Factorization-based Hierarchical Reconstruction for Circular Motion

H. Zhong and Y. S. Hung
Department of Electrical and Electronic Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong
{hzhong, yshung}@eee.hku.hk

Abstract

A new practical method is developed for 3D reconstruction from an image sequence captured by a camera with constant intrinsic parameters undergoing circular motion. We introduce a method for enforcing the circular constraint in a factorization-based projective reconstruction. This is called a circular projective reconstruction. Given a turntable sequence, our method uses a hierarchical approach to reconstructing the objects and cameras, which first computes a circular projective reconstruction of a sub-sequence and then extends the reconstruction to the complete sequence. Camera matrix and the motion parameters, i.e. the rotation angles, are computed iteratively in a way that minimizes the 2D reprojection error. Thus, an optimal reconstruction is obtained upon convergence. The algorithm is evaluated using real image sequence.

1 Introduction

Considerable attention has been devoted to efficient 3D reconstruction of objects obtained from circular motion in recent years. Traditionally, the reconstruction process was first done by careful calibration [5,6,9]. More recently, this problem was dealt with directly from uncalibrated image sequences [2,3,7,10,12]. As shown in [3,11], from uncalibrated images alone, the reconstruction from circular motion can be only determined up to a two-parameter ambiguity, but the rotation angles can be determined uniquely. There exist several methods to compute the rotation angles [3,4,7,13]. Fitzgibbon et al. [3] computed the rotation angles using trifocal tensors. Mendonca et al. [7] used surface profiles to estimate the epipolar geometry, and then the rotation angles. A method based on fitting a conic section was presented in [4,13]. The conic fitting method is demonstrated to be simpler than the existing ones. However, the optimization is only defined to minimize the geometric distances of chosen points to the fitted conic, but does not involve an optimal estimate of the camera matrix. Zhong and Hung [8] showed that camera matrices could be upgraded from projective reconstruction to metric reconstruction with a two-parameter family by using the knowledge of rotation angles. This suggests that it is possible to estimate the rotation angles as well as the camera matrix simultaneously within the reconstruction process, instead of after a projective reconstruction, e.g. computation of fundamental matrix.

BMVC 2004 doi:10.5244/C.18.23

In an image sequence captured from circular motion, feature correspondences often appear in consecutive images, but are missing in others. The missing data generally hinders the singular value decomposition (SVD). Consequently a general projective reconstruction based on singular value decomposition cannot be applied to an image sequence with too many missing points. A hierarchical reconstruction approach where the complete sequence is divided into a number of sub-sequences, is appropriate in this case, as projective reconstructions are carried out on each sub-sequence which usually contains few, or even no missing data. Another advantage of hierarchical reconstruction is that error can be distributed evenly over the sequence [2,14].

In this paper, we combine the methods of [1] and [8] and propose a new algorithm to estimate the camera matrix and the rotation angles by minimizing the 2D reprojection error. In this algorithm, a hierarchical approach in which the complete sequence is partitioned into several sub-sequences is adopted to avoid the problem caused by the missing data and to make the algorithm more efficient. The factorization-based method in [1] is modified and the circular constraint is explicitly enforced to obtain a circular projective reconstruction for one sub-sequence. It is shown that computing the circular projective reconstruction of one sub-sequence is sufficient to obtain the circular projective reconstruction of the complete sequence. The algorithm ends with a bundle adjustment which is also conducted with the minimization of 2D reprojection error. Thus the algorithm ensures that the reconstruction is obtained in an optimal way.

The paper is structured as follows. In section 2 circular motion geometry is reviewed and the concept of circular projection reconstruction is introduced. In section 3 the computation of circular projection reconstruction is discussed. The new algorithm is then presented in section 4. Experimental results are given in section 5. Finally, some concluding remarks are given in section 6.

2 Circular motion geometry and circular projective reconstruction

Circular motion is a special kind of planar motion. In terms of screw decomposition, there is zero translation along the screw axis and the screw axes of each Euclidean action coincide in this motion [3]. It is also called turntable motion as it can be modelled as a rotating object viewed by a static camera. For cameras with constant intrinsic parameters, the mathematical model of camera matrices **P** is very simple, as is shown in the following.

