A Reflectance Model for Radar Shape From Shading

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Abstract

This paper describes work aimed at developing a practical shape-from-shading process for terrain analysis from radar imagery. The paper commences by providing an analysis of the radar reflectance properties of terrain structures. By using ground truth elevation data, we provide an empirical study of the radar reflectance characteristics for large-scale terrain features. The main conclusion of this study are twofold. Firstly, we show that radar has a strong backscatter component. Secondly, we show that the radar noise has a tail which extends to large reflectance values. Based on these observations we develop a semi-empirical shape-from-shading algorithm. We illustrate the effectiveness of the algorithm in extracting surface orientation information from radar images of a mountainous area of terrain in North Wales.

1 Introduction

Radar shape-from-shading provides an important route to automatic terrain analysis [4]. The process aims to use radar images to recover surface topography via an analysis of variations in scattering intensity. Historically, the method has attempted to borrow and adapt ideas developed for recovering local surface orientation using photometric shape-from-shading. Here it is well known that the observed surface luminosity depends on a number of factors including the reflectance, the light source direction and the viewing angle. More formally, photometric shape from shading is posed as the recovery of the needle map from the image irradiance equation. In general, this problem is ill-posed since at each image location two components of surface normal must be recovered from a single luminance value. For this reason a number of additional constraints must be applied in order to solve for the surface shape. The most common of these is that the direction of the local surface normals vary smoothly across the recovered surface. In addition, the image irradiance equation is a simplistic physical model since it assumes that the reflectance function is known (usually Lambertian) and is constant across the surface. Moreover, the direction of the light source must be known in advance.

Horn[1] was the first to address the shape from shading problem using a characteristic strip method. The method is notoriously sensitive to image noise. To limit the problems of noise, Ikeuchi and Horn[2] search for solutions of the image irradiance equation in which the surface normals vary smoothly. Their method is typical of a large group of regularisation methods which involve the global optimisation of a criterion which includes a 'data closeness' term and a 'penalty term' which penalises non-smooth solutions. Such approaches are robust to a certain degree of noise. However, a careful choice of smoothing kernel must be made[3] to prevent over-smoothing.

BMVC 1999 doi:10.5244/C.13.16

The extension of these ideas to the radar domain is by no means straightforward. The main obstacles are as follows. Firstly, and as demonstrated by Frankot and Chellappa [4] and many others [7, 5], the standard Lambertian model is a poor approximation for radar reflectance. The physical reason for this is that radar reflectance is better modelled by multiple scattering centres. As a result the reflectance depends strongly on the material composition of the object under study and the scattering angle [8]. Secondly, the levels of noise associated with radar data are usually much higher than those at visible wavelengths. This is mainly due to speckle effects in the image formation process. Moreover, the noise in some radar systems is far from Gaussian. For instance. some studies have found multiplicative noise in images [12, 13]. Thirdly, the relatively large wavelength of radar means that shading analysis is only practical for surfaces of large physical dimensions. For shading variations to become apparent the surface must be 10's to 100's of metres in size. This limits the usefulness of radar shape-from-shading to remotely sensed terrain. Finally, the reflectance characteristics are very different at radar wavelengths.

However, despite these difficulties there are features of radar which simplify certain aspects of shading analysis. For instance, since the radar itself illuminates the scene, we always have accurate knowledge of the direction of illumination. Moreover, the viewing angle is identical to the illumination angle. This considerably simplifies the reflectance model. Unfortunately, in spotlight synthetic aperture radar the illumination direction varies to some extent during image formation, leading to some blurring of the surface reflectance.

