Detection of Partial Ellipses Using Seperate Parameters Estimation Techniques

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Abstract

Ellipses are a powerful feature and are useful in computer vision. Most techniques for ellipse fitting can be divided into two main classifications: one is the least square estimation (LSE) method and the other is the Hough transform (HT). They either make certain assumptions about the type of noise distribution, or require input parameters. This often prevents the techniques working robustly over a large range of data. A simple algorithm for partial ellipse detection using seperate parameters estimation techniques is proposed. The new method consists of two steps: (1) detect the elliptical center using its skewed symmetry properties; (2) estimate the other three parameters (A, B, θ) using the median of the intercepts (MI). The new method has been tested on both the synthetic and real images. The experimental results show that the method is reliable and accurate.

1 Introduction

The ellipses are frequently seen in the real world. They provide the most important clues to 3D interpretation of images next to straight lines. One reason for this is that many man-made objects have circular or spherical parts, and circles and spheres are projected onto ellipses. Thus, if an ellipse on image plane is supposed as the projection of a 3D circle and the five parameters of the ellipse such as the center coordinates(x_c, y_c), long semi-axis A, short semi-axis B and rotating angle θ have been detected, the 3D position and orientation of the circle can be estimated easily.

For ellipse detection, many methods have been proposed. They can be assessed by the following criteria: robustness to noise and incomplete data, accuracy, and computational efficiency. Two of the most common fitting techniques used in computer vision are the least squares estimation (LSE) method [1,2,3,4] and the Hough transform (HT) [5,6,7] based techniques and

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its variations. However, both are unsatisfactory concerning the above criteria, especially for detecting partial ellipses. Although the LSE methods perform well in the presence of Gaussian noise it requires that the contour or shape of an ellipse to be detected be complete [2]. Porrill proposed a method of fitting ellipses by using Kalman filter [11], but care must be taken to account for bias. In addition it needs to solve a five rank linear equations in which each element is the sum $\sum_{i}^{N} x_i^p y_i^q$ {p,q = 0,1,2,3, 4, p+q < 4, (x_i, y_i) are the detected elliptical points}, so it is easy to cause computing overflow and it becomes unreliable. The Hough transform (HT) is a power tool in shape analysis and is insensitive to outliers, but it requires large computer storage and processing time. The ellipse finding algorithm requires a 5D accumulator array in the standard HT method. Furthermore, it is not easy to choose a suitable peak finding rule to the multiple peaks problem. Therefore it is rather impractical to implement these methods.

In the past most of these techniques usually estimate the five parameter (x_c, y_c, A, B, θ) of an ellipse simultaneously. They are inefficient and unreliable to detect broken or partial ellipses. Here, we propose a new algorithm to detect partial ellipses using the seperate parameter estimation techniques. It is very simple and intuitive.The new method consists of two steps: (1) detect the center (x_c, y_c) of an ellipse using its skewed symmetry. (2) estimate the other three parameters (A, B, θ) by the median of the intercepts (MI).

The remains of this paper are organized as follows. In Section 2 we analyse the skewed symmetrical properties of ellipse. According to them, an algorithm for detecting the center of ellipse is proposed. We describe the method for estimating the three parameters (A, B, θ) by the median of the intercepts (MI) in Section 3. Then some situations are discussed and the experimental results are presented in Section 4. They show that the algorithm we design is very efficient, reliable and fast. Finally, some conclusions are given in the last section.

2 Center Detection of Ellipses

This section describes the skewed symmetrical properties of an ellipse. We use them to detect the center of an ellipse. The five parameters of an ellipse are denoted as: (x_c, y_c) , the coordinate of the center; A and B, the lengths of the two semi–axes; and θ , the rotating angle.

2.1 Theoretical Bases

The psychological experiment indicated that human beings interpret a skewed symmetry as a oriented real symmetry. When 3–D bilateral and orthogonal symmetry objects are perspectively projected onto 2–D images, the 2–D shapes mostly are the skewed symmetry. Now, we give the definition of skewed symmetry shape.