Without loss of generality, we can define the motion plane as the XY plane of the world coordinate system. Then, the motion of each camera with respect to the first camera can be described by a rotation angle θ about the Z axis [8]. Suppose the first camera C_I is at position r on the world X axis and the rotation of C_I about its centre relative to the world frame is **R**. Then, the projection matrix of C_I may be written as

$$\mathbf{P}_I = \mathbf{KR}\left[\mathbf{I} \mid \mathbf{t}\right]$$

where **K** is an upper triangular matrix representing the camera calibration matrix, **I** is the identity matrix, and $\mathbf{t} = \begin{bmatrix} -r & 0 & 0 \end{bmatrix}^T$.

Referring to this camera, a second camera rotated by θ_i from C_I about the Z axis is given by

$$\mathbf{P}_i = \mathbf{P}_I \mathbf{Q}_i \tag{1}$$

where

$$\mathbf{Q}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

is a rotation matrix about the Z axis in homogeneous coordinates.

Equation (1) gives a concise description of the geometry of circular motion. Projection matrices can be expressed in terms of the first camera matrix and a rotation matrix. Suppose we have obtained a reconstruction that satisfies (1). Such a reconstruction has a property defined in Euclidean space, i.e. θ_i. However, it is not fully Euclidean because the first camera matrix $\mathbf{P}_1$ is only determined up to a two-parameter family [7]. Therefore this is the nearest metric reconstruction we can obtain from images alone. We will call this circular projective reconstruction to indicate the recovered circular constraint θ_i while differentiating it from Euclidean reconstruction, and correspondingly, $\mathbf{P}_1$ as the reference camera matrix. One of the advantages of carrying out circular projective reconstruction over general projective reconstruction is that the results are usually more reliable and robust. This has been discussed in [14] and is also demonstrated in our experimental results.

3 Computing circular projective reconstruction

This section describes the computation of circular projective reconstruction. Unlike the method given in [3] where two-view and three-view tensors were computed to obtain a reconstruction, we use a multiple view approach. The main idea is to parameterize the reconstruction (1) using a common technique, so that the reference camera matrix and the motion parameters are estimated iteratively. We first briefly introduce the factorization-based projective reconstruction method. The circular projection reconstruction is then formulated.

3.1 General projective reconstruction

Let $\mathbf{x}_{ij} = \begin{bmatrix} u_{ij} & v_{ij} & 1 \end{bmatrix}^T$ be the image of a 3D point $\mathbf{X}_j = \begin{bmatrix} x_j & y_j & z_j & 1 \end{bmatrix}^T$ on the i[th] view, and λ_{ij} be the corresponding projective depth to $\mathbf{x}_{ij}$, the perspective projection is formulated as

$$\lambda_{ij}\mathbf{x}_{ij} = \mathbf{P}_i\mathbf{X}_j$$

where $\mathbf{P}_i$ is a 3 x 4 projection matrix. In a factorization-based projective reconstruction method, all the views, image points and 3D points are considered together to form three matrices such that

$$\left\{\lambda_{ij}\mathbf{x}_{ij}\right\} = \mathbf{PX} \tag{3}$$

where the matrix $\left\{\lambda_{ij}\mathbf{x}_{ij}\right\}$ consisting of all image points and their corresponding projective depths is known as the scaled measurement matrix, $\mathbf{P} = \left[\mathbf{P}_1^T, \mathbf{P}_2^T, \ldots, \mathbf{P}_m^T\right]$ is called the joint projection matrix, and $\mathbf{X}=[\mathbf{X}_1,\mathbf{X}_2,\ldots\mathbf{X}_n]$ the projective shape matrix. If the projective depths λ_{ij} are known, then the scaled measurement matrix may be factorized into the joint projection matrix and the projective shape matrix, which is at most rank 4. The factorization is done by singular value decomposition.

Tang and Hung [1] turned this problem into a problem of minimizing the 2D reprojection error by writing (3) as

$$\min_{\mathbf{P},\mathbf{X},\lambda}\sum_{i,j}\left\|\begin{bmatrix}1\\1\\\gamma_{ij}\end{bmatrix}\cdot\left\{\mathbf{x}_{ij}-\beta_{ij}\mathbf{P}_i\mathbf{X}_j\right\}\right\|_F^2 \tag{4}$$

where $\gamma_{ij}=\max\left(|u_{ij}|,|v_{ij}|\right)$ is the weighting factor and $\beta_{ij}=\dfrac{1}{\lambda_{ij}}$. The minimization problem is solved by iteratively estimating $\mathbf{P}_i$, $\mathbf{X}_j$ and λ_{ij} using weighted least squares (WLS) method.

