Autocalibration in the Presence of Critical Motions

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Abstract

Autocalibration is a difficult problem. Not only is its computation very noise-sensitive, but there also exist many critical motions that prevent the estimation of some of the camera parameters. When a "stratified" approach is considered, affine and Euclidean calibration are computed in separate steps and it is possible to see that a part of these ambiguities occur during affine-to-Euclidean calibration.

This paper studies the affine-to-Euclidean step in detail using the real Jordan decomposition of the infinite homography. It gives a new way to compute the autocalibration and analyzes the effects of critical motions on the computation of internal parameters. Finally, it shows that in some cases, it is possible to obtain complete calibration in the presence of critical motions.

1 Introduction

This article raises the problem of autocalibration of a camera undergoing rigid motions under the assumption of constant intrinsic camera parameters.

Many methods of autocalibrating monocular and stereo sensors have been developed in the recent years. Faugeras, Luong and Maybank [5] propose to solve the Kruppa equations from points matches in 3 images. However, this requires non-linear resolution methods. An alternative solution consists to first recover affine structure and then solve for the camera calibration using this. This "stratified" approach [4] can be applied to a single camera motion [8] or to a stereo rig in motion [3] and requires no knowledge about the observed scene.

Affine calibration has already been studied by many authors and amounts to recovering the equation of the plane at infinity, or equivalently the infinite homographies between the views. Many classes of motions have been treated and theoretically solved : pure translation [9], rotations around the camera's center of projection [6], planar motions [1] [2] and general displacements [11] [7].

The infinite homographies then allow the Euclidean calibration to be computed and it is well known that this computation is possible when at least 2 motions with non zero rotations and non parallel rotation axes are available. However, it is not always possible to have such motions. One solution is to add a constraint on the internal parameters (e.g. that the image axes are perfectly orthogonal or that the aspect ratio is known). But, even in

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this case there exist some critical motions [10] which prevent an unambiguous calibration.

The main contribution of this article is a detailed analysis of affine-to-Euclidean autocalibration, based on the real Jordan decomposition of the infinite homographies. This provides a new way to calculate the Euclidean calibration. Critical motions (where the intrinsic parameters can not all be recovered) are also studied. However, in some cases, if the correct constraint is applied, the problem can be solved and all of the intrinsic parameters can be calculated.

2 Preliminaries

A pinhole camera projects a point $\boldsymbol{M}$ from the 3-D projective space onto a point $\boldsymbol{m}$ of the 2-D projective plane. This projection can be written as a 3×4 homogeneous matrix $\mathbf{P}$ of rank equal to 3 :

$$\boldsymbol{m} \simeq \mathbf{P}\boldsymbol{M}$$

where $\simeq$ is the equality up to a scale factor. If we restrict the 3-D projective space to the Euclidean space, then it is well known that $\mathbf{P}$ can be written as :

$$\mathbf{P} = (\mathbf{KR}\ \mathbf{K}\boldsymbol{t})$$

$\mathbf{R}$ and $\boldsymbol{t}$ are the rotation and translation that link the camera frame to the 3-D Euclidean one. The most general form for the matrix of intrinsic parameters $\mathbf{K}$ is :

$$\begin{pmatrix} \alpha & r\alpha & u_0 \\ 0 & k\alpha & v_0 \\ 0 & 0 & 1 \end{pmatrix}$$

where α is the horizontal scale factor, k is the ratio between the vertical and horizontal scale factors, r is the image skew and u_0 and v_0 are the image coordinates of the center of projection.

3 From affine to Euclidean

The affine calibration enables to calculate the infinite homography $\mathbf{H}_\infty$ between the images taken with a camera to calibrate, before and after the rigid motion. Once this infinite homography is obtained, it is possible to recover $\mathbf{K}$ thanks to the relation :

$$\mathbf{H}_\infty = \mathbf{KRK}^{-1} \tag{1}$$

where $\mathbf{R}$ is the rotation of the motion (the Euclidean frame is chosen to be the camera frame). A classical way to solve this equation was first proposed by R.Hartley [6] and consists to solve the equation :

$$\mathbf{H}_\infty^T \mathbf{C} \mathbf{H}_\infty = \mathbf{C} \tag{2}$$

where $\mathbf{C} = \mathbf{K}^{-T}\mathbf{K}^{-1}$ is the image of the absolute conic. $\mathbf{K}$ is then obtained by Cholesky decomposition of $\mathbf{C}$. It is well known that the solutions $\mathbf{C}$ of (2) define a 1-parameter family. This ambiguity can be eliminated if at least 2 motions with non parallel rotation axes are considered.