Definition 1. If the mapping of two shapes is one–to–one each other, the connecting lines of two corresponding points always parallel, and their midpoints locate in the same line, then the two shapes are called the skewed symmetry and their symmetry axis is the connecting line of midpoints, shown in Fig.1.

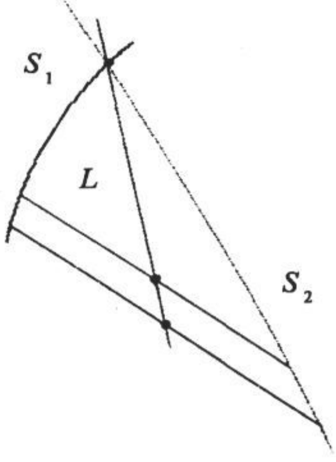


Fig.1. Curve S_1, S_2 are the skewed symmetry and their axis is line L.

Theorem 1. An ellipse has the countless skewed symmetry axes.

Theorem 2. If a closed shape has several the skewed symmetry axes (the axes number $>$ 2), then all axes cross at the center of the closed shape.

The proof of above theorems is omitted, please refer to [9]. We prove the following corollary.

Corollary 1. The skewed symmetry axes of an ellipse cross at its enter.

Proof: Because the skewed symmetry of an ellipse is invariant by rotating and translating, for simplicity, we assume that the equation of ellipse is standard:

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \qquad (1)$$

Two paralleing lines are $y = kx + b_1$ and $y = kx + b_2$. They interact at points $p_1(x_1, y_1)$, $p_2(x_2, y_2)$, $p_3(x_3, y_3)$, and $p_4(x_4, y_4)$, with the ellipse shown in Fig.2.

$p_1(x_1, y_1)$ and $p_2(x_2, y_2)$ satisfy the following equations

$$\begin{cases} \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \\ y = kx + b_1 \end{cases} \qquad (2)$$

We obtain the midpoint $M_1(x_{01}, y_{01})$ of p_1p_2

$$\begin{cases} x_{01} = -\dfrac{2kb_1A^2}{B^2 + A^2k^2} \\ y_{01} = kx_{01} + b_1 \end{cases} \quad (3)$$

Similarly, the midpoint $M_2(x_{02}, y_{02})$ of p_3p_4

$$\begin{cases} x_{02} = -\dfrac{2kb_2A^2}{B^2 + A^2k^2} \\ y_{02} = kx_{02} + b_2 \end{cases} \quad (4)$$

The linking line of M_1M_2 is

$$y = \frac{y_{02} - y_{01}}{x_{02} - x_{01}}x + \frac{x_{02}y_{01} - x_{01}y_{02}}{x_{02} - x_{01}} \quad (5)$$

Wherein

$$m = \frac{x_{02}y_{01} - x_{01}y_{02}}{x_{02} - x_{01}} = 0$$

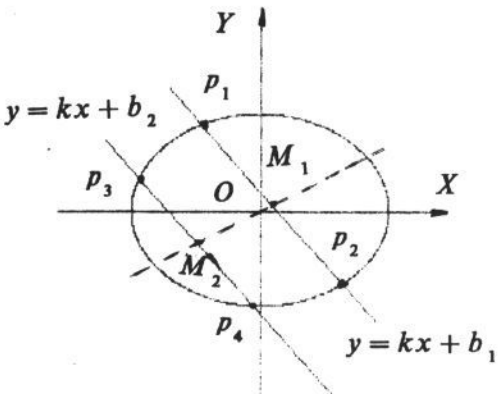


Fig.2 The skewed symmetry axis of an ellipse passes through its center.

This result indicates that M_1M_2 passes through the coordinate origin (0,0) which is the center of the ellipse. Due to the arbitrary of k, b_1 and b_2, all of the skewed symmetry axes of an ellipse cross at its center.