3.2 Circular projective reconstruction

Consider a circular projective reconstruction given by (1). By substituting (1) into (4), we get another minimization problem

$$\min_{\mathbf{P}_1,\mathbf{X},\lambda,\mathbf{Q}}\sum_{i,j}\left\|\begin{bmatrix}1\\1\\\gamma_{ij}\end{bmatrix}\cdot\left\{\mathbf{x}_{ij}-\beta_{ij}\mathbf{P}_1\mathbf{Q}_i\mathbf{X}_j\right\}\right\|_F^2 \tag{5}$$

Note that the number of variables in the joint projection matrix is now reduced from $11m$ to $m+11$ where m is the number of views involved.

Although λ_{ij}, $\mathbf{X}_j$, and $\mathbf{P}_1$ can be readily computed in (5), e.g. using WLS method, the computation of the rotation matrix $\mathbf{Q}_i$ is less obvious. To see this, assume λ_{ij}, $\mathbf{X}_j$, and $\mathbf{P}_1$ are already known and denote the 3D structure by $\mathbf{X}=[\mathbf{X}_1,\mathbf{X}_2,\ldots\mathbf{X}_n]$, then we may write (5) in matrix form as

$$\min_{\mathbf{Q}_i}\left\|\mathbf{M}_i-\mathbf{W}_i.*\left(\mathbf{P}_1\mathbf{Q}_i\mathbf{X}\right)\right\|_F^2 \tag{6}$$

where $\mathbf{M}_i$ is the weighted 2D point matrix of the i^{th} view and $\mathbf{W}_i$ is the corresponding weighting matrix. Using (2), (6) can be transformed equivalently to

$$\min_{\theta_i}\left\|\mathbf{A}_i\begin{bmatrix}\cos\theta_i\\\sin\theta_i\end{bmatrix}-\mathbf{V}_i\right\|_2^2. \tag{7}$$

Let $\mathbf{p}^i$ be the i^{th} column of $\mathbf{P}_1$ and $\mathbf{X}=\begin{bmatrix}\tilde{\mathbf{X}}_1^T & \tilde{\mathbf{X}}_2^T\end{bmatrix}^T$ with $\tilde{\mathbf{X}}_1$ containing the first two rows of $\mathbf{X}$ and $\tilde{\mathbf{X}}_2$ the last two rows of $\mathbf{X}$. Then it is easy to see that in (7), $\mathbf{A}_i$ is a $3n$ x 2 matrix formed by rows of $\mathbf{W}_i.*\left(\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2\end{bmatrix}\tilde{\mathbf{X}}_1\right)$ and $\mathbf{W}_i.*\left(\begin{bmatrix}\mathbf{p}_2 & -\mathbf{p}_1\end{bmatrix}\tilde{\mathbf{X}}_1\right)$, and $\mathbf{V}_i$ is a $3n$ x 1 vector generated by taking elements of $\mathbf{M}_i-\mathbf{W}_i.*\left(\begin{bmatrix}\mathbf{p}_3 & \mathbf{p}_4\end{bmatrix}\tilde{\mathbf{X}}_2\right)$ row by row.

In (7), we might take $\cos\theta_i$ and $\sin\theta_i$ as two independent variables and use least squares method to obtain a solution. However, there is no guarantee that the computed

$\cos\theta_i$ and $\sin\theta_i$ satisfy $(\cos\theta_i)^2 + (\sin\theta_i)^2 = 1$. Consequently, the computed $\mathbf{Q}_i$ is not necessarily a rotation matrix. In the following, we propose to apply householder transformations to obtain the desired rotation matrix $\mathbf{Q}_i$.

