A 2D+t Feature-preserving Non-local Means Filter for Image Denoising and Improved Detection of Small and Weak Particles

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Denoising of images containing small and weak particle-like objects has many important applications in both civil and military areas [1,2]. A feature-preserving non-local means (FP-NLM) filter has been developed recently for denoising such images [3]. It explores the commonly used non-local means (NLM) filter to employ two similarity measurements taken in the original greyscale image and a feature image which measures the particle probability in the original image. In this paper, we report a new approach to image mapping for constructing the feature image by incorporating both spatial and temporal (2D+t) characteristics of objects. We present a 2D+t FP-NLM filter based on the improved particle probability image. Experiments show that the new filter can achieve better balance between particle enhancement and background smoothing for images under severe noise contamination and has a greater capability in detecting particles of interest in such environments.

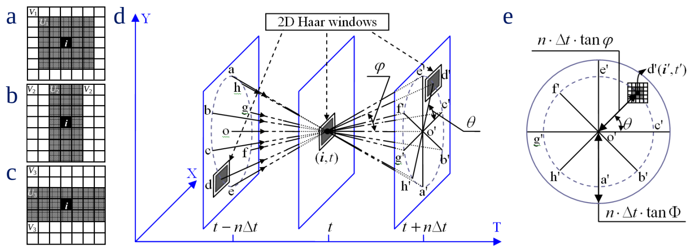

Figure 1: (a-c) 2D Haar windows $H_k(\boldsymbol{i},s)$ $(k=1,2,3)$ [3]; (d-e) 2D+t Haar-like features: (d) a moving particle in $N=2n+1$ consecutive frames that pass through pixel $\boldsymbol{i}$ at time t; (e) parameters at time $t+n\Delta t$.

Motion continuity is an unambiguous property for identifying objects of interest in an image sequence. We here make use of such property to improve the accuracy of particle classifications based on the spatial (2D) Haar-like features (HLFs) as depicted in Fig. 1(a-c) [3]. Assuming constant velocity and direction of particle-like objects in neighbouring frames, trajectories of a particle in N consecutive frames that pass through the pixel $\boldsymbol{i}$ at time t are depicted in Fig. 1(d). For a trajectory that has the coordinates $(\boldsymbol{i}',t')$ at time point t', we can apply the 2D Haar windows to compute local contrasts at this pixel: $H_k(\boldsymbol{i}',t',s)=M_{U_k}(\boldsymbol{i}',t',s)-M_{V_k}(\boldsymbol{i}',t',s)$, where s is a given scale, M_{U_k} and M_{V_k} are the means of the pixel values in the shaded area, U_k, and in the white area, V_k, centred at this pixel. We define 2D+t HLFs as

$$H_k(\boldsymbol{i},t)=\max_{s,\theta,\varphi}\{\eta(\boldsymbol{i},\theta,\varphi,t,s)\cdot H_k(\boldsymbol{i},t,s)\}, \tag{1}$$

k=1,2,3, where the coefficient

$$\eta(\boldsymbol{i},\theta,\varphi,t,s)=\exp\left(-\frac{1}{N}\cdot\sum_{t'=t-n\Delta t}^{t+n\Delta t}\left(\frac{H_k(\boldsymbol{i}',t',s)-\overline{H}_k(\boldsymbol{i},\theta,\varphi,t,s)}{\overline{H}_k(\boldsymbol{i},\theta,\varphi,t,s)}\right)^2\Big/2\gamma^2\right) \tag{2}$$

measures the consistency of the local contrasts computed for all possible trajectories in N frames, where $\overline{H}_k(\boldsymbol{i},\theta,\varphi,t,s)=\frac{1}{N}\sum_{t'=t-n\Delta t}^{t+n\Delta t}H_k(\boldsymbol{i}',t',s)$ is the mean value. The parameter γ controls the sensitivity to penalize false trajectories. Thus, $H_k(\boldsymbol{i},t)$ in Eq. 1 is the consistency weighted maximum values of the local contrasts for all possible scales of the Haar windows (s), all possible directions of motion (θ) and all possible velocities of motion (φ). The three 2D+t HLFs $H_k(\boldsymbol{i},t)$ can be combined in a linear way same as in the case of 2D HLFs [3], and each pixel in the image is then classified as particle class if