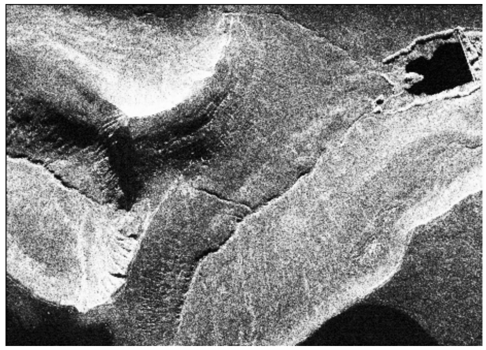
We have recently embarked on a programme of work aimed at developing improved methods for radar terrain analysis. This paper reports the first stages of the work and is concerned with developing a practical radar scattering model. The approach adopted is an empirical one. Using a digital elevation map, we investigate the scattering characteristics of the radar returns using data with known ground truth. The resulting scattering model is found to give good results when combined with the relatively simple Ikeuchi and Horn shape-from-shading scheme [2].

The outline of this paper is as follows. In section 2 we present the radar data and ground truth information used in this study. In section 3 we discuss the reflectance functions of surfaces illuminated by radar. In section 4, we analyse the noise properties of the images. Finally in section 5 we apply a shape from shading algorithm to recover surface normals for a remotely sensed area of terrain, and compare to a ground truth elevation map for the same region.

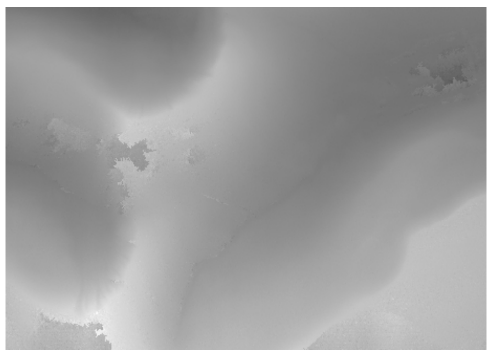
2 Radar Data

The data presented in this study was collected using a C-band SAR radar which has a frequency of 5.7GHz, a bandwidth of 90MHz and peak power of 9.4 W. The imagery was taken at a nominal antenna depression angle of 20 degrees. This radar collects dual-antenna images for use with SAR interferometry. From this data-set it is possible to directly reconstruct a DEM (digital elevation map). It is this DEM which provides the ground truth for our shape-from-shading study. The SAR image and the DEM used in our study are shown in Figure 1.

The two images have been processed to 2m resolution in azimuth and are sampled in the digital image plane at 1.905m in azimuth and 1.499m in range. They have a pixel reso-



(a) SAR image.



(b) Digital elevation map.

Figure 1: Data used in the study.

lution of 1850 by 1320. Each image pixel is represented by a complex number containing amplitude and phase information. Radar reflectance is represented on a logarithmic scale. In the process of deriving the associated height map, the imagery has been averaged by a factor of 4 in both azimuth and range, to alleviate noise problems. As a result, the reduced size of the DEM data array is 462 by 330 pixels. The sampling in the DEM is, therefore, 7.62m in azimuth by 5.996m in range.

3 Reflectance model

Standard shape-from-shading methods assume a Lambertian reflectance model which applies to matte surfaces. As a result, the observed intensity is independent of viewing angle and depends only on the angle of illumination. If $\mathbf{n}$ is the unit surface normal and $\mathbf{s}$ is the unit vector in the light-source direction, then the reflectance function $R(\mathbf{n}.\mathbf{s}) = \mathbf{n}.\mathbf{s}$. In other words, the reflected intensity is uniformly distributed over the viewing angle.

However, this model is unlikely to be useful for representing the angular dependence of radar reflectance. Study of the literature reveals very little discussion of such reflectance models. One of the reasons for this is that a typical angular reflectance pattern for objects smaller than 10's of metres appears to be semi-random. The angular dependence is only likely to be stable for very large objects, such as the terrain features we are studying here.

