From Subspace to Submanifold Methods

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Abstract

Twenty years ago it was not obvious that subspace approximations would be such a successful representation for faces and other phenomena whose measurement-space manifolds exhibit clear nonlinearities. Now PCA is ubiquitous in computer vision and, although its globally linear view of the data manifold can be a liability as systems scale up, it is not obvious that one can reliably construct a better nonlinear data model.

This question motivates a rapidly growing literature of graph-theoretic and tensorial approximations that view the data manifold as linear only in locales and slices, respectively. These limited-linearity "submanifold methods" offer much richer descriptions of the data manifold, and may ultimately replace subspace methods in computer vision. First, problems with suboptimality, solution instability, and sample complexity must be overcome. Some of the obstacles to "industrial-strength" manifold modeling are discussed, and some methods that offer significant improvements in robustness and accuracy are introduced which eschew the common assumptions about the manifold geometry - in particular, a generalization of PCA that can exactly recover the intrinsic coordinate systems of a significant class of nowhere-linear manifolds. Connecting these data models to image analysis and synthesis algorithms is only slightly more complicated than using subspace methods. Some applications to tracking, recognition, and editing of objects and people in video are demonstrated.