Application of Characteristic Function Method in Target Detection

Mohammad H Marhaban and Josef Kittler
Centre for Vision, Speech and Signal Processing
University of Surrey
Surrey, GU2 7XH, UK
eep5mm@ee.surrey.ac.uk

Abstract

Target detection is one of the important elements of Automatic Target Recognition (ATR) systems. In this paper, we propose a new approach to detect outliers in radar returns, based on modelling the background using an empirical distribution rather than a parametric distribution. The key innovation lies in the use of the Characteristic Function (CF) to describe the distribution. The experimental results show a promising performance improvement in terms of detection rate and lower false alarm rate, compared with the conventional Gaussian model which employs the Mahalanobis metric as a distance function.

1 Introduction

Automatic Target Recognition (ATR) is a process of detection, tracking and recognition of targets in a cluttered background. Typically, target acquisition and classification are accomplished by a computer analysis of data, usually in the form of image sequence, which is extracted from any of a variety of sensors and platforms. These images are subjected to numerous processing steps and algorithms, involving a wide range of tools [2]. An ideal system should keep the false alarm rate to its minimum, while maintaining high detection of true targets. This is a challenge that has to be tackled in the target detection stage which is one of the most important steps in constructing a reliable system.

Pattern recognition is the most popular approach that has been used to solve the ATR problem. The method is based on the hypothesis that target features lie in regions of a multidimensional feature space easily separable from its background [1]. Statistical pattern recognition uses a probabilistic model to underpin classification algorithms where the most widely used model is the Gaussian distribution. Often this assumption is made purely for the convenience of mathematical tracktability.

The novelty of this work is three fold: First of all we adopt the empirical distribution as a model of the background, rather than making unwarranted assumptions. Second, we propose a novel method of estimating the distribution based on the characteristic function approach. The features used are extracted using Principle Component Analysis developed in [5, 6]. The target detection problem is viewed as an outlier detection problem, as in [5, 6]. A novel metric is developed to detect outliers directly in the support domain of the

BMVC 2003 doi:10.5244/C.17.31

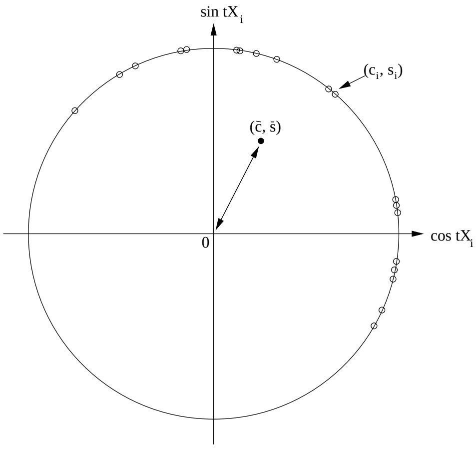


Figure 1: The operation of empirical characteristic function.

characteristic function. A few modifications have been made to the technique presented in [5, 6] so as to improve performance.

2 Characteristic Function

Recently, there is growing interest in applying methods using characteristic function (CF) among the signal processing community [7]. The interest stems from the need to apply signal models more complex than the Gaussian, which are more conveniently characterised through a CF rather than probability density function (PDF). The CF, $\phi(t)$ is by definition a Fourier transform of a density function $p(x)$ [8]

$$\phi(t) = \int_{-\infty}^{\infty} p(x) \cdot \mathrm{e}^{jtx} dx \tag{1}$$

where e^{jtx} denotes a complex exponential. The simplest estimator of the CF is the sample or empirical characteristic function (ECF) which is defined as

$$\hat{\phi}(t) \triangleq \frac{1}{N} \sum_{i=1}^{N} \mathrm{e}^{jtx_i} \tag{2}$$

where $x_i; i = 1, 2, ..., N$ represent independent and identically distributed (IID) random variable (RV) with CF $\phi(t)$. The ECF is directly calculated from the empirical distribution and $\hat{\phi}(t)$ is computable for all value of $t \in \mathbb{R}$. At a given t, $\hat{\phi}(t)$ is an RV and $\hat{\phi}(t), -\infty < t < \infty$ is a stochastic process. The ECF of an RV X can be represented by the cosine and sine function as

$$\hat{\phi}(t) = \frac{1}{N} \sum_{i=1}^{N} \cos(tX_i) + j \sin(tX_i). \tag{3}$$

