

# Pattern Recognition Methods for Object Boundary Detection

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## Abstract

Boundary extraction is a data representation problem: image features are segmented and approximated by a parametric curve or a sequence of model points. However, the use of classic Pattern Recognition methods in boundary detection is unusual when compared with more popular approaches, e.g., active contours. This can be partially explained by their inability to separate boundary edges from other image strokes. This paper presents modified versions of several clustering and neural networks algorithms (c-means, fuzzy c-means, Kohonen maps, elastic nets), enhanced with dynamic data segmentation capabilities. This is achieved by using a noise model. The noise model consists of a virtual unit equidistant of all data points, which can be geometrically interpreted as a noise plane parallel to the image plane. The proposed technique extends the unified framework recently proposed by Abrantes and Marques [1] in the context of edge linking with constrained clustering techniques. Results are provided in the paper to illustrate the segmentation capability of the novel methods in the analysis of images with undesired strokes (e.g., inside the object). It is concluded that the noise model proposed in this paper allows a widespread use of classic constrained clustering algorithms in the context of shape analysis.

## 1 Introduction

Boundary detection is a data segmentation and representation problem: we wish to approximate the object boundary by a parametric curve or a sequence of points. Many Pattern Recognition methods were proposed to approximate a generic data set by a sequence

of prototypes. Well known examples are clustering techniques (e.g., Isodata, Lloyd-Max algorithm), fuzzy systems (fuzzy c-means) and neural networks (Kohonen maps, elastic nets). However, the application of these methods in object boundary detection has been restricted. This can be partially explained by their inability to discriminate between the edge points belonging to the object boundary from the others (inner edges, edges from other objects, etc).

Well succeeded methods in the context of edge linking and boundary detection are the active contour models which try to minimize an image dependent energy function [6]. Some energy functions are directly computed from the image and its derivatives. However, other energies are defined using image features only (e.g., edge points) [3]. In this case, the role of Active Contours is similar to the goal of constrained clustering algorithms: they try to approximate data points detected in the image (or a subset of it) by a sequence of prototypes. A unified framework to allow a joint study of several Pattern Recognition and Active Contour methods was recently proposed in [1]. Unfortunately, this has not allowed a widespread use of unsupervised learning techniques in image shape analysis: constrained clustering methods are not able to segment the data.

This paper presents modified versions for several Pattern Recognition methods which encompass dynamic data segmentation. A noise model is used to represent data points which are far from the object boundary. This is achieved by adding a virtual unit to the boundary model and introducing the necessary modifications in the fuzzy energy proposed in [1]. The virtual unit can be interpreted as a noise plane parallel to the image plane. The distance between both planes controls noise discrimination power. All the properties associated to the unified framework (e.g., the attraction regions, centroids and Hooke constants) are still valid in the class of modified algorithms described in this paper.

The rest of the paper is organized as follows: section 2 briefly describes the main concepts of the unified framework; section 3 discusses the introduction of a noise model in this class of algorithms; section 4 presents some numerical results to illustrate the segmentation ability of the modified algorithms and section 5 concludes the paper.

## 2 Unified Framework

This sections presents a class of Pattern Recognition algorithms which can be applied to the extraction of object boundaries given a set of feature points (edge points) detected in the image. Let  $P$  denote a set of image points and let  $V = [v_1, v_2, \dots, v_M]^T$ ,  $v_k \in R^2$ , be a sequence of prototypes (shape model) which tries to represent the data<sup>1</sup>. In the unified framework the estimation of  $V$  is obtained by the minimization of an energy function

$$E = E_i + E_e \quad (1)$$

where  $E_i$  is an internal energy which depends on the model configuration (the most usual shapes have low energies while the most unusual correspond to high energies) and  $E_e$  is a data dependent energy (external energy) which measures how well is the data represented by the model. In the context of the unified framework the internal energy is a simple quadratic form,  $E_i = \text{tr} \{V^T A V\}$  where  $A$  is a semi-positive definite matrix;  $E_e$  is a

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<sup>1</sup>in edge linking problems the model should only represent data points belonging to the object boundary and discard the rest. This operation is often known as data segmentation.

fuzzy energy defined by

$$E_e = \sum_{p \in \mathcal{P}} \sum_{k=1}^M w_k(p) d_k(p) \quad (2)$$

where  $w_k(p)$  is a weighting factor which measures the influence of the edge point  $p$  on the  $k$ -th model unit and  $d_k(p) = \|v_k - p\|^2$  is the square of the Euclidean distance from  $p$  to the  $k$ -th unit. It is assumed that the weighting function  $w_k(p)$  may depend on all distances of point  $p$  to the model prototypes, i.e.,  $w_k = f_k(d_1, \dots, d_M)$ . Well known algorithms minimizing energies contained in this framework are the snakes (with Cohen energy [3]) and several Pattern Recognition methods (Kohonen maps [7], elastic nets [5] and fuzzy c-means [4] [2]). The weighting functions associated with these algorithms are shown in Table 1.

