Catadioptric camera model with conic mirror

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Catadioptric systems consist of the combination of lenses and mirrors [1]. From them, central panoramic systems stand out because they provide a unique effective viewpoint, leading to the well-known unifying theory for central catadioptric systems [4], [2]. This work considers catadioptric systems consisting of a conic mirror and a projective camera. Although a system with conic mirror does not possess a single projection point, it has some advantages as the cone is a very simple shape to produce, it has higher resolution in the peripheral and adds less optical distortion to the images [5], [3]. The contributions of this work are the model of this noncentral system by means of projective mappings from a torus to a plane, and the definition of the conic fundamental matrix with a role similar to that of perspective cameras. Additionally, a procedure to compute the relative motion between two views from the conic fundamental matrix is presented.

The geometry of the imaging system with the different parameters and coordinate systems involved are depicted in Fig. 1. The vertex of the conic mirror is $\mathbf{O_m}$, and $\mathbf{O_c}$ is the centre of the projective camera. Some geometric parameters described in the paper are τ , f, f_m or α . The viewpoint of this camera system lies on a circular locus of radius f_x vertically translated f_z with respect the vertex of the cone. We consider a world point \mathbf{p} with coordinates $(X,Y,Z)^T$ in a general reference system with origin $\mathbf{O_m}$ and z-axis aligned with the camera optical axis. If we align the x-axis of the reference with the vertical projection of the world point, the new coordinates (p_x, p_y, p_z) of \mathbf{p} are obtained as $p_x = \sqrt{X^2 + Y^2}$, $p_y = 0$ and $p_z = Z$. The coordinates (x_m, z_m) of the point $\mathbf{p_m}$ on the mirror can be derived and projected on the image plane as

$$x_c = x_m - x_m (f + f_m + z_m) / (f_m + z_m),$$
 (1)

which is a radial coordinate that determines the image projection $\mathbf{p_c}$ together with $\varphi = \arctan 2(Y,X)$.

The unitary torus model is inspired on the idea of the unified sphere model for central cameras presented in [4], [2]. In central cameras, a section of the unitary sphere along the symmetry axis gives a unitary circle. In our case, the revolution of the unitary circle in each viewpoint across the circular locus yields a torus. We define the coordinate system (x^t, z^t) with origin $\mathbf{O_t}$ as shown in Fig. 1. The section of the torus is also represented in Fig. 1 with two symmetric circles, where the centre of the tube is the circle of radius f_x (i.e. the circular locus on which the viewpoint of the camera system lies) and the radius of the tube is the unit. In the paper, the procedure for reprojecting a point onto the torus given its coordinates x_c and φ in the image is developed, and it can be summarized in the following steps:

- 1- Coordinate transformation: $x_c^t = (x_c f \tan \alpha)/(1 + (x_c/f) \tan \alpha)$
- 2- Reproject to the unitary torus obtaining the $\mathbf{p_t^t}$ with coordinates (x_t^t, z_t^t) in the reference system at $\mathbf{O_t}$: $x_t^t = \lambda x_c^t/f$, and $z_t^t = \lambda$ with $\lambda = 1/\sqrt{1+(x_c^t/f)^2}$
 - 3- Coordinate transformation:
 - $x_t = x_t^t \cos \alpha z_t^t \sin \alpha$, and $z_t = x_t^t \sin \alpha + z_t^t \cos \alpha$.

The result is a point $\mathbf{p_t}$ projected onto the torus (x_t, z_t, φ) , playing a similar role as the point in the unitary sphere for central cameras. The model obtained with this procedure is a unified representation of the points in the unitary torus.

Taking advantage of the camera model defined, a novel conic fundamental matrix is presented with a role similar to the fundamental matrix for perspective cameras. Then, it can be proved that there exists a 5×5 matrix ${\bf F}$ that we call the conic fundamental matrix satisfying

$$\begin{pmatrix} \cos \varphi' \\ \sin \varphi' \\ x'_t \cos \varphi' \\ x'_t \sin \varphi' \\ 1 \end{pmatrix} \mathbf{F} \begin{pmatrix} \cos \varphi \\ \sin \varphi \\ x_t \cos \varphi \\ x_t \sin \varphi \\ 1 \end{pmatrix} = 0.$$
 (2)

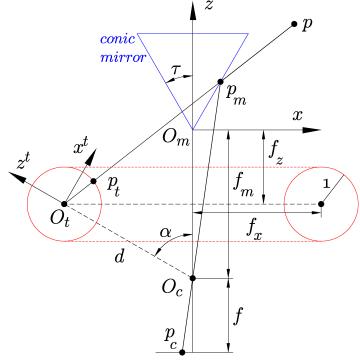


Figure 1: Geometry of the conic mirror and camera system. A world point \mathbf{p} is projected in the image plane \mathbf{p}_c and the unitary torus \mathbf{p}_t . The coordinate systems and geometric parameters are described in the paper.

The entries of **F** only depend on the two camera matrices and the conic fundamental matrix is given by

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & f_{13} & f_{14} & f_{15} \\ 0 & 0 & f_{23} & f_{24} & f_{25} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} \end{bmatrix},$$
(3)

The conic fundamental matrix can be computed from a set of point correspondences, without knowledge of the relative camera positions, using the constraint (2) by solving a linear system of equations. The lifted coordinates are obtained from the normalized points of the unitary torus: $(x_t, y_t, 1)$ and $(x'_t, y'_t, 1)$. The relative motion between two projective cameras can be computed from the decomposition of the fundamental matrix, and a procedure for the motion extraction from the conic fundamental matrix \mathbf{F} (3) is described in the paper.

In summary, a novel model for catadioptric camera systems with conic mirrors has been presented. The contributions are the conic mirror imaging model based on the unitary torus model, the conic fundamental matrix definition and the procedure to compute relative motion between two conic views.

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