2.2 Procedures of Estimating the Center of Ellipse

1. Find the two farthest points M_0N_0 of ellipse, shown in Fig.3.

2. Determine the peak point P_0 whose distance to M_0N_0 is the longest.

3. Make the paralleling lines with M_0N_0 from P_0 to M_0N_0, obtain the midpoints of line segment. We use the least squre estimation(LSE) mehtod to form these points into a skewed symmetry axis P_0O_0 of ellipse.

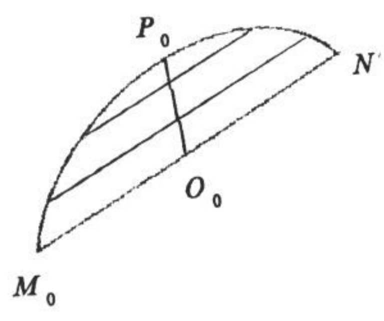
4. Move N_0 to N_i, find the new peak point P_i and make the paralleling lines with M_0N_i from P_i to M_0N_i, obtain a series of axes P_iO_i.

5. Similarly, move M_0 to M_j, find the peak point P_j, and Make the paralleling lines with M_jN_0 from P_j to M_jN_0 and obtain another series of axes P_jO_j.

6. Repeat Step 4 and 5, get more skewed symmetry axes.

7. Calculate the crosspoints of these skewed symmetry axes.

8. Estimate the center of ellipse by averaging the crosspoints.



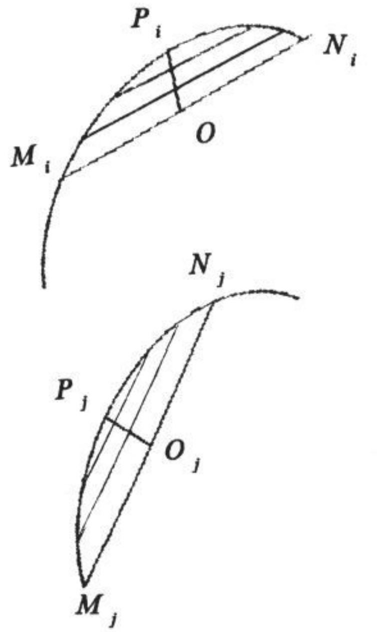


Fig.3. Finding the skewed symmetry axes of an ellipse.

3 Estimating Parameters A, B, θ by the Median of the Intercepts (MI) Method

Recently, robust fitting techniques using medians have become popular [8,10]. Their advantage is that they can tolerate many large outliers (i.e., they have a breakdown points) without requring any arbitrary parameters and are reliable (e.g., there are no problems with convergence as in the iterative reweighted least squres method). Kamgar–Parsi et al[8] proposed a new method for fitting the straight line to a set of points called the median of intercepts (MI). Rosin described an extension of this technique to ellipse fitting in [10], but he accumulated the five parameters of all perfect ellipse fits. This paper simplifies his method and proposes a new algorithm which can detect the partial ellipses, because the center coordinates of ellise have been determined seperately.

After the center coordinates of an ellipse have been detected, the equation of an ellipse with three parameters(A, B, θ) is written as

$$ax^2 + bxy + cy^2 = 1 \qquad (6)$$

where

$$a = \frac{cos^2\theta}{A^2} + \frac{sin^2\theta}{B^2}$$

$$b = 2sin\theta cos\theta(\frac{1}{A^2} - \frac{1}{B^2})$$

$$c = \frac{sin^2\theta}{A^2} + \frac{cos^2\theta}{B^2}$$

It is apparent that at least three points of ellipse must be known to obtainA, B, θ.