The minimization of (1) can be performed by several methods. A popular solution is the use of a gradient algorithm<sup>2</sup>. The gradient of the energy (2) can be expressed in terms of the differences between the model prototypes and corresponding data centroids. The update of the  $k$ -th unit by the gradient algorithm is given by [1]

$$v_k \leftarrow v_k + \Delta v_k \quad (3)$$

$$\Delta v_k = -\alpha (AV)_k + \beta \mu_k (\xi_k - v_k) \quad (4)$$

where  $\alpha, \beta$  are update gains,  $(M)_k$  is the  $k$ -th line of matrix  $M$ ,  $\xi_k$  is the centroid of the data associated with the  $k$ -th unit and  $\mu_k$  measures the corresponding amount of data (mass). The mass and centroid associated with the  $k$ -th model unit are given by

$$\mu_k = \sum_p \vartheta_k(p) \quad (5)$$

$$\xi_k = \frac{\sum_p p \vartheta_k(p)}{\mu_k} \quad (6)$$

where  $\vartheta_k(p)$  controls the attraction force generated by pattern  $p$  on  $v_k$ . The second term of (4) is the total force, obtained assuming that all data points are connected to the unit  $v_k$  by springs with the Hooke constants given by  $\vartheta_k(p)$ . Constants  $\vartheta_k(p)$  are related to the weighting functions  $w_k(p)$  by [1]

$$\vartheta_k(p) = w_k(p) + \sum_j d_j(p) \frac{\partial w_j(p)}{\partial d_k(p)} \quad (7)$$

### 3 Robust Estimation with Noise Plane

The unified framework summarised in the previous section allows a common description of Snakes and several Pattern Recognition methods and suggests the use of these methods in adaptive shape representation (only Snakes are usually applied in this context). Unfortunately, Pattern Recognition methods try to represent all the data points, assuming that a

<sup>2</sup>not all the algorithms, which were mentioned before, use the gradient algorithm to minimize the energy.

Algorithm	$w_k(p)$	$\vartheta_k(p)$
Snakes	$2\sigma^2 \frac{1-\phi_\sigma(d_k)}{d_k}$	$\phi_\sigma(d_k)$
Kohonen Maps	$\Lambda_\theta(k, k^*)$	$\Lambda_\theta(k, k^*)$
Elastic Nets	$\frac{-2\sigma^2 \log \sum_{j=1}^M \phi_\sigma(d_j)}{M d_k}$	$\frac{\phi_\sigma(d_k)}{\sum_{j=1}^M \phi_\sigma(d_j)}$
Fuzzy C-means	$\left( \sum_{j=1}^M \left( \frac{d_k}{d_j} \right)^{\frac{1}{q-1}} \right)^{-q}$	$\left( \sum_{j=1}^M \left( \frac{d_k}{d_j} \right)^{\frac{1}{q-1}} \right)^{-q}$
C-means	$\delta(k - k^*)$	$\delta(k - k^*)$

Table 1: Weighting functions and Hooke constants for snakes, Kohonen maps, elastic nets, fuzzy c-means and c-means. In this table  $\phi_\sigma(d_k) = \exp(-d_k/2\sigma^2)$ ,  $k^*$  denotes the nearest model unit,  $\delta$  is the delta function and  $\Lambda$  is the neighboring function used in Kohonen networks.

prior segmentation was performed to discard edges not belonging to the object boundary (far away edges produce significant attraction forces on the model units). This is not a reasonable assumption. Adaptive segmentation mechanisms have to be incorporated to avoid long distance attraction forces.

This problem is similar to the one addressed in robust statistics: how to avoid the influence of outliers on the estimates. However, the class of algorithms studied in this paper is defined in a deterministic context and different types of solutions are therefore required.

To overcome this difficulty, data points belonging to other objects or strokes (e.g., inner edges, boundary of other objects) are considered as noise and they will be modelled by adding a virtual unit  $v_{M+1}$  to the shape model  $V$ . It will be assumed that the distance of  $v_{M+1}$  to all data points is a constant,  $\eta$ , i.e.,

$$d_{M+1}(p) = \eta^2, \forall p \in P \quad (8)$$

The virtual unit used to represent noise data has a simple geometric interpretation. It is not a model point (prototype) in the image, but instead it is a plane parallel to the image plane being  $\eta$  the distance between both planes (see Fig. 1).