Let $\mathbf{v} \in \mathbb{R}^n$ be a nonzero vector. An n x n matrix $\mathbf{H}$ of the form

$$\mathbf{H}_v = \mathbf{I} - 2\mathbf{v}\mathbf{v}^T / \mathbf{v}^T\mathbf{v}$$

is called a Householder matrix and the vector $\mathbf{v}$ is called a Householder vector. An important property of a Householder matrix is that it is orthogonal. Suppose $\mathbf{H}$ is a Householder matrix such that

$$\mathbf{HA}_i = \begin{bmatrix} a_1 & a_2 \\ 0 & a_4 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \text{ and } \mathbf{HV}_i = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \end{bmatrix}.$$

Then, a solution minimizing the norm $\left\| \mathbf{HA}_i \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix} - \mathbf{HV}_i \right\|_2^2$ will also be a solution to (7) since $\mathbf{H}$ is orthogonal.

Henceforth within this section, we assume such a Householder matrix is already multiplied to (7). We will use c and s to represent $\cos\theta_i$ and $\sin\theta_i$ for simplicity. With this notation, the norm we want to minimize is given by

$$n = (a_1 c + a_2 s - v_1)^2 - (a_4 s - v_2)^2 . \qquad (8)$$

Let $t = \tan(\theta_i)$ and $c = 1/\sqrt{1+t^2}, s = t/\sqrt{1+t^2}$. Solutions of (7) are the roots of the differential equation of (8) with respect to θ_i, which satisfy

$$a(t^2 - 1) + bt = -(d + et)\left(\sqrt{1+t^2}\right)$$

where $a = -a_1 a_2, b = a_2^2 - a_1^2 + a_4^2, d = -v_1 a_2 - v_2 a_4, e = v_1 a_1$.

Taking square on both sides of the above equation gives rise to a quartic polynomial equation in t

$$p_4 t^4 + p_3 t^3 + p_2 t^2 + p_1 t + p_0 = 0 \qquad (9)$$

with $p_4 = a^2 - e^2, p_3 = 2ab - 2de, p_2 = b^2 - 2a^2 - e^2 - d^2, p_1 = -2ab - 2de, p_0 = a^2 - d^2$.

Note that two solutions for c and s can be obtained from a solution t to (9), namely $c = 1/\sqrt{1+t^2}, s = t/\sqrt{1+t^2}$ and $c = -1/\sqrt{1+t^2}, s = -t/\sqrt{1+t^2}$. Thus there are totally 8 possible solutions. The correct result is then determined by substituting all these 8 solutions back into (7) and choosing the one resulting in the smallest norm.

We have shown that a rotation matrix which is a solution to (5) can be computed by solving a quartic polynomial equation. This enables the rotation angle to be determined in four quadrants since the cosine and sine of the angle are computed ultimately. Now that there is a way to compute the rotation angle, a circular projective reconstruction can be obtained by enforcing the circular motion constraint in a general projective reconstruction. In particular, we estimate λ_{ij}, $\mathbf{P}_1$, $\mathbf{X}_j$ and $\mathbf{Q}_i$ iteratively. In next section, we will discuss the hierarchical reconstruction of an image sequence by means of circular projective reconstruction.

4 Hierarchical reconstruction with circular constraint

The circular projective reconstruction algorithm presented in last section can be applied to a sequence with any numbers of images. As a result, a simplified hierarchical approach can be employed to obtain the complete reconstruction by just computing one circular projective reconstruction from a sub-sequence, and then extending to the whole sequence. After that, bundle adjustment can be used to refine the reconstruction.

4.1 Obtaining circular projective reconstruction

To compute a circular projective reconstruction, we need a rough estimate of the reference camera matrix $\mathbf{P}_I$ as well as the rotation angles. This can be achieved by first computing a general projective reconstruction and then rectifying it to a circular projective reconstruction.

As shown in section 3, assuming the errors are normally distributed in 2D images, the general projective reconstruction can be obtained by minimizing the 2D reprojection error [1]. After that, we can use the conic fitting method presented in [4] to compute the rotation angles. It is recommended to conduct a general projective reconstruction prior to computing the rotation angles, as this process gives a better estimate of the rotation angles because 2D reprojection error is minimized during the reconstruction.

After we have obtained a general projective reconstruction and an estimate of the angles of rotation, we are able to compute the reference camera matrix $\mathbf{P}_I$ for the sub-sequence of interest. This can be done by using the rectification algorithm given in [8]. The underlying idea is that a circular projective reconstruction is related to a general projective reconstruction by a projective transformation. Combining at least three projective cameras, the non-singular 4 x 4 rectification matrix and the reference camera matrix $\mathbf{P}_I$ can be computed by the least squares method. The circular projective reconstruction is then estimated using the method given in section 3.2.