We propose another way to solve (1) with an analysis which allows us to identify critical motions and to partially solve for calibration in these particular cases. This analysis is based on the real Jordan decomposition which has already been studied in [2] in the case of projective displacements. The approach takes into account the fact that (1) defines $\mathbf{H}_\infty$ as the conjugate of a rotation $\mathbf{R}$. In an appropriate frame, $\mathbf{R}$ can be reduced to the simple form $\mathbf{J}_\theta$:

$$\mathbf{J}_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then, there exists a 3×3 matrix $\mathbf{S}$ such that :

$$\boxed{\mathbf{H}_\infty = \mathbf{S}\mathbf{J}_\theta\mathbf{S}^{-1}} \tag{3}$$

This is a real Jordan decomposition of $\mathbf{H}_\infty$.

3.1 Analysis of the real Jordan decomposition

3.1.1 Ambiguity

In (3), θ is uniquely determined. However, $\mathbf{S}$ is not. Indeed, if $\mathbf{S}$ satisfies (3) and $\mathbf{P}$ is any invertible matrix which commutes with $\mathbf{J}_\theta$, we have :

$$\begin{aligned} \mathbf{H}_\infty &= \mathbf{S}\mathbf{J}_\theta\mathbf{P}\mathbf{P}^{-1}\mathbf{S}^{-1} \\ &= \mathbf{S}\mathbf{P}\mathbf{J}_\theta\mathbf{P}^{-1}\mathbf{S}^{-1} \\ &= (\mathbf{S}\mathbf{P})\mathbf{J}_\theta(\mathbf{S}\mathbf{P})^{-1} \end{aligned}$$

So, $\mathbf{S}' = \mathbf{S}\mathbf{P}$ satisfies (3) too. The converse is also true. That is, if $\mathbf{S}_1$ and $\mathbf{S}_2$ both satisfy (3), then $\mathbf{S}_2^{-1}\mathbf{S}_1$ is a matrix which commutes with $\mathbf{J}_\theta$. It can be easily shown that such a matrix can be written :

$$\mathbf{P}_{a,b,c} = \begin{pmatrix} a & -b & 0 \\ b & a & 0 \\ 0 & 0 & c \end{pmatrix}$$

Let $\mathbf{Q}$[1] be an orthogonal matrix such that $\mathbf{R} = \mathbf{Q}\mathbf{J}_\theta\mathbf{Q}^T$. (1) gives a real Jordan decomposition for $\mathbf{H}_\infty$:

$$\mathbf{H}_\infty = \mathbf{K}\mathbf{Q}\mathbf{J}_\theta(\mathbf{K}\mathbf{Q})^{-1}$$

Furthermore, if $\mathbf{S}$ has been calculated by a real Jordan decomposition of $\mathbf{H}_\infty$ (we will see later how to obtain it), we have, with respect to what has been shown previously, the following relationship :

$$\mathbf{S}\mathbf{P}_{a,b,c} = \mathbf{K}\mathbf{Q} \tag{4}$$

Then,

$$(\mathbf{K}\mathbf{Q})(\mathbf{K}\mathbf{Q})^T = (\mathbf{S}\mathbf{P}_{a,b,c})(\mathbf{S}\mathbf{P}_{a,b,c})^T$$

And so, by $\mathbf{Q}\mathbf{Q}^T = \mathbf{I}$,

[1] $\mathbf{Q}$ is the transformation which enables to express the rotation in a canonic frame where the rotation axis is the z-axis.

$$\mathbf{K}\mathbf{K}^T = \mathbf{S}\begin{pmatrix} a^2+b^2 & 0 & 0 \\ 0 & a^2+b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}\mathbf{S}^T$$

Let be $\lambda = a^2 + b^2$ and $\nu = c^2$. We have finally :

$$\boxed{\mathbf{K}\mathbf{K}^T = \mathbf{S}\begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{pmatrix}\mathbf{S}^T} \tag{5}$$

3.1.2 Particular forms of S

For any motion, the relation (4) implies that the form of $\mathbf{S}$ is :

$$\mathbf{S} = \mathbf{K}\mathbf{Q}\mathbf{P}_{a,b,c}^{-1} = \mathbf{K}\mathbf{Q}\mathbf{P}_{a',b',c'}$$

where $\mathbf{P}_{a',b',c'} = \mathbf{P}_{a,b,c}^{-1}$ commutes with $\mathbf{J}_\theta$.