The use of a virtual unit is an elegant way to introduce the segmentation ability in Pattern Recognition methods. The expression of the energy with noise model is the same as before (see 2) but with  $M$  replaced by  $M + 1$ ,

$$\tilde{E}_e = \sum_p \sum_{k=1}^{M+1} w_k(p) d_k(p) \quad (9)$$

Furthermore, the weights  $w_k(p)$  and the Hooke constants  $\vartheta_k(p)$  associated with each algorithm have the same expressions (see Table 1). This does not mean that the weighting functions and Hooke parameters associated with the model units have the same values as before. In fact, the additional unit modifies the values of all these variables. This is a consequence of the competitive learning nature of all previous methods (with the exception of Snakes): all model units compete to represent the data; if a data point is far from all model units it will be represented by the virtual unit.

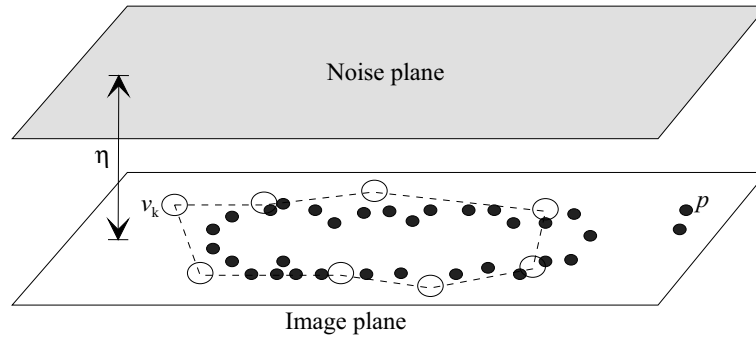


Figure 1: Geometric representation of image and noise planes.

Figures 2 and 3 illustrate the influence of the virtual unit on the spring constant  $\vartheta_k(p)$  and on the evolution of the attraction regions<sup>3</sup> when the plane distance  $\eta$  decreases (the first column corresponds to  $\eta = \infty$ ). Two algorithms are considered in these figures: the fuzzy c-means and the elastic nets. In both cases, the use of a noise plane allows an efficient control of the attraction region scope. It should be stressed that the algorithms are modified in a natural way since they are still members of the unified framework and minimize a fuzzy energy given by (9). Similar examples could be provided for the other methods mentioned before.

## 4 Experimental Results

Experimental tests were carried out to assess the performance of the modified algorithms in the presence of undesired edge strokes (inner edges, multi-objects, complex background, etc).

Figures 4, 5 show the results obtained with fuzzy c-means and elastic nets, respectively, with and without the noise model. The classic versions of the algorithms (without noise model) try to represent all the edge points detected in the image and they are attracted towards inner strokes. The modified algorithms with a noise model are able to discriminate the data and provide acceptable approximations of the object boundary. The ability of fuzzy c-means and elastic nets to model the boundary concavities must be emphasized since they perform much better than Snakes in these cases (the snake attraction regions are local and have the ability to segment the data but they are not able to approximate deep concavities).

These examples show that the noise model allows an effective segmentation of the edge points detected in the image, enabling the algorithms to discard the data points far from the model boundary. If the model is properly initialized it will converge to the object boundary, discarding the influence of the undesired edges.

The class of algorithms described in sections 2, 3 can also be used in tracking applications. Experimental tests were conducted to evaluate the performance of the elastic nets, Kohonen maps, fuzzy c-means and hard c-means in the tracking of non rigid objects. Fig-

<sup>3</sup>the attraction region associated with the k-th model unit is the set of points which attract  $v_k$  with Hooke constant greater than a threshold  $T$  ( $\vartheta_k(p) > T$ )