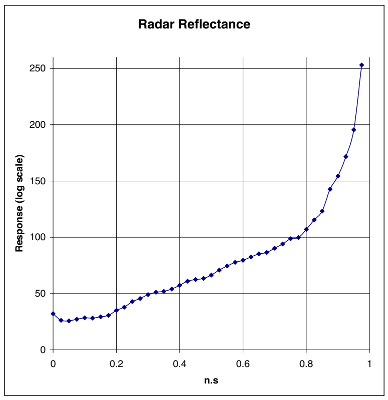
Even so, most of the work on radar reflectance has focused more on identifying terrain classes from their radar reflectance patterns, rather than establishing an analogue of the Lambertian model for radar shape-from-shading. Such reflectance patterns are ideal for locating specific terrain types, since there is a high degree of inter-class variability between different surfaces. Unfortunately, this property poses serious obstacles to developing practical radar shape-from-shading schemes for terrain analysis. The reason for this is that we must identify the terrain class and select the appropriate reflectance model prior to performing shading analysis to recover surface normal information. However, those studies which have been performed reveal a very non-linear pattern to the angular dependence of radar reflectance [8]. It should be noted that for radar data incidence angle and viewing angle are identical due to the geometry of the radar image formation process. In other words, we do not need to separately model the viewing angle dependance of the reflectance function. This clearly simplifies the modelling process.

Equipped with our ground-truth DEM and knowledge of the radar illumination direction, we can analyse the reflectance characteristics of our sample SAR data in an empirical manner. The idea is to use the distribution of observed radar intensity with incidence angle to estimate the reflectance function. The procedure is as follows: Firstly we differentiate the DEM in the x- and y-directions. We then use the resulting directional gradients to reconstruct ground-truth surface normals for the scene. Since we know the illumination direction, it is a simple matter to calculate the angle between illumination direction and the surface normal. Using the angles computed in this way we can then estimate the angular reflectance distribution.

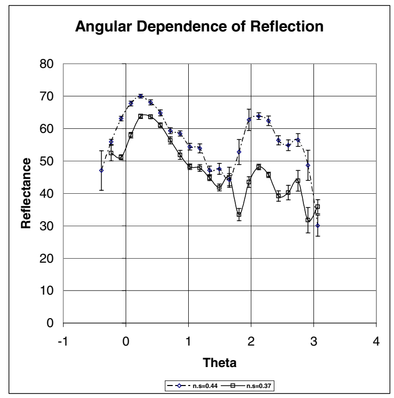
Figure 2a shows the result of applying this technique to one of our SAR images. The overall shape of this function is very similar to that found by other studies [8]. We will denote this reflectance distribution as $R(\mathbf{n} \cdot \mathbf{s})$ where $\mathbf{n}$ and $\mathbf{s}$ are unit surface normal and light direction vectors respectively.

The shape of the reflectance function deserves further comment. Firstly, it is strongly peaked close to $\mathbf{n}.\mathbf{s} = 1$. In other words the peaking is in the direction of the illumination and the strongest amplitudes are associated with backscatter. This is in marked contrast to Lambertian reflectance where the angular distribution of amplitudes is uniform with respect to the illumination direction. Secondly, the reflectance curve is relatively flat for intermediate and small values of $\mathbf{n}.\mathbf{s}$. As a result the variation of radar amplitude at large angles with the illumination direction is relatively slow. This will limit our capacity to detect shading variations associated with highly inclined surfaces.

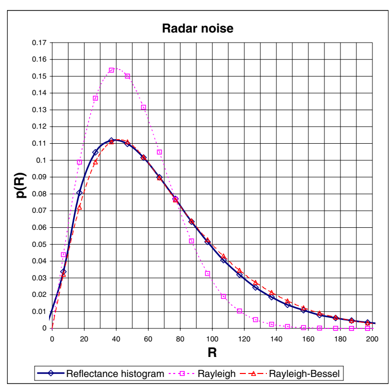
This analysis assumes that the terrain under study is isotropically reflecting and all of a homogeneous terrain class. We can examine the assumption of isotropy more critically by looking at the dependence of reflectivity on the pose of the surface. We examine locations where the surface has a particular orientation with respect to the radar direction. We can then plot the relationship between the angle of the surface normal ($= tan - 1(y/x)$ for a normal of direction $(x, y, z)^T$) and the reflectance values. Figure 2b shows that there is some angular dependence; the terrain under study here is not completely isotropic.