Now we have a set of n points P{p_1, p_2, ..., p_n}. Every three points selected from P determine a group of three parameters A, B, θ.

$$a = \frac{\Delta_1}{\Delta_0},\ b = \frac{\Delta_2}{\Delta_0},\ c = \frac{\Delta_3}{\Delta_0} \qquad (7)$$

where

$$\Delta_0 = \begin{vmatrix} x_1^2 & x_1y_1 & y_1^2 \\ x_2^2 & x_2y_2 & y_2^2 \\ x_3^2 & x_3y_3 & y_3^2 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & x_1y_1 & y_1^2 \\ 1 & x_2y_2 & y_2^2 \\ 1 & x_3y_3 & y_3^2 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} x_1^2 & 1 & y_1^2 \\ x_2^2 & 1 & y_2^2 \\ x_3^2 & 1 & y_3^2 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} x_1^2 & x_1y_1 & 1 \\ x_2^2 & x_2y_2 & 1 \\ x_3^2 & x_3y_3 & 1 \end{vmatrix}$$

then

$$\begin{cases} \theta = 0.5arctan\dfrac{b}{a-c} \\ A^2 = \dfrac{cos2\theta}{acos^2\theta - csin^2\theta} \\ B^2 = \dfrac{cos2\theta}{asin^2\theta - ccos^2\theta} \end{cases} \quad (8)$$

For N points, there are $C_N^3 = N(N-1)(N-2)/6$ groups of three paremeters altogether which provide (at most) C_N^3 pairs of estimates for the intersections. It is not necessary to consider all combinations of P, because the nearer two points are, the greater error will be caused.In practice, we only select those points whose distances satisfy a proper threshold value and the computing complexity can be reduced greatly.

4 Experimental Results

In order to test the proposed method, some ellipses that are either internally generated by a computer or the part of an object externally grabbed from a CCD camera were used for evaluation.

1. We choose perfect ellipses to test shown Fig. 4.1. For these ellipses the algorithm we design can be simplified. In estimating the center coordinates we make two groups of lines paralleling to x–axis and y–axis respectiely, obtain two skewed symmetry axes and determine their crosspoint (i.e., the center of ellipse). Then we select some extreme points which are very important to estimate the other three parameters (A, B, θ) of an ellipse. The left of Fig. 4.1 is the original image and the right is the estimated result which shows that the error is very small. Tabel 1 is the experimental data.

2. For broken ellipses shown in Fig 4.2, we make eight groups of paralleling lines whose orientation angles with the x–axis vary from 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, obtain eight skewed symmetry axes and calculate the average of their crosspoints as the center coordinate of the broken ellipses.

3. Fig.4.3 shows several partial el-

ellipses which include an extreme point of the ellipses. We make more than ten groups of paralleling lines to estimate the centers of the ellipse. During estimating the other three parameters of ellipses it must select more pairs from the set of points to assure the accuracy.

4. At last we use the algorithm designed to detect the upper and lower ellipses of natural images of a cylinder. The results demonstrate that the method can properly detect elliptical objects.

Tabel.1 The Experimental Data

	Original Data					Detected Data					Error				
	x_c	y_c	A	B	θ	x_c	y_c	A	B	θ	Δx_c	Δy_c	ΔA	ΔB	$\Delta\theta$
1	100	100	40.0	80.0	145.0	99	100	38.8	80.7	145.5	1	0	1.2	−0.7	−0.5
2	200	100	80.0	20.0	30.0	199	99	81.1	19.4	30.3	1	1	−1.1	0.6	−0.3
3	300	100	40.0	30.0	130.0	298	100	41.2	29.4	129.1	2	0	−1.2	0.6	0.9
4	100	100	40.0	80.0	145.0	100	101	39.1	81.7	147.6	0	−1	0.9	−1.7	−2.6
5	200	100	80.0	20.0	30.0	199	99	81.1	19.4	30.3	1	1	−1.1	0.6	−0.3
6	300	100	40.0	30.0	130.0	299	101	42.0	29.0	128.3	1	−1	−2.0	1.0	1.7
7	120	200	140.0	60.0	130.0	125	204	142.7	65.1	128.6	−5	−4	−2.7	−5.1	1.4
8	270	200	40.0	100.0	30.0	265	195	39.1	105.6	25.3	5	5	0.9	−5.6	4.7
9	370	300	80.0	50.0	30.0	370	304	76.3	51.2	36.7	−5	−4	−3.7	−1.2	−6.7
10	170	220	70.0	35.0	5.0	169	217	67.8	36.0	5.87	1	3	2.2	−1.0	−0.87
11	165	360	65.0	35.0	2.0	163	359	63.5	36.0	−1.81	2	1	1.5	−1.0	3.81