4.2 Extending the reconstruction

In a turntable sequence captured by a camera with constant internal parameters, the reference camera matrix of the circular projective reconstruction obtained from one sub-sequence is also constant throughout the complete sequence. Therefore after computing a circular projective reconstruction, it remains to identify the rotation angles between successive views in other sub-sequences. The circular projective reconstruction just obtained can be used to accomplish this task too, as is discussed in the following.

Consider three images k, k+1 and k+2 in which correspondences have been established. Suppose the rotation angle between image k+1 and k+2 is known and denoted by θ_{k+1}, and the reference camera matrix is $\mathbf{P}$. We now want to estimate the rotation angle between image k and k+1, i.e. θ_k.

Let $\mathbf{x}_i = \begin{bmatrix} x_i & y_i & 1 \end{bmatrix}^T$ be a point on image k and $\mathbf{X}_i = \begin{bmatrix} X_i & Y_i & Z_i & T_i \end{bmatrix}^T$ be the reconstructed 3D point, then we have

$$\mathbf{x}_i \cong \mathbf{P}\mathbf{Q}_k\mathbf{X}_i \,. \tag{10}$$

where $\mathbf{Q}_k$ is a rotation matrix as defined in (2). Note that the rotation angle θ_k is defined with respect to the reference camera matrix **P** and is a relative quantity. Therefore the value depends on how $\mathbf{X}_i$ is reconstructed.

Because equation (10) is defined up to an unknown scale factor, only two of the three equations are linearly independent. We may use cross product to eliminate the homogeneous scale factor, which yields

$$\begin{bmatrix} x_i\mathbf{p}^{3T} - \mathbf{p}^{1T} \\ y_i\mathbf{p}^{3T} - \mathbf{p}^{2T} \end{bmatrix} \begin{bmatrix} \cos\theta_k X_i - \sin\theta_k Y_i \\ \sin\theta_k X_i + \cos\theta_k Y_i \\ Z_i \\ T_i \end{bmatrix} = 0 \tag{11}$$

where $\mathbf{p}^{iT}$ are the rows of **P**.

This is an inhomogeneous linear equation in $\cos\theta_k$ and $\sin\theta_k$. It can be verified that by writing

$$\mathbf{A}_i = \begin{bmatrix} x_i\mathbf{p}^{3T} - \mathbf{p}^{1T} \\ y_i\mathbf{p}^{3T} - \mathbf{p}^{2T} \end{bmatrix} = \begin{bmatrix} -1 & 0 & x_i \\ 0 & -1 & y_i \end{bmatrix}\mathbf{P} = \begin{bmatrix} \mathbf{a}_{1i} & \mathbf{a}_{2i} & \mathbf{a}_{3i} & \mathbf{a}_{4i} \end{bmatrix},$$

(11) can be written as

$$\begin{bmatrix} \mathbf{b}_{1i} & \mathbf{b}_{2i} \end{bmatrix} \begin{bmatrix} \cos\theta_k \\ \sin\theta_k \end{bmatrix} = \mathbf{v}_i \tag{12}$$

where $\mathbf{b}_{1i} = \begin{bmatrix} \mathbf{a}_{1i} & \mathbf{a}_{2i} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \end{bmatrix}, \mathbf{b}_{2i} = \begin{bmatrix} \mathbf{a}_{1i} & \mathbf{a}_{2i} \end{bmatrix} \begin{bmatrix} -Y_i \\ X_i \end{bmatrix}, \mathbf{v}_i = \begin{bmatrix} -\mathbf{a}_{3i} & -\mathbf{a}_{4i} \end{bmatrix} \begin{bmatrix} Z_i \\ T_i \end{bmatrix}$

In the above derivation, we assume the 3D point $\mathbf{X}_i$ is known. Indeed, because the camera matrices of image k+1 and k+2 are already known, i.e. **P** and $\mathbf{PQ}_{k+1}$, respectively, the 3D points visible to these two images can be reconstructed using the optimal triangulation method [14]. The rotation angle θ_k is then computed by solving (12) using the method proposed in section 3.2. The θ_k computed in this manner has an inversed sign to θ_{k+1} because image k is preceding image k+1 and **X** is reconstructed in such a way that the camera matrix of image k+1 is used as the reference camera matrix.