When the rotation component of the displacement is performed around an axis parallel to the basis axes of the camera, $\mathbf{S}$ takes special forms :

- If the rotation axis is **parallel to the horizontal axis** of the camera :

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{S} \simeq \mathbf{S}_x \simeq \begin{pmatrix} * & * & 1 \\ * & * & 0 \\ * & * & 0 \end{pmatrix} \tag{6}$$

- If the rotation axis is **parallel to the vertical axis** of the camera :

$$\mathbf{Q} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{S} \simeq \begin{pmatrix} * & * & \frac{r}{k} \\ * & * & 1 \\ * & * & 0 \end{pmatrix}$$

  In practice, r is often negligible in comparison with k and we can consider that :

$$\mathbf{S} \simeq \mathbf{S}_y \simeq \begin{pmatrix} * & * & 0 \\ * & * & 1 \\ * & * & 0 \end{pmatrix} \tag{7}$$

- Finally, if the rotation axis is **orthogonal to the image plane**, $\mathbf{Q}$ is the identity and :

$$\mathbf{S} \simeq \mathbf{S}_z \simeq \begin{pmatrix} * & * & u_0 \\ * & * & v_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

We can observe that, in these 3 cases, the structure of $\mathbf{S}$ is independent of any ambiguity in the real Jordan decomposition. It will be shown later that these cases correspond to critical motions for affine-to-Euclidean calibration.

3.2 Affine-to-Euclidean calibration

The real Jordan decomposition can easily be obtained from the eigenvectors of $\mathbf{H}_\infty$.

Indeed, let $\{e^{i\theta}, e^{-i\theta}, 1\}$ be the eigenvalues of $\mathbf{H}_\infty$[2] and $\boldsymbol{u_1}$, $\boldsymbol{u_2} = \bar{\boldsymbol{u_1}}$ and $\boldsymbol{u_3}$ be the associated eigenvectors.

If $\boldsymbol{v_1} = \boldsymbol{u_1} + \boldsymbol{u_2}$, $\boldsymbol{v_2} = i(\boldsymbol{u_1} - \boldsymbol{u_2})$ and $\mathbf{S} = (\boldsymbol{v_1}\boldsymbol{v_2}\boldsymbol{u_3})$, then we have $\mathbf{H}_\infty = \mathbf{S}\mathbf{J}_\theta\mathbf{S}^{-1}$.

3.2.1 Resolution of (5)

As we have seen previously, given an infinite homography $\mathbf{H}_\infty$ between the images of the same camera, it is possible to calculate its real Jordan decomposition and hence a matrix $\mathbf{S}$ such that $\mathbf{H}_\infty = \mathbf{S}\mathbf{J}_\theta\mathbf{S}^{-1}$. Then, it was shown that $\mathbf{S}$ should satisfy (5). The calibration process consists of solving this equation which can be written as a system of 6 equations with 7 unknowns (a, k, u_0, v_0, r, λ and ν) :

$$
\begin{aligned}
\alpha^2 + \alpha^2 r^2 + {u_0}^2 - {S_{1,1}}^2\,\lambda - {S_{1,2}}^2\,\lambda - {S_{1,3}}^2\,\nu &= 0 & (9)\\
k^2\,\alpha^2 + {v_0}^2 - {S_{2,1}}^2\,\lambda - {S_{2,2}}^2\,\lambda - {S_{2,3}}^2\,\nu &= 0 & (10)\\
1 - {S_{3,1}}^2\,\lambda - {S_{3,2}}^2\,\lambda - {S_{3,3}}^2\,\nu &= 0 & (11)\\
r\,k\,\alpha^2 + u_0\,v_0 - S_{1,1}\,\lambda\,S_{2,1} - S_{1,2}\,\lambda\,S_{2,2} - S_{1,3}\,\nu\,S_{2,3} &= 0 & (12)\\
u_0 - S_{1,1}\,\lambda\,S_{3,1} - S_{1,2}\,\lambda\,S_{3,2} - S_{1,3}\,\nu\,S_{3,3} &= 0 & (13)\\
v_0 - S_{2,1}\,\lambda\,S_{3,1} - S_{2,2}\,\lambda\,S_{3,2} - S_{2,3}\,\nu\,S_{3,3} &= 0 & (14)
\end{aligned}
$$

It is clear that this system cannot be solved just as it is (there is one unknown too many). We must either consider a constraint on one unknown, or add equations from several motions (with non parallel rotation axes).

If we want to calibrate with a single motion or with planar motions (where all rotation axes are parallel), a constraint on the internal parameters must be imposed. Commonly used constraints are either $k = k_0$ or $r = 0$.