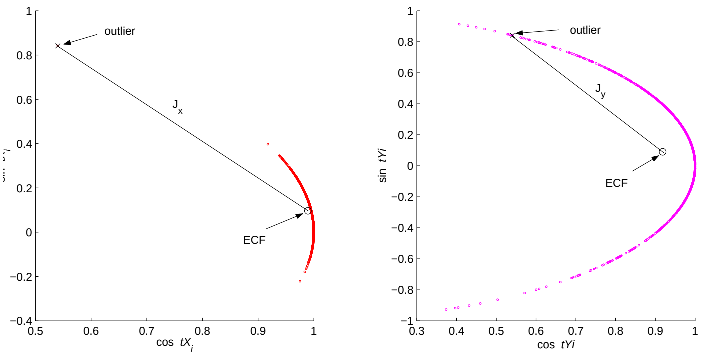


Figure 2: A comparison of distance between outlier and ECF for data X and data Y.

The operation of the estimator ECF is illustrated in Figure 1 [4]. For data $x_1, x_2, \ldots, x_N$ and a given t, the points $(c_i, s_i) = (\cos tX_i, \sin tX_i)$ fall on the unit circle. The ECF of an RV is thus the coordinatewise average of these points which is denoted in Figure 1 as $(\bar{c}, \bar{s})$. The modulus of the ECF $|\hat{\phi}(t)|$ is simply its distance from the origin, also known as the resultant length. From the figure, it can be easily seen that at $t = 0$ the coordinate $(\bar{c}, \bar{s}) = (1, 0)$ and at $-\infty < t < \infty$, $|\hat{\phi}(t)| \leq 1$.

Without loss of generality, let us select a small value for t and consider two sets of data X and Y , having the same mean but different variance, in such a way that

$$\mathrm{Var}[X] < \mathrm{Var}[Y], \tag{4}$$

and assign an outlier to the data, for example $x_1 = y_1 = 10$. The plot of point c_i against s_i for these data is shown in Figure 2.

A small value of t was selected to maximise the spread between the outliers and the background of the data in this complex plane. As expected, the outliers of the data (which are denoted by $\times$ in the figure) fall at the 'tail' of the plot, while its background clusters together. It is interesting to note that the Euclidean distance J_x between the outlier and its ECF for the first data is larger than the Euclidean distance J_y between the outlier and its ECF for the second data. This method thus can be used as a distance measure, where a distance from any random variable x_i to its background in one dimensional case can be calculated using the Pythagoras theorem,

$$J_i^2 = (\bar{c} - c_i)^2 + (\bar{s} - s_i)^2 \tag{5}$$

where $c_i = \cos\, tx_i$ and $s_i = \sin\, tx_i$ are calculated from the sample. Since ECF is an unbiased estimator of the corresponding CF, the point $(\bar{c}, \bar{s})$ could either be the ECF or theoretical CF when the underlying distribution of the RV is known.

In multidimensional analysis, the scaled Euclidean distance has been used as it weights the difference in a given dimension according to its mean and covariance. By normalising the data to its mean and covariance matrix, the distance is calculated as [3]

$$d_M(\mathbf{x}, \mathbf{y}) = \left[\sum_{i=1}^{m} \frac{(x_i - y_i)^2}{\sigma^2} \right]^{\frac{1}{2}} \tag{6}$$