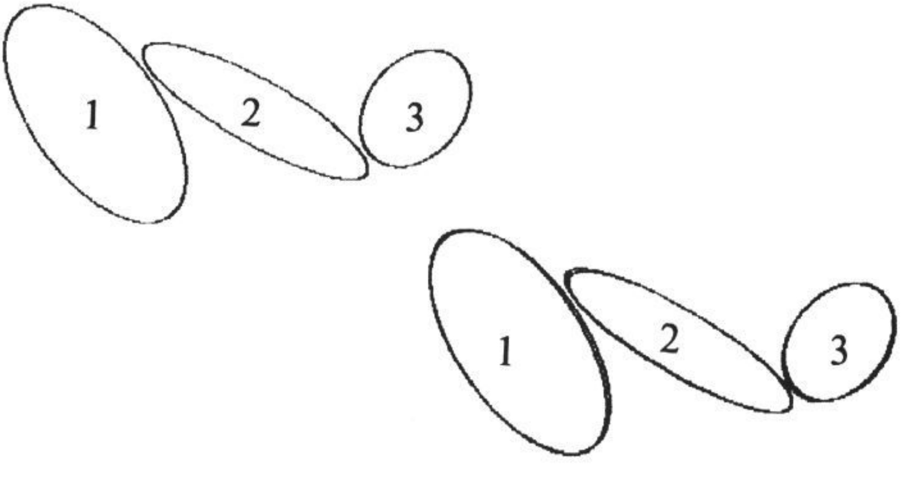


Fig4.1. Detecting complete ellipses of synthetic image.

Fig4.2. Detecting broken ellipses of synthetic image.

