well known Geometric Heat Flow (GHF) equation [2] as

$$\frac{\partial I}{\partial t} = I_{\tau\tau} = \frac{(I_{xx}I_y^2 - 2I_{xy}I_xI_y + I_{yy}I_x^2)}{(I_x^2 + I_y^2)} \tag{3}$$

GHF derives its name from the fact that, under this flow, the feature boundaries of the image evolve in the normal direction in proportion to their curvature. Thus GHF decreases the curvature of level-sets of the image while removing noise to obtain sharp edges.

On the other hand, omitting the tangential diffusion, while keeping the normal diffusion in the heat equation yields

$$\frac{\partial I}{\partial t} = I_{\eta\eta} = \frac{(I_{xx}I_x^2 + 2I_{xy}I_xI_y + I_{yy}I_y^2)}{(I_x^2 + I_y^2)} \tag{4}$$

Normal diffusion does not preserve edges since it diffuses across them in the image. Because of this, it did not take much attention for image enhancement or for edge detection. Here, we choose the normal diffusion as a main tool for medial function generation in binary and gray-scale images for skeleton extraction purpose. The normal diffusion can preserve prominent curvatures of the shapes along the level-sets in images while removing noise, since it does not diffuse along the level-sets (in tangent direction). However, we do not completely omit tangential diffusion and let it contribute slightly depending on the user, since it can also remove noise along the feature boundaries and contribute to obtain smoother skeleton. The proposed anisotropic heat diffusion problem for medial function generation, in images, is given below,

$$\frac{\partial I}{\partial t} = I_{\eta\eta} + c\, I_{\tau\tau} \quad , \qquad 0 \le c \le 0.4 \tag{5}$$

$$I(\mathbf{x}, t=0) = F(\mathbf{x}), \quad \text{initial condition}$$

$$\frac{\partial I(\mathbf{x}, t)}{\partial n} = 0, \quad \text{boundary condition}$$

where $0 \le c \le 0.4$ is the diffusion coefficient which is a positive constant and bounded to prevent excess smoothing in direction tangent. Note that in our experiments $c = 0.2$. $\mathbf{x} = (x, y)$ is the space vector and $F(\mathbf{x})$ is the input binary or gray-scale image, which represents initial condition of the diffusion problem. The boundaries of the image are insulated with homogeneous Neuman condition, $dI/dn = 0$, which means there is no heat flow in, or out of the image domain definition. Note that in boundary condition n represents direction normal to the image boundary.

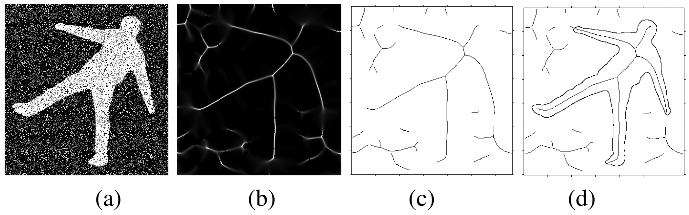

Figure 1: Experimentation with respect to salt and pepper noise (density=0.3) in the binary image of size 350×335. (a) Original image. (b) Skeleton strength map for brighter regions $SSM < 0$. (c) Thin and binary skeleton, (d) Binary skeleton with the subject shape in the same image.

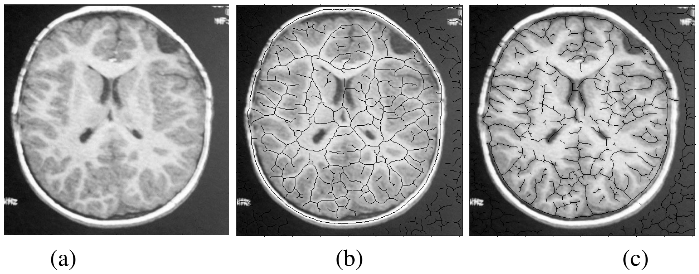

Figure 2: Skeletonization of gray-scale brain image of size 413×404. (a) The human brain, (b) Binary skeleton for $SSM < 0$, (c) Binary skeleton for $SSM > 0$.

The skeleton strength map is likelihood for each pixel to be a skeleton point. In our algorithm, the skeleton strength map (SSM) is the mean curvature measure of the level-sets that is computed after terminating diffusion,

$$SSM = \nabla \cdot \left(\frac{\nabla I(\mathbf{x}, t_s)}{|\nabla I(\mathbf{x}, t_s)|} \right) \tag{6}$$

where the $SSM < 0$ represents SSM for brighter regions and the $SSM > 0$ represents SSM for darker regions. t_s is the number of iterations (diffusion time). The overall process is completed by non-maxima suppression to make SSM thin, and hysteresis thresholding to observe binary skeleton. Figure 1 illustrates skeletonization of the binary image of the human, which is corrupted by salt and peper noise (density=0.3). Figure 2 shows skeletons computed in the gray-scale brain image.

The proposed anisotropic diffusion provides high quality skeleton features, while denoising the image and the object boundary. Results indicate that proposed diffusion generates better medial function than distance transform [3], Poisson equation (linear diffusion) [1] and Gaussian filtering (linear diffusion) [4]. Results also show that our technique can be applied to both binary and gray-scale images.

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