TV-Based Multi-Label Image Segmentation with Label Cost Prior

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The minimum description length principle (MDL) is an important concept of information theory. It states that any regularity in a given set of data can be used to compress the data, i.e. to describe it using fewer symbols than needed to describe the data literally [4]. Zhu and Yuille [6] proposed to segment images based on the continuous formulation of MDL principle, which boils down to the minimization of the following energy function:

$$\min_{\Omega_i} \sum_{i=1}^{n} \Big\{ \int_{\Omega_i} \rho(\ell_i, x)\, dx + \lambda \int_{\partial\Omega_i} ds \Big\} + \gamma M\,, \tag{1}$$

where Ω_i, $i = 1, \dots, n$, are homogeneous segments corresponding to n models/labels ℓ_i, $M = \#\{1 \le i \le n \,|\, \Omega_i \neq \emptyset\}$ is the number of nonempty segments, and data fidelity function $\rho(\ell_i, x) = -\log P(I_x|\ell_i)$ is a negative log-likelihood for model ℓ_i at pixel x. The second term in (1) describes the total perimeter of segments and favours spatially regular segments with minimum length boundary. Constants λ and γ describe relative weights of spatial regularity and label cost prior, correspondingly. Zhu and Yuille applied a local searching method, namely region competition, to approximate the highly nonconvex optimization problem (1). Their method converges to a local minimum. Recent studies of joint MDL with $\alpha-$expansion appeared in [3, 5].

In this work, we use standard total-variation (TV) based approach to representing segment boundaries, which allows to rewrite equation (1) as:

$$\min_{u_i(x)\in\{0,1\}} \sum_{i=1}^{n} \Big\{ \int_{\Omega} u_i(x)\rho(l_i, x)\, dx + \lambda \int_{\Omega} |\nabla u_i|\, dx \Big\} + \gamma M \tag{2}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} u_i(x) = 1\,,\ \forall x \in \Omega$$

where u_i are indicator functions over x such that $\Omega_i = \{x \in \Omega \,|\, u_i(x) = 1\}$ and M is the number of appearing models $M = \#\{1 \le i \le n \,|\, u_i \neq 0\}$.

Given n labels $\{l_1, \dots, l_n\}$, we introduce the auxilliary indicating function $y_i \in \{0,1\}$, $i = 1, \dots, n$, which indicates if the label l_i appears in the segmentation result: $y_i = 1$ when l_i appears in the final segments and $y_i = 0$ otherwise. Therefore, we have $\sum_{i=1}^{n} y_i = M$.

Clearly, the indicating variable y_i is related to the labeling function $u_i(x) \in \{0,1\}$, $i = 1, \dots, n$, such that $\max_{x\in\Omega} u_i(x) = y_i$, $i = 1, \dots, n$.

Therefore, we can reformulate (2) by

$$\min_{u_i(x)\in\{0,1\}} \sum_{i=1}^{n} \Big\{ \int_{\Omega} u_i(x)\rho(l_i, x)\, dx + \lambda \int_{\Omega} |\nabla u_i|\, dx + \gamma y_i \Big\} \tag{3}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} u_i(x) = 1\,, \quad u_i(x) \le y_i\,, \quad \forall x \in \Omega\,; \tag{4}$$

or equivalently

$$\min_{u_i(x)\in\{0,1\}} \sum_{i=1}^{n} \Big\{ \int_{\Omega} u_i(x)\rho(l_i, x)\, dx + \lambda \int_{\Omega} |\nabla u_i|\, dx + \gamma \max_{x\in\Omega} u_i(x) \Big\} \tag{5}$$
$$\text{s.t.} \quad \sum_{i=1}^{n} u_i(x) = 1\,, \quad \forall x \in \Omega\,.$$

In this paper, we propose to solve (2) or (5) by relaxing the integer constraints $u_i(x) \in \{0,1\}$ to be $[0,1]$, like [1, 2], and reformulating them as a convex functional as:

$$\min_{u(x)\in S,\, x\in\Omega} \sum_{i=1}^{n} \Big\{ \int_{\Omega} u_i(x)\rho(l_i, x)\, dx + \lambda \int_{\Omega} |\nabla u_i(x)|\, dx + \gamma \max_{x\in\Omega} u_i(x) \Big\}\,. \tag{6}$$

where S is the pointwise simplex constraint for the labeling functions. The infinity norm of $u_i(x)$ provides a convex analogue for the label cost term in (1) and (2). Energy function (6) is convex and can be globally optimized.

The effectiveness of the proposed convex label cost prior of (6) can be shown by a simple example (see Fig. 1): the method without considering either the label cost term or the length cost term fails to discover the reasonable result, i.e. valid segment boundaries and proper number of segments. In contrast, the proposed approach based on (6), by adding both smoothness and label cost terms, captures the optimal segmentation result with both reasonable boundaries and proper number of segments.

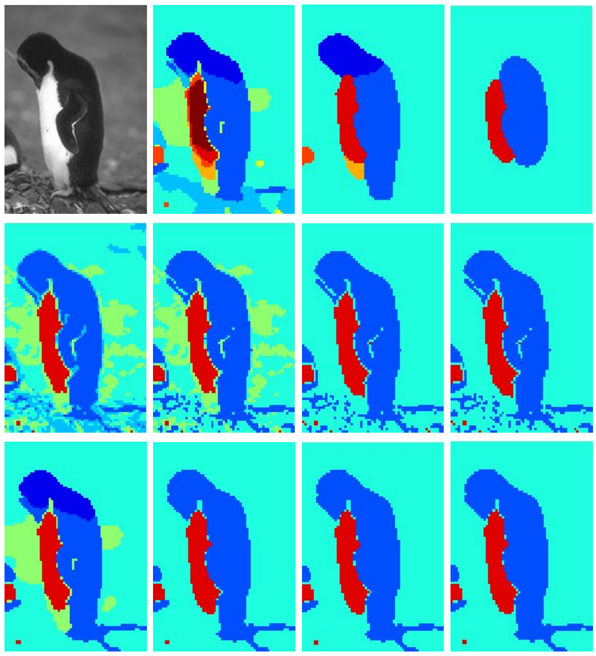

Figure 1: **At the first row (from left to right):** *1st figure* shows the given image to be labeled by 11 labels $\{0, 0.1, \dots, 1\}$. *2nd - 4th figures* show the labeling results without the proposed label cost prior, where the total-variation penalty parameter $\lambda = 0.05, 0.25, 0.65$ respectively and the result is colorized by matlab; different color is associated to different color. **At the second row (from left to right):** the figures show the result computed by the proposed approach only regularized by the label cost prior, i.e. without the total-variation term, where $\lambda = 0$ and the label-cost parameter $\gamma = 25, 43, 50, 100$. **At the third row (from left to right):** the figures show the result computed by the proposed MDL approach (6) with the label cost prior, where λ is fexed to be 0.05 and the label-cost parameter $\gamma = 10, 25, 50, 100$ respectively.

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