Similarly, suppose that the last two images in the current sub-sequence are l and l+1, then the rotation angle between image l+1 and image l+2 can be computed in the same way. This process is repeated against other images in the preceding and subsequent sub-sequences, and sub-sequence by sub-sequence, until all rotation angles in the complete sequence are computed. After that, we can use the trifocal tensor to remove outliers in each image triplet.

4.3 Bundle adjustment

In bundle adjustment, we combine all the estimated rotation angles, the reconstructed structure, and the reference camera matrix to refine the result. The missing data problem no longer exists since we have already obtained a circular projective reconstruction for the complete sequence. The effectiveness of bundle adjustment is justified by the connection between consecutive sub-sequences, i.e. the common 3D points captured in the last few images of one sub-sequence and the first few images of the subsequent sub-sequence. The bundle adjustment is carried out as another circular projective reconstruction. Hence the method in section 3.2 can be used again. But this time, it is run with the complete set of images.

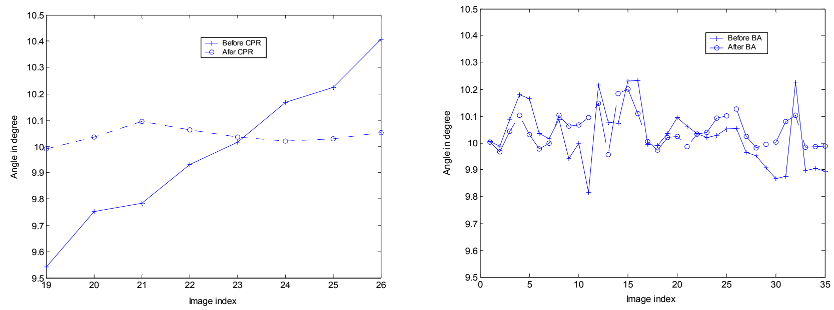
4.4 Algorithm summary

Suppose feature points are already tracked with some outliers. The complete algorithm is summarized as follows.

1. Partition the complete sequence into sub-sequences.
2. Compute a general projective reconstruction on the sub-sequence with the highest numbers of matched points and views.
3. Compute the circular projective reconstruction for that sub-sequence.
4. Compute the rotation angles between views in other sub-sequences using the circular camera matrix $\mathbf{P}_I$ and the structures reconstructed in step 3 and remove outliers via the trifocal tensor.
5. Apply bundle adjustment to the complete sequence.

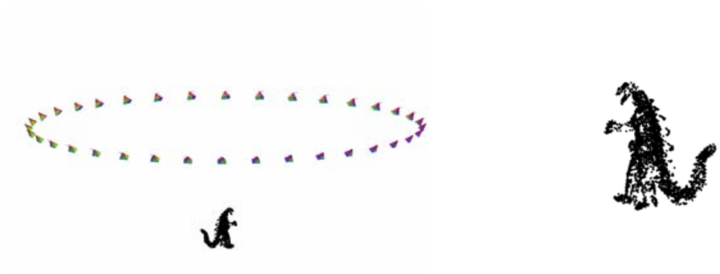
5 Experimental results

The reconstruction algorithm is tested on the popular dinosaur sequence from University of Hannover by courtesy of Oxford Vision Geometry Group. We first choose points that are visible to more than 8 views and form a sub-sequence containing views from 18 to 26 with 25 points and 4 missing data. This sub-sequence is used to compute a circular projective reconstruction. The estimated relative rotation angles before and after circular projective reconstruction are shown in figure 1 (a), which shows that enforcing the circular constraint improves the estimation results significantly. Usually, the general projective reconstruction gives smaller 2D reprojection error than the circular projective reconstruction due to the constrained motion of camera in the later situation. In our case, the 2D reprojection error of general projective reconstruction is 0.32 pixels whereas that of circular projective reconstruction is 0.4 pixels. However, the circular constraint is important to the reconstruction and should always be enforced. Other sub-sequences are then formed with at least 15 points seen in each sub-sequence and two views overlapping in successive sub-sequences. Totally 9 sub-sequences are produced. The circular projective reconstruction is then extended to the complete sequence followed by a bundle adjustment. Figure 1 (b) shows the results of estimated relative angles of rotation of the complete sequence before and after bundle adjustment. The RMS errors of the two results before and after bundle adjustment are 0.11° and 0.07°, demonstrating the accuracy of the algorithm. The reconstructed cameras and dinosaur model are given in figure 2.