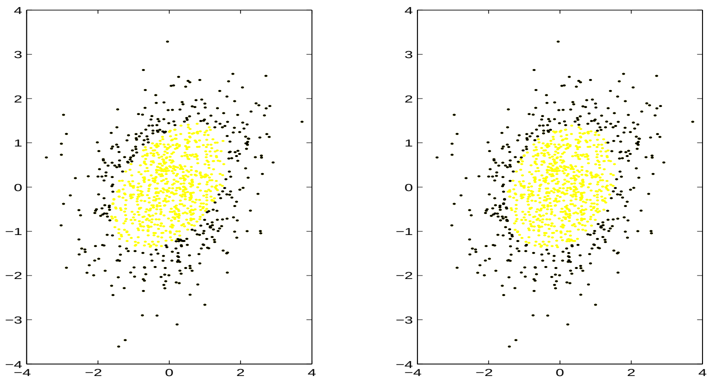


Figure 3: Contour of equal distance for bivariate data. Proposed method is on the left and Mahalanobis is on the right.

which is known as the Mahalanobis metric. Thus following from the univariate case, the distance for multidimensional data can be calculated by

$$J_i = \sum_{i=1}^{m} (\Phi - \cos t\hat{\mathbf{X}}_i) \tag{7}$$

where $\hat{\mathbf{X}}_i$ is the normalised $m-$dimensional vector, and Φ is the real component of the ECF or theoretical CF, as CF of a zero mean process is always real. In many cases, features can be assumed independent, thus instead of using the multi-dimensional CF, the vector of one-dimensional characteristic function can be used, which is a great advantage for a system employing forward feature selection method. Figure 3 shows the contour of equal distance of the proposed distance function for a 2-dimensional case. The bivariate data was generated for $\mathbf{X}_{N=3000} \sim \mathcal{N}(0,1)$, with a positive correlation. The comparison shows that the proposed method performs similarly to the Mahalanobis distance for bivariate normal data.

3 Target Detection

The ATR problem that we are facing is to detect a multiple target on the sea surface. The data was made available by DERA Farnborough. It consists of a sequence containing about twenty frames which have been artificially generated using a standard ray-tracing package. It represents the scenario of a sensor attached to a ship looking out over the sea with five targets inserted into this sequence. The locations of these targets are given by the ground truth (training data). The target detection problem is viewed as an outlier detection problem, where anything that does not normally occur in the background can be considered as a potential target. The three basic steps proposed in [6] have been applied here:

- **Model generation** involving feature extraction by Principle Component Analysis.

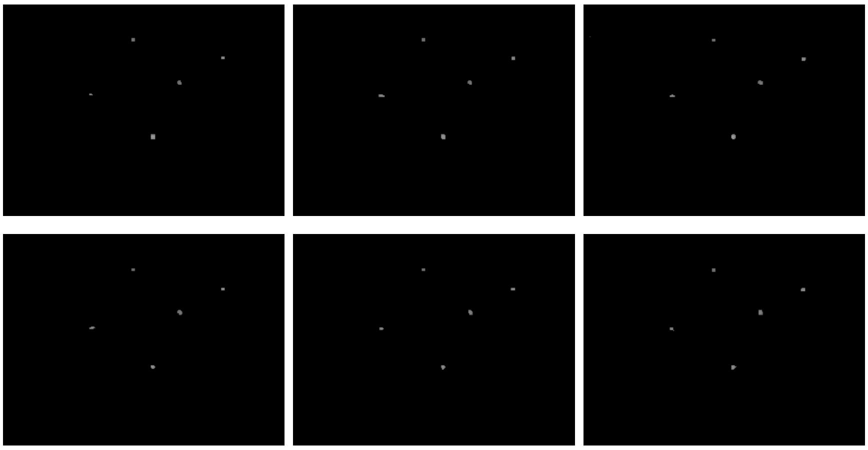


Figure 4: The results for the first 6 frames obtained using the proposed method (5 frames temporal average).

- **Model optimisation** whereby the model and model size are optimised using a sequential floating forward selection search based on training data.
- **Target Detection** where outliers are identified on a pixel by pixel basis by checking whether the features are accurately described by the model.