(a) Reflectance model.



(b) Angular distribution



(c) Noise distribution

Figure 2: Empirical distributions.

4 Noise Properties of Radar data

Speckle noise is a significant problem in most radar images. This noise derives from the occurrence of multiple reflecting centres in each radar pixel. The magnitude of this noise is related to the roughness of the surface under study. Some studies, for example those of Zito [12] have suggested that such noise follows a Rayleigh distribution. This distribution follows when the multiple targets produce constant amplitude and random phase responses. However, if the pixels are re-sampled in such a way as to correlate neighbouring pixel sites, this distribution may be modified. For example, if the pixels are re-sampled in complex space (i.e. with a representation $x + iy$ for image pixels), the distribution of amplitude A is modified to a product of Rayleigh and Bessel-function distribution [13]. If I_0 is the zero-order modified Bessel function, then the probability distribution for the radar amplitude is given by

$$p(A) = KA \exp\left[-\frac{A^2}{2\nu^2}\right] I_0 \left[\frac{A^2}{\sigma^2}\right] \quad (1)$$

where K is a constant of normalisation, ν is the modal radar amplitude and σ^2 is the variance parameter of the correlated noise component.

To demonstrate that the product of Rayleigh and Bessel functions accurately models the distribution of noise we have conducted the following experiment. Commencing from the DEM, we have computed ground-truth surface normals. Using the known radar illumination direction $\mathbf{s}$ we have used the ground-truth normals to compute the resulting values of the inner product $\mathbf{n}.\mathbf{s}$ at each pixel location. The radar reflectance function $R(\mathbf{n}.\mathbf{s})$ returns the predicted mean-value ν for the radar amplitude A. We have histogrammed the observed radar amplitudes for all pixels that are predicted to to have a given mean-amplitude ν according to the reflectance function. Figure 2c shows the resulting histogram for pixels with predicted reflectance value $R(\mathbf{n}.\mathbf{s}) = 40$. The plot shows the histogram bin-contents as a solid curve with square markers. Also shown in the plot, as a dotted curve, is the pure Rayleigh distribution with mode $\nu = 40$. This curve fails to predict the high reflectance tail of the radar amplitude histogram. By contrast, the Rayleigh-Bessel distribution, which is shown as a dashed curve, provides a significantly more accurate description of the amplitude histogram. In other words, the histogram strongly indicates that the noise process is Rayleigh-Bessel and that the components of the radar response are partially correlated.

5 Shape from shading

In this section we use the reflectance model developed earlier in this paper to model the brightness process in shape-from-shading.

5.1 Shape-From-Shading Algorithm

To perform shape-from-shading on our radar data, we adopt the variational framework of Horn and Brooks [9]. Whilst there are other, more recent and in some cases more elegant, approaches to the problem [10, 11], the Horn and Brooks formulation offers a technique which we can easily adapt to suit the needs of our radar shape-from-shading application.

The original Horn and Brooks error functional is defined to be

$$E = \int\int \left\{ \left(I(x,y) - \mathbf{n}\cdot\mathbf{s} \right)^2 + \lambda\left(|\frac{\partial \mathbf{n}}{\partial x}|^2 + |\frac{\partial \mathbf{n}}{\partial y}|^2 \right) + \mu(|\mathbf{n}|^2) \right\} dxdy \tag{2}$$

where μ and λ are Lagrange multipliers. This energy function can be minimised applying variational calculus and solving the Euler equation:

$$(I(x,y,) - \mathbf{n}\cdot\mathbf{s})\mathbf{s} + \lambda\nabla^2\mathbf{n} - \mu\mathbf{n} = 0 \tag{3}$$

