

Multi-Scale Depth from Slope with Weights

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The *integration of a slope map* to yield a height (or depth) map is a critical step in many machine vision techniques such as shape-from-shading [4] and multiple-light photometric stereo [5]. These methods have inherent advantages over competing techniques such as laser and stereo triangulation including their low cost, high resolution and their ability to recover albedo information. Major obstacles to the wider use of integration methods are the computational cost and the fragility of current approaches.

Efficient algorithms, such as those integrators based on Fourier filtering methods [3] or path-oriented algorithms [2, 6], are unable to efficiently handle issues that appear in slopes obtained with real world data. The Fourier filtering approach is robust against noise but does not take into account the occurrence of discontinuities on the surface or unreliable data on the computed slope map. While path-oriented methods avoid undesired regions and cliffs, the presence of noise produces spurious height differences between adjacent pixels since the accumulated error propagates along the paths. Weight maps can be used to exclude regions where the slope information is missing or untrusted, and to allow for the integration of height maps with linear discontinuities (such as along object silhouettes) which are not recorded in the slope maps. The weight maps can be determined by external information or by cliff or shadow detection algorithms. Integrators based on the weighted Poisson Equation offer good quality reconstructions in presence of the aforementioned problems, given a suitable weight map. However, local iteration or direct system solving [1] methods are too costly and are therefore impractical for high-resolution slope maps.

In this paper, we describe a robust integrator in which the key feature is the fast solution of the weighted Poisson-like integration equations. The equations are solved by a multi-scale iterative technique using the Gauss-Seidel (or Gauss-Jacobi) algorithm. Each equation states the equality between two estimates of the Laplacian $\mathbf{L}(z) = \nabla \cdot (\nabla z)$ at the point $q[u, v]$: one computed from the unknown heights (the left-hand side), and one from the given slope values (the right-hand side). In our implementation we use the equation $-\mathcal{L}(z)[u, v] = -\mathcal{D}(f, g)[u, v]$, which can be expressed by

$$-\mathcal{L}(z)[u, v] = z[u, v] - \frac{w_{-0}}{w_{00}} z_{-0} - \frac{w_{+0}}{w_{00}} z_{+0} - \frac{w_{0-}}{w_{00}} z_{0-} - \frac{w_{0+}}{w_{00}} z_{0+} \quad (1)$$

$$\begin{aligned} -\mathcal{D}(f, g) &= \frac{w_{--}}{w_{00}} f_{--} + \frac{w_{-+}}{w_{00}} f_{-+} - \frac{w_{+-}}{w_{00}} f_{+-} - \frac{w_{++}}{w_{00}} f_{++} \\ &+ \frac{w_{-0}}{w_{00}} g_{-0} + \frac{w_{+0}}{w_{00}} g_{+0} - \frac{w_{0-}}{w_{00}} g_{0-} - \frac{w_{0+}}{w_{00}} g_{0+} \end{aligned} \quad (2)$$

Here w_{rs} , z_{rs} , f_{rs} and g_{rs} are the weight, height and slopes at point $q[u+r, v+s]$ where all $r, s \in \{-, +\} = \{-1, +1\}$.

Unlike other local iterative methods, it obtains the initial estimate by recursively solving a reduced scale version of the problem. Namely, it reduces the given slope maps f, g and the weight map w to one half of their original width and height, recursively computes from them a reduced-scale height map z , expands the latter to twice its size, and uses the Gauss-Seidel iteration to adjust this map according to the full-scale slope data. The recursion stops at a level m where the slope maps are so small that the iteration will quickly converge from any initial guess. See figure 1.

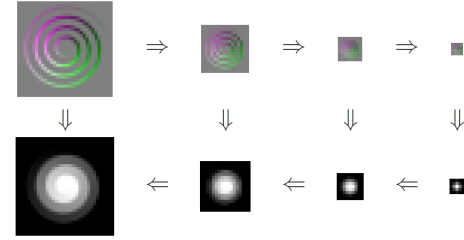


Figure 1: The multiscale integration method.

The central part of our algorithm is the recursive procedure below:

ComputeHeights(f, g, w)

1. If size of f is small enough then
 2. $z \leftarrow (0, 0, \dots, 0)$;
 3. else
 4. $f' \leftarrow \text{ShrinkSlopes}(f, w)$; $g' \leftarrow \text{ShrinkSlopes}(g, w)$;
 5. $w' \leftarrow \text{ShrinkWeights}(w)$;
 6. $z' \leftarrow \text{ComputeHeights}(f', g', w')$;
 7. $z \leftarrow \text{ExpandHeights}(z')$;
 8. $A, b \leftarrow \text{BuildSystem}(f, g, w)$;
 9. $z \leftarrow \text{SolveSystem}(A, b, z)$;
 10. Return z .
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Except for pathological cases, the memory and time costs of our method are typically proportional to the number of pixels N . This cost is asymptotically optimal, and significantly better than that of the best previous weighted integrators, which solve Poisson equation system by Gaussian or Cholesky factorization. These require time proportional to $N^{1.5}$ and memory proportional to $N^{1.5}$. Yet the accuracy and resilience to noise of our method is comparable to that of other Poisson-based integrators, and much better than that of path-based and Fourier-based integrators. Tests show that our method is as accurate as the best weighted slope integrators, but substantially more efficient in time and space.

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