In the target detection stage, the proposed distance function employing the ECF of the data has been used. The probability of detection was set to 0.99 to ensure that targets are not missed in the feature selection stage. In order to reduce the comparatively high false alarm, the geometric mean of 5 consecutive thresholded images is calculated, which is given by

$$\mu_g = \left[\prod_{i=1}^{t} \mathbf{x}_i \right]^{\frac{1}{t}} \tag{8}$$

where $\mathbf{x}_i$ is the dilated and thresholded image in i frame. This method of temporal tracking has an advantage that the resultant image is already linearly scaled between 0 and 255.

4 Results

Figure 4 shows the temporally averaged results by using the proposed method. The corresponding results using the Mahalanobis distance are given in Figure 5. It can be seen that all five targets have been detected by using both methods, as the probability of detection was set close to 100%. However, the probability of false alarm for Mahalanobis distance is much higher than for the proposed method. The average false-positive rate for Mahalanobis distance is 10.36 which decreases dramatically to 1.45 by using the proposed distance function. A plot of false positive versus frame number is given in Figure 6(a).

The good result achieved by using the proposed method motivate us to use a smaller number of frames in the temporal averaging stage. This greatly improves the response time, thus providing us with the earliest possible warning. In the next experiment, three

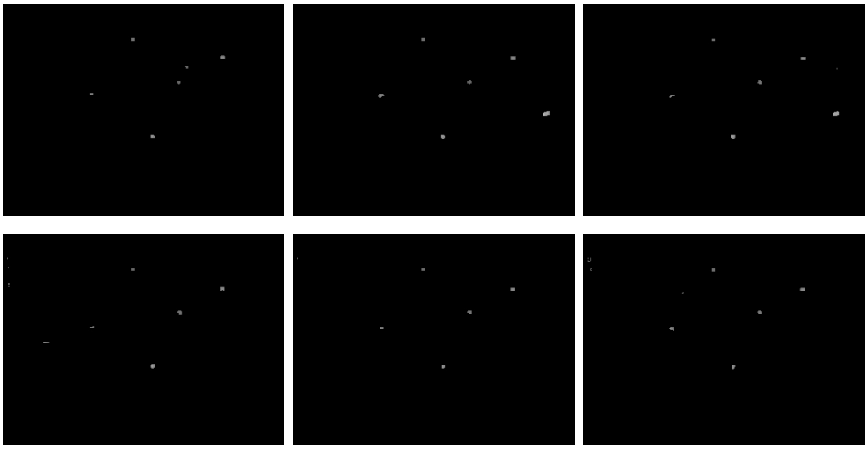


Figure 5: The results for the first 6 frames obtained using Mahalanobis distance (5 frames temporal average).

frames were used in the temporal averaging stage with the results shown in Figure 7 and 8 for the proposed method and Mahalanobis distance, respectively. It can be seen that the proposed method performed better than the Mahalanobis distance. The average false-positive rate for the proposed method is 2.54 while for the Mahalanobis distance, the average is 61.82. A plot of false positive versus frame number is given in Figure 6(b).

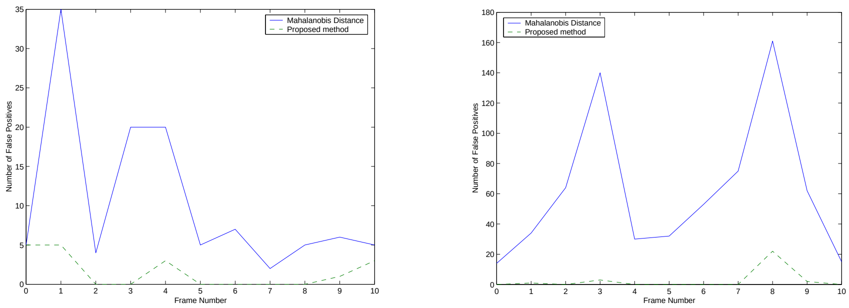
5 Conclusion

A new statistical distance function to measure the relationship of a data point to its background has been proposed. The method not only utilises the mean and variance of the data, but also takes into account the characteristic function of the data. A comparison of global distance for bivariate normal data shows that the proposed method performs similarly to the Mahalanobis distance.