For the purposes of our radar shape-from-shading application, we make two modifications to the Horn and Brooks algorithm. Firstly, and most importantly, Equation 3 adopts a Lambertian model of surface reflectance. As we have seen, this model is not appropriate for radar reflectance. Consequently, we adopt a more general reflectance model. The associated regularised energy function is

$$E_R = \int\int \left\{ \left(I(x,y) - R(\mathbf{n}\cdot\mathbf{s}) \right)^2 + \lambda\left(|\frac{\partial \mathbf{n}}{\partial x}|^2 + |\frac{\partial \mathbf{n}}{\partial y}|^2 \right) + \mu(|\mathbf{n}|^2) \right\} dxdy \tag{4}$$

The corresponding Euler equation is

$$(I(x,y) - R(\mathbf{n}\cdot\mathbf{s}))\frac{\partial R}{\partial \mathbf{n}} + \lambda\nabla^2\mathbf{n} - \mu\mathbf{n} = 0 \tag{5}$$

Our second departure from the standard Horn and Brooks algorithm is to modify the means by which the Laplacian $\nabla^2\mathbf{n}$ is estimated. Specifically, the Laplacian is estimated using the following differencing mask applied to the full 3x3 neighbourhood of each image pixel

$$L = \frac{1}{20\epsilon}\begin{pmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{pmatrix} \tag{6}$$

This term of the Euler equation represents a regularizer which imposes a smoothness constraint on the solution. This constraint is particularly important when dealing with noisy data. There are other choices of smoothness regularizer. For instance Worthington and Hancock [3] use robust error kernels to deal with noise and outliers.

The Euler equation is solved numerically using a quasi-Newton method. In practice, this involves computing the derivatives of the reflectance function R off-line and storing them in a lookup table. The update equation for updating the surface normal between iterative epochs k and $k+1$ at the pixel location (x,y) is

$$\mathbf{n}^{k+1}_{x,y} = \hat{\mathbf{n}}^{k}_{x,y} + \eta\left\{ I(x,y) - R(\mathbf{n}^{k}_{x,y}.\mathbf{s}) \right\} \frac{\partial R(\mathbf{n}^{k}_{x,y}.\mathbf{s})}{\partial \mathbf{n}} \tag{7}$$

where $\hat{\mathbf{n}}^{k}_{x,y}$ is the weighted average of the neighbouring surface normals obtained using the Laplacian mask and η is the step-size for the quasi-Newton method.

5.2 Boundary Conditions

In order to solve the Euler equation for shape-from-shading, we require one additional ingredient. Boundary conditions are required to produce a unique solution to the Euler equation. These boundary conditions are provided by points in the image at which the directions of the surface normals are already known. In typical shape from shading problems, such points are provided by either occluding boundaries, where the normals lie in the image plane, or by critical points, where the surface normals are perpendicular to the image plane. In terrain data there are no occluding boundaries. We can however, use some unique topographic terrain features to provide the necessary constrained surface normals.

- **Ridge lines:** The radar data being studied covers a mountainous region in Wales. In fact the main topographic structure is Penn-y-Gynt. One of the useful features in such terrain are crest-lines or ridges. These are contours connecting local height maxima and are characterised by zero Gaussian curvature and non-zero mean curvature. In other words, they will be highly curved in one direction and relatively flat in the perpendicular direction. As a result crest-lines present a unique intensity profile in the shading map. Such features are associated with rapid changes in shading. In addition, the local surface at the apex of a ridge-line is parallel to the image plane and provides us with points of known surface orientation. The sharp intensity changes associated with the ridge profileq can be detected by searching for zero crossings of the Laplacian.