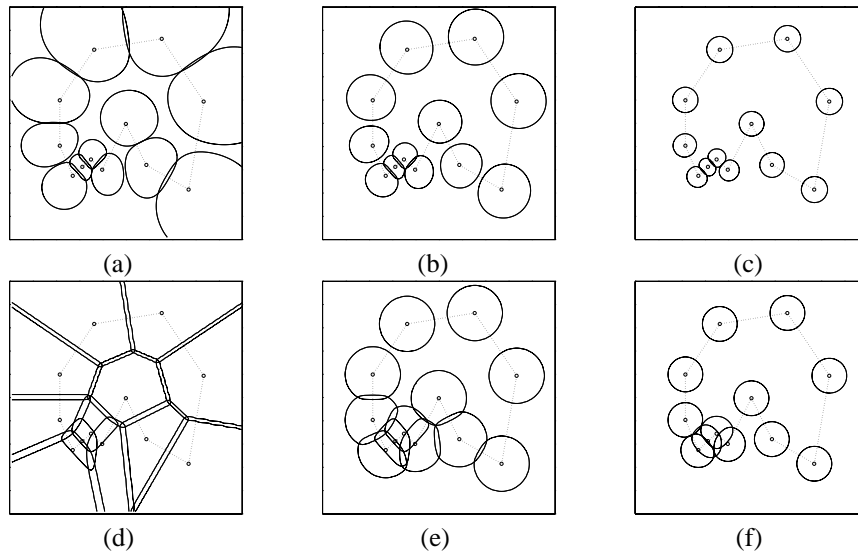


Figure 2: Shape evolution of the attraction regions when  $\eta$  decreases, for a small contour model with 12 units. The (a-c) represent the attraction regions of fuzzy c-means and (d-f) the attraction regions of elastic nets.

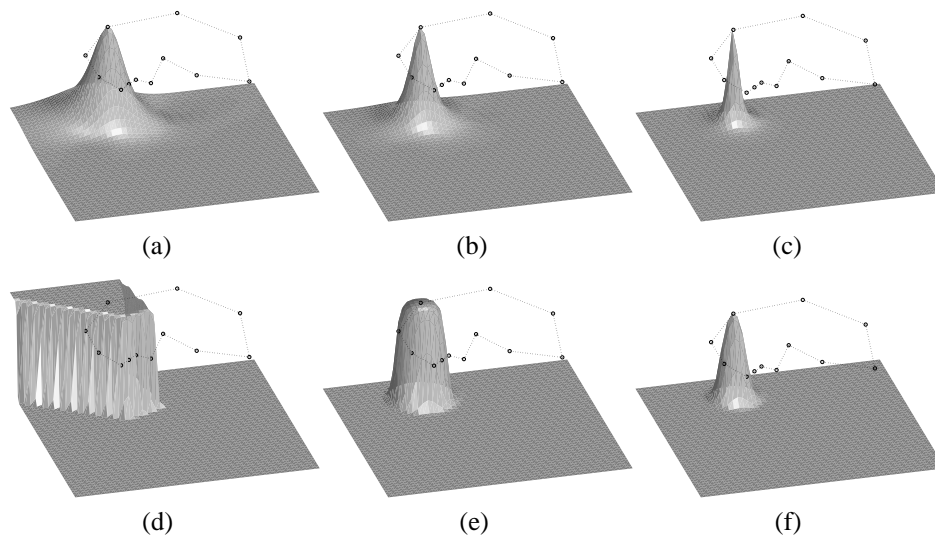


Figure 3: Evolution of the Hooke constant when  $\eta$  decreases, for a specific unit, using the model of figure 2. The (a-c) show the Hooke constants of fuzzy c-means algorithm and (d-f) the Hooke constants of the elastic nets.

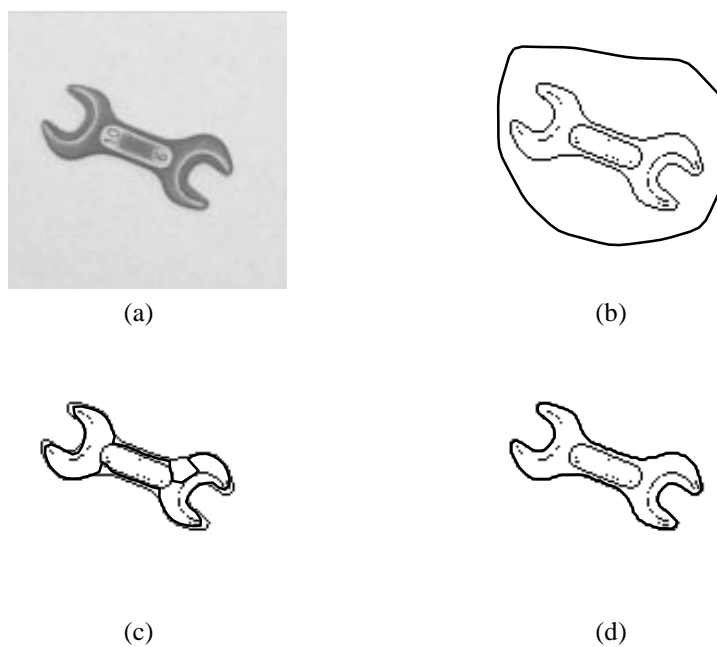


Figure 4: Boundary extraction with fuzzy c-means: (a) original image; (b) edge image and initial model configuration; (c,d) results obtained without noise plane and with the noise plane, respectively.