General resolution

If the matrix $\mathbf{S}$ does not have one of the forms expressed in 3.1.2, the system can then be solved :

- if the constraint $r = 0$ is used, the intrinsic parameters are uniquely defined.
- if the constraint $k = k_0$ is chosen, there are 2 sets of solutions : they correspond to the 2 solutions λ of a second degree equation. We keep only the one with smallest $|r|$.

Now consider the degenerate cases.

Degenerate cases

- **Horizontal rotation axis.** We have $\mathbf{S} \simeq \mathbf{S}_x$ and so, $S_{3,3} = 0$ and $S_{2,3} = 0$ (see (6)) : (11) gives λ and u_0 and v_0 can then be computed with (13) and (14).

[2]We suppose that $\mathbf{H}_\infty$ is normalized such that its determinant is equal to 1.0. Since it is conjugate to a rotation matrix, it has one real and one complex conjugate pair of eigenvalues, all of unit modulus.

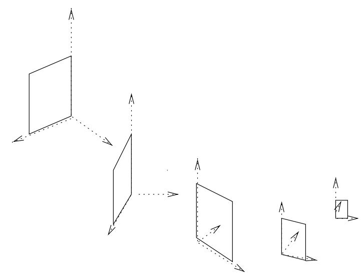

Figure 1: example of critical motion : vertical rotation axis

  - if we impose $r = 0$, (10) gives $k\alpha$. However, (9) can't give either α or ν.
  - if $k = k_0$ is considered, $\alpha^2 + \alpha^2 r^2$ and $\alpha^2 r$ can be calculated directly and solved to give r, $\alpha_u = \alpha$ and $\alpha_v = k\alpha$.

- **Vertical rotation axis.** This is the case when $\mathbf{S} \simeq \mathbf{S}_y$, which is similar to the previous one. We have $S_{3,3} = 0$.
  - if $r = 0$, we have $S_{1,3} = 0$ and then all parameters except k and $k\alpha$ can be evaluated.
  - if $k = k_0$, $S_{1,3} = \frac{r}{k}$ leads to a total resolution of calibration

- **Rotation axis orthogonal to the image plane.** We have $\mathbf{S} \simeq \mathbf{S}_z$ ($S_{3,1} = 0$ and $S_{3,2} = 0$).

  It is the worst critical case in so far as u_0, v_0, k and r can be calculated, but α remains always undetermined, whatever the constraint may be.

As a conclusion, we saw that the problem of affine-to-Euclidean calibration could be easily solved in particular cases (single motion, all parallel axes rotations[3]).

We also showed that using the constraint $k = k_0$ allowed us to avoid critical cases : there remains then just one real critical motion (rotation axes orthogonal to the image plane).

4 Experiments and results

4.1 Description

In this section we apply our autocalibration algorithms to synthetic data in order to analyse the effect of different kind of motions on the computation of autocalibration. 3-D points were generated and projected onto the cameras of a virtual stereo rig[4] performing different kind of motions. Gaussian noise of 1-pixel standard deviation was added to the data.

[3] In this case, it is possible to calculate a matrix $\mathbf{S}$ that satisfies the real Jordan decomposition of each infinite homography.

[4] Intrinsic parameters of each camera were constant.

For simplicity, we show results only for the calibration of the left camera of the rig. The actual intrinsic parameters are :

$$\mathbf{K} = \begin{pmatrix} 715 & 0 & 140 \\ 0 & 995 & 275 \\ 0 & 0 & 1 \end{pmatrix}$$

The aim of this experiment is not to obtain accurate computation of intrinsic parameters, but to show that if the constraint $k = k_0$ is used, there is only one critical kind of motions for the affine-to-Euclidean calibration (instead of three) : motions whose rotation axes are orthogonal to the image plane.

4.2 Results

First, projective displacements D_{proj} are calculated from point correspondences and epipolar geometry with the method described in [7]. Then, the equation of the plane at infinity is calculated and the infinite homographies $\mathbf{H}_\infty$ associated to the left camera are derived. Our affine calibration algorithms are similar to [1] and [7] and cope with general and planar motions (in this case, we need at least 2 motions). Finally, the real Jordan decomposition of each $\mathbf{H}_\infty$ is calculated and the resolution of (5) enables us to obtain the complete camera calibration.