(a) (b)

Figure 1: Estimated relative rotation angles (a) of the sub-sequence containing views 18 to 26 before and after circular projective reconstruction (CPR), and (b) of the complete sequence before and after bundle adjustment (BA).



(a) (b)

Figure 2: Circular projective reconstruction of the dinosaur sequence. (a) Recovered camera positions and 3D reconstruction of the dinosaur. (b) Another view of the dinosaur 3D reconstruction.

6 Conclusions

This paper introduces a novel reconstruction algorithm for structure from motion under circular motion. The annoying missing data problem is avoided using the hierarchical reconstruction approach. The circular constraint is inherently enforced in the reconstruction process as well as in the bundle adjustment. This enforcement significantly improves the accuracy, robustness, and efficiency of the estimation, as is verified by the real experiment. In particular, we have shown that computation expenses can be greatly reduced for circular motion as the complete reconstruction can be obtained by extending a reconstruction of only one sub-sequence.

7 Acknowledgements

The work described in this paper is partially supported by a grant from the Research Grant Council of the Hong Kong Special Administrative Region, China (Project No. HKU 7058/02E) and partially supported by CRCG of the University of Hong Kong.

References

[1] W. K. Tang and Y. S. Hung, *A Factorization-based method for Projective Reconstruction with minimization of 2-D reprojection errors*, DAGM, pp.387-394, 2002.

[2] A. W. Fitzgibbon and A. Zisserman, *Automatic Camera Recovery for Closed or Open Image Sequences*, ECCV, pp. 311-326, 1998.

[3] A. W. Fitzgibbon, G. Cross, and A. Zisserman, *Automatic 3D Model Construction for Turn-Table Sequences*, SMILE'98, LNCS 1506, pp. 155-170, 1998.

[4] G. Jiang, H. T. Tsui, L. Quan, and S. Q. Liu, *Recovering the Geometry of Single Axis Motions by Conic Fitting*, CVPR, Vol. 1, pp. I-293 – I-298, 2001.

[5] E. Boyer, *Object Models From Contour Sequences*, ECCV pp. 109-118, 1996.

[6] S. Sllivan and J. Ponce, *Automatic Model Construction and Pose Estimation From Photographs Using Triangular Splines*, PAMI, Vol. 20, No. 10, pp. 1091-1097, 1998.

[7] P. R. S. Mendonca, K. Y. K. Wong, and R. Cipolla, *Epipolar Geometry from Profiles under Circular Motion*, PAMI, Vol. 23, No. 6, pp. 604-616, 2001.

[8] H. Zhong and Y. S. Hung, *Conjugate Epipole-based Self-Calibration of Camera under Circular Motion*, M2VIP, pp. 239-246, 2003.

[9] W. Niem and R. Buschmann, *Automatic Modelling of 3D Natural Objects from Multiple Views*, Yakup Paker and Sylvia Wilbur: Image Processing for Broadcast and Video Production, Workshops in computing series, Springer, Hamburg, 1994.

[10] P. R. S. Mendonca and R. Cipolla, *Estimation of Epipolar Geometry from Apparent Contours: Affine and Circular Motion Cases*, CVPR, pp. 9-14, 1999.

[11] Y. Liu, H. T. Tsui, and C. K. Wu, *Resolving Ambiguities of Self-calibration in Turntable Motion*, Proceeding of the 15th Int. Conf. on Pattern Recognition, IEEE, pp. 865-868, 2000.

[12] K. Y. K. Wong, P. R. S. Mendonca, and R. Cipolla, *Structure and Motion Estimation from Apparent Contours under Circular Motion*, Image and Vision Computing 20 (2000), pp. 441-448, 2000.

[13] G. Jiang, H. T. Tsui, L. Quan, and A. Zisserman, *Single Axis Geometry by Fitting Conics*, ECCV 2002, LNCS 2350, pp. 537-550, 2002.

[14] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge University Press, 2002.