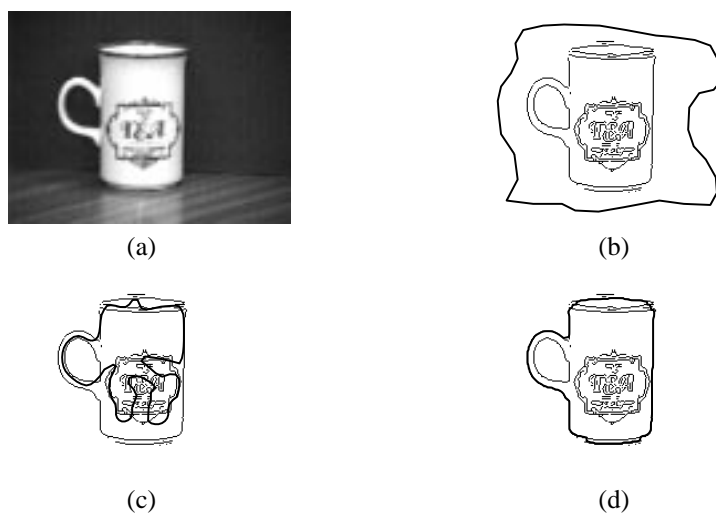


Figure 5: Boundary extraction with elastic nets: (a) original image; (b) edge image and initial model configuration; (c,d) results obtained without noise plane and with noise plane, respectively.

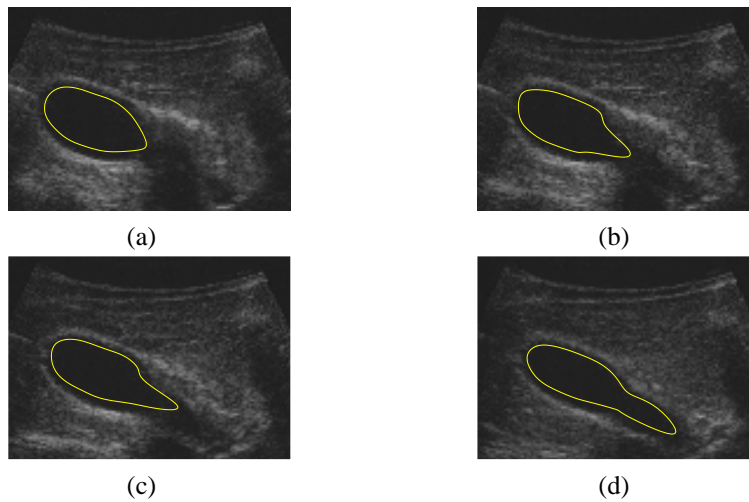


Figure 6: Tracking of a deformable organ using fuzzy c-means with noise model.

Figure 6 shows a set of ultrasound images of a gall bladder, extracted from a video sequence. The fuzzy c-means algorithm with a noise model was used in this example to track the organ cross section.

A systematic comparison of the performance of these five algorithms in a large set of images was not performed yet (the choice of automatic criteria to evaluate the performance of adaptive shape models is still an open question). However, the experimental tests conducted in this study suggest that the fuzzy c-means algorithm (with noise model) achieves the best performance, becoming an advantageous alternative to the use of active contours.

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## 5 Conclusion

This paper presented a set of Pattern Recognition algorithms for object boundary detection. These algorithms are modified versions of classic data representation methods proposed in the context of unsupervised learning, fuzzy systems and neural networks. The modified algorithms overcome the main difficulty of the classic clustering techniques in object boundary extraction: the lack of segmentation mechanisms to discriminate the boundary features from the others. A noise model is used in this paper to deal with the undesired features. This model has a simple geometric interpretation in the context of the unified framework proposed in [1]: it corresponds to a virtual plane parallel to the image plane. The distance between both planes controls the scope of the attraction region associated with each model unit while the local shape of the attraction region is still dependent on the specific method considered. Experimental results are presented to illustrate the performance of the modified algorithms in images with inner edges. It is concluded that the



noise model provides an effective segmentation of image features, allowing a widespread use of these algorithms in the context of shape analysis.

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