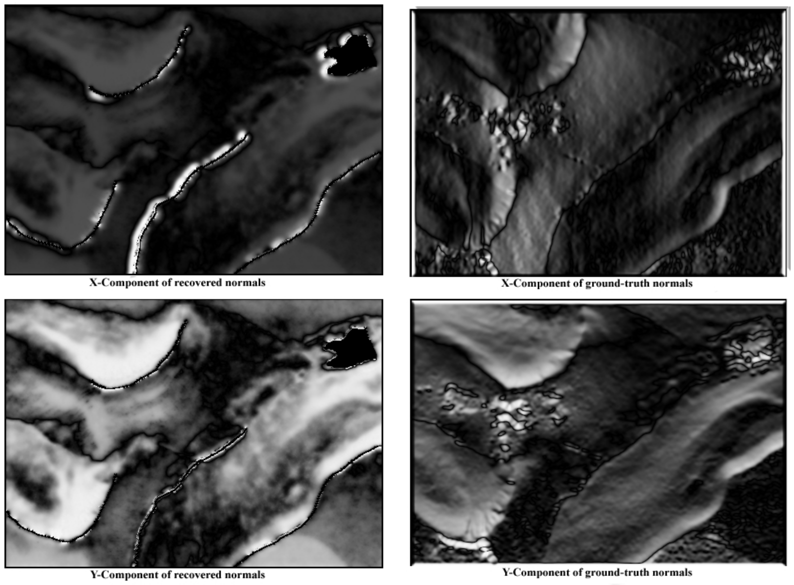
- **Water:** The images here have lakes, ponds and other areas of water in them. These areas have very low radar response compared to other terrain types are hence are easily identified. Since the surface of water is flat and horizontal, the associated surface normals are perpendicular to the the image plane.

With the constrained surface normals provided by crest-lines and areas of water, the Euler equation may be solved by a number of methods. Here we use numerical iteration based on finite differences. The method requires sufficient iterations to spread information from the points of known surface orientation to the remaining points in the image. In practice some 200-300 iterations are required.

6 Results

Figure 3a shows the x and y components of the surface normals resulting from applying the shape-from shading algorithm to the SAR image. For comparison, the ground truth values are shown in Figure 3b.

Examination of these figures shows good agreement between the normal maps. However, there are a number of artifacts worth mentioning. The experimental normal map is clearly over-smoothed when compared with the ground-truth data. This highlights the difficulty of controlling both the pre-smoothing step and the regularization term of the optimisation approach to shape from shading. One major criticism of the variational approach is that it tends to over-smooth the resulting normal map. There may be ways of overcoming this limitation by applying a more sophisticated model of needle-map smoothness [3, 10, 11].



(a) Shape-from-shading (b) Elevation map.

Figure 3: Surface normals.

Another observation from the data is that the y-direction normals appear to be more accurate than their x-direction counterparts. This is an artefact of the illumination direction, which is aligned closely with the vertical axis of the image. The problem of low accuracy along the illumination direction is something which plagues all shape-from-shading techniques.

7 Discussion

In this paper we have examined the noise properties of SAR images of terrain and established a Rayleigh-Bessel model for noise in the raw radar reflectance distribution. Empirical study of the reflectance function for the data reveals that we must adopt a non-Lambertian model for surface reflectance. The experimental reflectance map also reveals that the response is relatively flat at intermediate angles, which may lead to difficulties in detecting changes in surface normal direction at these orientations. Finally, we have established the feasibility of shape-from-shading in radar terrain images by applying a simple modification of the Horn and Brooks [9] algorithm. The resulting surface normal components are in good agreement with ground-truth normal information.

Our future plans revolve around incorporating the radar reflectance model developed in this paper into a more elaborate shape-from-shading scheme. One candidate is the robust regularisation framework recently reported by Worthington and Hancock [3]. Here the simple averaging of the neighbourhood surface normals is replaced by a robust error kernel which weights against outliers. This new needle-map consistency process has been demonstrated to offer advantages in terms of improving the recovery of sharp curvature detail. In the domain of terrain analysis, it should facilitate the recovery of ravines and drainage channels.

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