The advantage of the proposed method is that it does not require one to assume that the data is normally distributed. One can choose to use the theoretical characteristic function, when the distribution of the data is known a priori. In other cases, the empirical characteristic function can be used which provides a distribution-free distance function.

We have demonstrated that the proposed method performed better than the Mahalanobis distance, for target detection application. With a similar parameter setting, a significantly lower false-positive rate was observed. It is interesting to note that the proposed method performs similarly (in some cases better) to the 3D-PCA method described in [5]. In the 3D-PCA method, d consecutive frames were used to construct a three dimensional filter in the feature extraction stage. Therefore, in a real life situation, one has to wait until three frames are captured before any feature processing step can be made. With the development of the CF/ECF based method, the 2D-PCA filter described [5] can be used, yet the results are similar to the 3D-PCA.

With the great performance improvement shown by the proposed method, the number of frames used in the temporal averaging stage can be reduced. With a reduced number



(a) 5 frames (b) 3 frames

Figure 6: Plots of false positive.

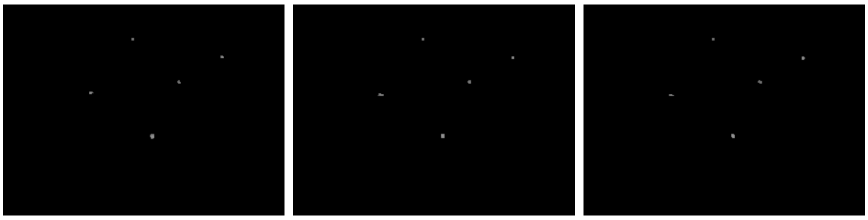


Figure 7: The results for the first 3 frames obtained using the proposed method (3 frames temporal average).

Figure 8: The results for the first 3 frames obtained using Mahalanobis distance (3 frames temporal average).

of frames, the movement of the targets from one frame to another may not be too significant, thus avoiding the need to dilate the thresholded image. This will not only assist the computation, but also the target detection using the original 'undilated' image will reduce the false alarm rate significantly. It has been found that some of the non-targets appear in approximately the same location in the consecutive frames, thus dilation will not only reward the real targets, but also the false ones as well. Furthermore, a fewer frame will mean a quicker detection (albeit computational complexity) thus providing an earliest possible warning.

References

[1] B. Bhanu. **Automatic target recognition: State of the art survey**. *IEEE Transactions on Aerospace and Electronic Systems*, 22(4):364–379, July 1986.

[2] B. Bhanu and T. Jones. **Image understanding research for automatic target recognition** . *IEEE AES Systems Magazine*, pages 15–22, October 1993.

[3] P.C. Mahalanobis. **On the generalized distance in statistics**. *In Proceedings of the National Institute of Science, Calcutta India*, 12:49–55, 1936.

[4] M. Markatou, J.L. Horowitz, and R.V. Lenth. **Robust scale estimation based on the empirical characteristic function** . *Statistics* & *Probability Letters*, 25:185–192, 1995.

[5] K. Messer, D. de Ridder, and J. Kittler. **Adaptive texture representation methods for automatic target recognition** . *Proceedings of the 10th British Machine Vision Conference*, 2:443–52, 1999.

[6] K. Messer and J. Kittler. **Data and decision level fusion of temporal information for automatic target recognition** . *Proceedings of the 11th British Machine Vision Conference*, 2:506–15, 2000.

[7] C.L. Nikias. **Signal processing with alpha-stable distributions and applications**. John Wiley & Sons, 1995.

[8] N.G. Ushakov. **Selected Topics in Characteristic Function** . Coronet Books, 1999.