$$\overline{H}(\boldsymbol{i},t)\geq\lambda\,. \tag{3}$$

A particle probability image (PPI) based on 2D+t HLF classifications is defined as the ratio

$$P(\boldsymbol{i})=(\Delta N/N_{tot})_{A_{\boldsymbol{i}}} \tag{4}$$

where N_{tot} is the total number of pixels in a given area $A_{\boldsymbol{i}}$ centred at $\boldsymbol{i}$; ΔN is the number of pixels satisfying Eq. (3) and spatially connected.

In our new 2D+t FP-NLM filter, the processed grey values of a noisy image F at pixel $\boldsymbol{i}$ are given as the weighted average of all pixel greyscale values in a search window $W_{\boldsymbol{i}}$ centred at $\boldsymbol{i}$

$$FPNLM_{2D+t}(F)(\boldsymbol{i})=\sum_{\boldsymbol{j}\in W_{\boldsymbol{i}}}\omega(\boldsymbol{i},\boldsymbol{j})F'(\boldsymbol{j}) \tag{5}$$

where F' is a pre-processed image of F (eg. mean filtered) and

$$\omega(\boldsymbol{i},\boldsymbol{j})=\frac{1}{A(\boldsymbol{i})}\exp\left(-\frac{\left\|\mathbf{V}(N_{\boldsymbol{i}})-\mathbf{V}(N_{\boldsymbol{j}})\right\|_{2,a}^2}{h^2}-\frac{\left\|\mathbf{P}(N_{\boldsymbol{i}})-\mathbf{P}(N_{\boldsymbol{j}})\right\|_{2,a}^2}{g^2}\right) \tag{6}$$

the weight, $A(\boldsymbol{i})$ the normalization constant, $\mathbf{V}(N_{\boldsymbol{i}})$ and $\mathbf{P}(N_{\boldsymbol{i}})$ the vectors of the pixel grey values and the particle probability values taken from the neighborhood $N_{\boldsymbol{i}}$, respectively. The first term in Eq. 6 measures the similarity of pixel grey values, as in the NLM filter [4], whereas the second term measures the similarity of particle probabilities between the same neighborhoods but taken from the PPI. By appropriately setting the parameters h and g, the two measurements can compensate each other to achieve more balanced feature preservation and background smoothing.

Fig. 2 shows the test results of a live image sequence of EB1-GFP, expressed in the *Drosophila* egg chamber where the microtubule cytoskeleton is complex and the imaging is challenging. The ROC curves demonstrate that 2D+t FP-NLM filtered image has higher sensitivity to pick out true particle pixels than several existing methods.

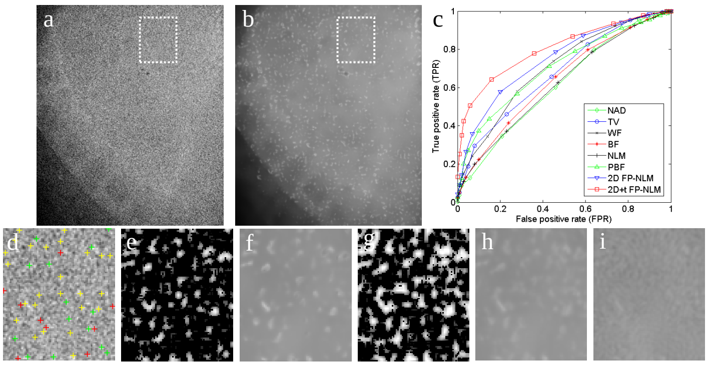

Figure 2: Test on a EB1-GFP image (a); (b) 2D+t FP-NLM filtered; (c) ROC curves; (d) manually identified particles from a subregion marked, green: strong, yellow: weak, red: not visible; (e-f) PPI by 2D+t HLFs and filtered; (g-h) PPI by 2D HLFs and filtered; (i) NLM filtered.

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