Algebraic Line Search for Bundle Adjustment

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Appendix 1

We give details about how to solve the Two-way ALS. We want to find the best magnitudes $\{\alpha_P^*, \alpha_Q^*\}$ for the camera displacement Δ_P and the structure displacement δ_Q. The algebraic error we want to minimize is:

$$\tilde{\varepsilon}(\mathbf{x}) = \sum_{i,j} v_{ij} \left\| \mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \left(\mathsf{P}_i + \alpha_P \Delta_{P_i}\right)\left(\mathbf{Q}_j + \alpha_Q \delta_{Q_j}\right)\right\|^2. \quad (1)$$

Nullifying partial derivatives of $\tilde{\varepsilon}(\mathbf{x})$ from 1 with respect to α_P and α_Q gives:

$$\begin{array}{ll} \frac{\partial\tilde{\varepsilon}}{\partial\alpha_Q} = \sum_{i,j}\left(2a + 2c\alpha_P + 2d\alpha_P^2 + 2\alpha_Q\alpha_P(2e + f\alpha_P) + 2g\alpha_Q\right) & = 0 \\ \frac{\partial\tilde{\varepsilon}}{\partial\alpha_P} = \sum_{i,j}\left(2b + 2c\alpha_Q + 2e\alpha_Q^2 + 2\alpha_Q\alpha_P(2d + f\alpha_Q) + 2h\alpha_P\right) & = 0 \end{array} \quad (2)$$

where

$$\begin{array}{l}
a = \sum_{i,j} v_{ij}\ (\mathsf{P}_i\mathbf{Q}_j)^\top(\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \mathsf{P}_i\delta_{Q_j}) \\
b = \sum_{i,j} v_{ij}\ (\mathsf{P}_i\mathbf{Q}_j)^\top(\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \Delta_{P_i}\mathbf{Q}_j) \\
c = \sum_{i,j} v_{ij}\ (\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \mathsf{P}_i\delta_{Q_j})^\top(\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \Delta_{P_i}\mathbf{Q}_j) + (\mathsf{P}_i\mathbf{Q}_j)^\top(\Delta_{P_i}\delta_{Q_j}) \\
d = \sum_{i,j} v_{ij}\ (\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \Delta_{P_i}\mathbf{Q}_j)^\top(\Delta_{P_i}\delta_{Q_j}) \\
e = \sum_{i,j} v_{ij}\ (\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \mathsf{P}_i\delta_{Q_j})^\top(\Delta_{P_i}\delta_{Q_j}) \\
f = \sum_{i,j} v_{ij}\ (\Delta_{P_i}\delta_{Q_j})^\top(\Delta_{P_i}\delta_{Q_j}) \\
g = \sum_{i,j} v_{ij}\ (\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \mathsf{P}_i\delta_{Q_j})^\top(\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \mathsf{P}_i\delta_{Q_j}) \\
h = \sum_{i,j} v_{ij}\ (\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \Delta_{P_i}Q)^\top(\mathsf{S}\left[\mathbf{q}_{ij}\right]_\times \Delta_{P_i}\mathbf{Q}_j)
\end{array}$$

Solving the polynomial system (2) with the MAPLE Buchberger's algorithm gave us the following solutions :

$$\{\alpha_P^*, \alpha_Q^*\} = \{p, \frac{-a - cp - dp^2}{g + 2ep + fp^2}\} \quad (3)$$

with $p = RootsOf(F(X))$ and $F : \mathbb{R} \rightarrow \mathbb{R}$ is a degree 5 polynomial:

$$
\begin{aligned}
F(X) = {} & (-hf^2 + d^2 f)\, X^5 \\
& + (3ed^2 - 4hef + cdf - bf^2)\, X^4 \\
& + (-4he^2 - 4bef - 2hfg + 4cde + 2d^2 g)\, X^3 \\
& + (c^2 e - 4heg - 4be^2 + 2ead - 2bfg - caf + 3cdg)\, X^2 \\
& + (c^2 g + 2dag - 4beg - fa^2 - hg^2)\, X \\
& - bg^2 - ea^2 + cag.
\end{aligned}
$$