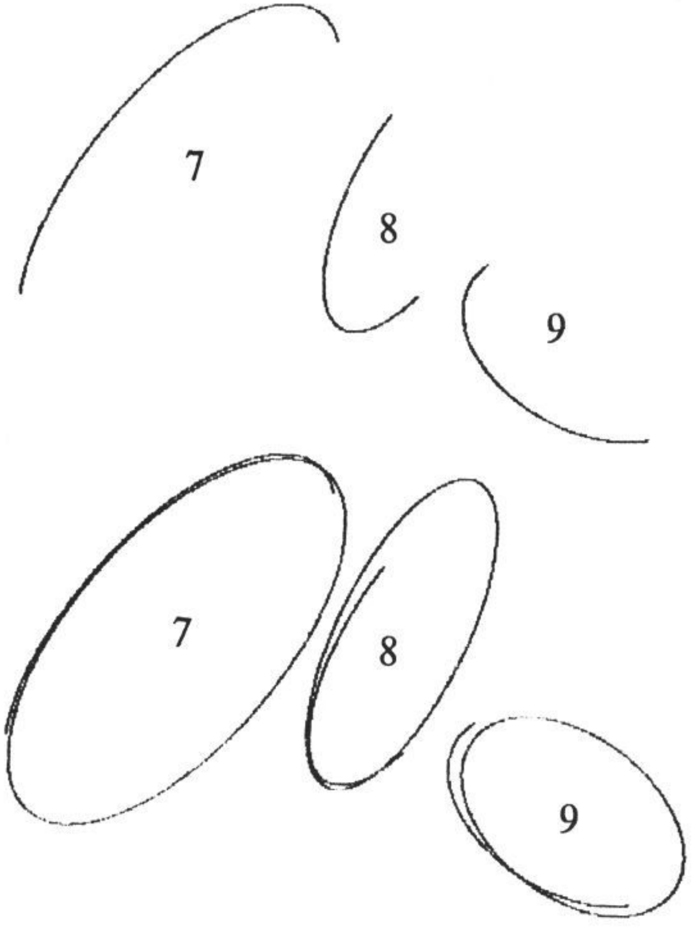
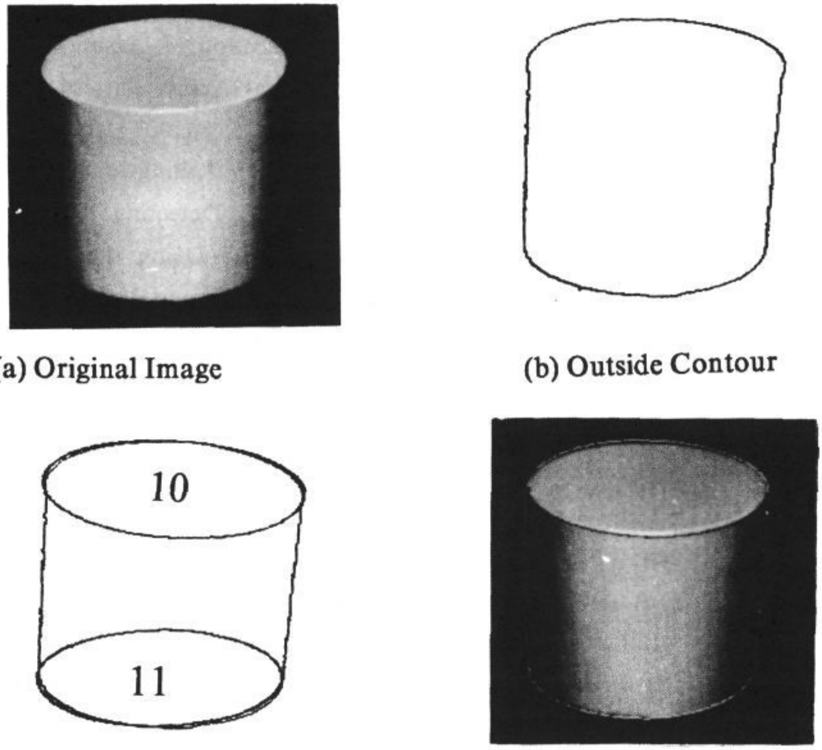


Fig4.3. Detecting partial ellipses of synthetic image.



(a) Original Image (b) Outside Contour

(c) Detected Ellipses (d) Result

Fig4.4. Detecting the ellipses of natural cylinder.

5 Conclusions

A new method for partial ellipse detection and fitting has been proposed in this paper. First, the method uses the skewed symmetry properties of an ellipse to locate its center. Then we estimate the other three parameters of an ellipse by the median of the intercepts (MI). It is based on accumulating many three–point ellipse fits to subsets of the data, sorting the parameters of these ellipse and selecting the medians of each parameter. This can reduce the computing complexity greatly.

The advantages of this technique are that no assumptions are made about the type of noise distribution, and no parameters are required. It robusts in the presence of a small number of outliers (i.e., the partial ellipse). This is in contrast to standard fitting techniques such as the least square estimations (LSE) and Hough transform (HT). The LSE method assumes that the noise distribution is Gaussian, causing it to be very sensitive to outliers. The results of the HT are sensitive to the bin sizes used to accumulate the evidence. Another merit of this method is that all the techniques employed in this detection algorithm are very simple in calculation and easy to implement.

Several experiments were designed to validate the proposal method. We have tested on both the synthetic and real images. The results indicate that the method is very effective and reliable. It is seen that the elliptical objects can be sucessfully detected and the detection outcomes are very satisfactory.

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