We show the results on 4 motion sequences (each consisting of 5 motions) :

- sequence 1 : non singular general motions
- sequence 2 : non singular planar motions
- sequence 3 : planar motion with a horizontal rotation axis
- sequence 4 : planar motion with a vertical rotation axis

The following table exhibits the matrices $\mathbf{H}_\infty$ and $\mathbf{S}$ obtained for the first motion of each sequence. It confirms the particular forms of $\mathbf{S}$ obtained for critical motions.

	$\mathbf{G}_\infty$	$\mathbf{S}$
sequence 1	$\begin{pmatrix} 0.778 & 0.135 & 315.085 \\ -0.703 & 0.768 & 354.918 \\ -0.000 & -0.000 & 1.018 \end{pmatrix}$	$\begin{pmatrix} 0.301 & -0.902 & 1.000 \\ 1.430 & -0.298 & -1.184 \\ 0.001 & 0.001 & 0.001 \end{pmatrix}$
sequence 2	$\begin{pmatrix} 1.014 & -0.086 & -161.587 \\ 0.254 & 1.006 & -129.024 \\ 0.000 & 0.000 & 0.891 \end{pmatrix}$	$\begin{pmatrix} 2.750 & -0.979 & 1.000 \\ 0.350 & -2.859 & -3.197 \\ -0.001 & -0.004 & 0.002 \end{pmatrix}$
sequence 3	$\begin{pmatrix} 1.003 & 0.034 & -8.576 \\ 0.008 & 1.044 & -297.574 \\ 0.000 & 0.000 & 0.875 \end{pmatrix}$	$\begin{pmatrix} -0.112 & 0.034 & 1.000 \\ -0.414 & -0.904 & 0.037 \\ -0.001 & 0.000 & 0.000 \end{pmatrix}$
sequence 4	$\begin{pmatrix} 1.009 & 0.003 & -214.246 \\ 0.116 & 1.003 & -26.163 \\ 0.000 & 0.000 & 0.903 \end{pmatrix}$	$\begin{pmatrix} 0.879 & -0.327 & -0.007 \\ -0.073 & -0.360 & 1.000 \\ -0.000 & -0.001 & 0.000 \end{pmatrix}$

The results of the computation of $\mathbf{K}$ from the previous matrices $\mathbf{S}$ are as follows :

- with the assumption $r = 0$, only the first two sequences allow us to calculate $\mathbf{K}$:

	α_u	α_v	$k = \frac{\alpha_v}{\alpha_u}$	u_0	v_0
sequence 1	722.5	1002.6	1.388	126.2	280.0
sequence 2	695.7	939.50	1.350	147.5	273.3
sequence 3	$\mathbf{821.0}$	978.8	$\mathbf{1.192}$	138.05	294.18
sequence 4	716.43	$\mathbf{702.28}$	$\mathbf{0.980}$	135.05	275.62

- the assumption $k = \frac{995}{715} \approx 1.39$ allows us to calculate $\mathbf{K}$ in all sequences :

	α_u	α_v	r	u_0	v_0
sequence 1	721.8	1007.8	-0.0164	139.7	255.4
sequence 2	691.8	966.0	-0.0137	153.9	241.0
sequence 3	700.8	978.6	-0.0272	117.6	293.4
sequence 4	712.5	994.5	0.0046	127.8	283.7

From these experiments, we can see clearly that the k-constraint allows us to calibrate even in the case of critical motions. With the r-constraint, we can see that significant errors are made in α_u for sequence 3, and in α_v for sequence 4.

5 Conclusion

We have described a method for solving the problem of affine-to-Euclidean calibration, based on the real Jordan decomposition of $\mathbf{H}_\infty$. This allows us to express the ambiguity in the computation of the absolute conic (and also, the intrinsic parameters) as a 1-parameter family. Although this ambiguity can be solved when many motions with non parallel rotation axes are used, it can not when motions are planar or when we dispose of a single motion. In these cases, we showed that an assumption should be made on one of the internal parameters r or k.

Besides, we showed the existence of 3 classes of critical motion. We saw that r and k didn't have the same role in the resolution of the equations : in particular, the knowledge of k allows us to cope with 2 of the 3 critical motions. Experiments on noisy synthetic data confirmed the theoretical results and proved it was possible to calibrate a camera in some of the special cases. Experiments we have made on real data (not shown) seem to confirm that in general resolution of the equations with the k-constraint is relatively stable even when the rotation axis is near to the horizontal or vertical axes of the camera.

However, the analysis we made here is only qualitative : we studied which parameters could be obtained for each kind of critical motion. We are currently studying more quantitative extensions to this work which try to analyze the precision and stability of the parameter computation as a function of the motion.

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