A Test of Camera Noise Models

Alan M. McIvor *

BP International Limited
Research Centre Sunbury
Chertsey Road
Sunbury–on–Thames
Middlesex TW16 7LN
United Kingdom

Models of the sensor noise added in the picture capture process are important components of sensor models for vision systems. Most studies assume that the noise can be described by an additive, independent, identically distributed Gaussian model. This paper presents a test of this hypothesis. It gives the results of applying the test to two camera and A/D converter setups. For both of these, the additive, independent, identically distributed Gaussian model was found to be unacceptable. Results are also presented from an analysis of the experimental data aimed at determining why this is so.

The picture signal generated by an image capture system is not exactly equal to the intensity pattern incident on the sensor array. The deviation from the ideal signal is due to the discrete nature of the sampling array and to errors, i.e., noise, introduced by the electronics. An accurate model for the noise is essential for developing optimized algorithms for such purposes as edge detection. Almost invariably, studies to develop such algorithms have assumed that the noise can be modelled as an additive, independent, identically distributed Gaussian process. The accuracy of such a noise model is fundamental to the claims of optimality for the resulting operators. Even so, few studies have been published which test the validity of this model.

In [1], experimental results are presented which show that the noise standard deviation increases linearly with mean intensity for both the CCD camera and the tube camera tested. This clearly indicates that an additive model is inadequate for these cameras. No information is given about spatial and temporal correlations. In [2], it is shown that for another CCD camera and A/D converter, the noise is almost constant over a fair range of intensities and the spatio-temporal correlation is low. For this setup, an independent, additive model may be adequate.

The purpose of this paper is to describe a test of the validity of the additive, independent, identically distributed Gaussian model. It also gives the results of two examples of its use. The paper is laid out as follows: After giving the details of the test, the results from applying the test to two systems are presented. Because the hypothesized model was so soundly rejected by the test in both cases, this section contains further discussion to highlight why the model is not acceptable. The paper is concluded with a summary.

*Some of this work was performed while the author was in the Oxford University Robotics Research Group.

NOISE MODEL TEST

The model for image noise that is commonly used is an additive, independent, identically distributed, Gaussian process. This can be more precisely stated as:

Hypothesis 1 *The noise in an image forms a spatio-temporal wide sense stationary, independent, identically distributed, Gaussian process.*

This can be tested against the alternative that is does not satisfy these conditions in the following way:

The camera system is set up to view a scene consisting of a grey board, over which the illumination is carefully arranged to be constant. A 32×32 central segment is recorded from each of 32 consecutive frames. This gives a $32 \times 32 \times 32$ spatio-temporal block of image data for testing.

The test of Hypothesis 1 consists of 3 parts [3]:

1. Test of spatio-temporal invariance.

   Assume that the pixel values in a block can be modelled by

   $$I(x,y,t) = \mu + \alpha x + \beta y + \gamma t + J(x,y,t) \quad (1)$$
   $$J(x,y,t) \overset{\text{iid}}{\sim} N(0,\sigma^2) \quad (2)$$

   Then a 3-way ANOVA test can be used to test the hypothesis of spatio-temporal invariance:

   Hypothesis 2

   $$\alpha = \beta = \gamma = 0 \quad (3)$$

   against the hypothesis that at least one of the parameters is non-zero.

BMVC 1990 doi:10.5244/C.4.63

2. Test of Correlation

   Given that the intensity distribution is spatio-temporally invariant, estimate the distribution statistics and in particular, the covariance, Σ. Using this and the assumption of a Gaussian distribution, test the independence hypothesis:

   Hypothesis 3 *For dx, dy, dt not all zero,*

$$\rho(dx, dy, dt) \triangleq \frac{\Sigma(dx, dy, dt)}{\Sigma(0,0,0)} = 0 \quad (4)$$

   against the hypothesis that it is non-zero. Note that zero correlation implies independence because a Gaussian distribution is assumed.

3. Test of Distribution

   Given that the intensity is spatio-temporally invariant and independent, test the hypothesis that the intensity distribution has a Gaussian distribution using a Kolmogorov–Smirnov test.

Using these three tests, Hypothesis 1 is rejected if more that 100α of the subtests are rejected at the $\alpha\%$ significance level. Note that Part 1 of this test verifies that the average scene brightness is constant.

EXPERIMENTAL RESULTS

The test described in the previous section has been applied to two camera/digitizer configurations. One setup consisted of a Panasonic WV-CD50 CCIR camera and a Datacube MaxScan A/D board. The other setup consisted of an Aqua HR 600 CCIR camera and a Datacube MaxScan A/D board[1]. For both of these configurations, several image sequence blocks were recorded, using a variety of reflective surfaces and lighting sources. For all of these, the test rejected the hypothesized noise model at the 1% significance level by a wide margin. For the Aqua HR 600, a data set was also obtained with the iris closed and the A/D offset adjusted to avoid clipping the signal. Even in this case, the test still rejected the hypothesized noise model.

The correlation test used above is not a strong test of independence because it assumes a Gaussian distribution. A contingency table test [4] is a better test of independence. The test statistic for this, at a spatio-temporal offset (i, j, k), is

$$T(i,j,k) = \Sigma_{u=L}^{U} \Sigma_{v=L}^{U} \frac{(n_{uv}(i,j,k) - N(i,j,k)\hat{\pi}_{uv}(i,j,k))}{N(i,j,k)\hat{\pi}_{uv}(i,j,k)} \quad (5)$$

where

L	=	minimum intensity in data block
U	=	maximum intensity in data block
$n_{uv}(i,j,k)$	=	number of observed cases where $I(x,y,t) = u$ and $I(x+i, y+j, t+k) = v$
$N(i,j,k)$	=	$\Sigma_u \Sigma_v n_{uv}(i,j,k)$
$\hat{\pi}_{uv}(i,j,k)$	=	estimated probability of observing $I(x,y,t) = u$ and $I(x+i, y+j, t+k) = v$

Then

$$T \sim \chi^2_{(U-L)^2} \quad (6)$$

The hypothesis of spatio-temporal independence becomes

Hypothesis 4

$$\hat{\pi}_{uv}(i,j,k) = \hat{\pi}_u \hat{\pi}_v \quad (7)$$

where

$\hat{\pi}_u$	=	estimated probability of intensity u being observed
	=	(no. pixels having intensity u) / (no pixels)

Such a test was applied to the data. It also rejected the hypothesis of independence. Various aspects of the covariance are plotted in Figures 1 — 5. From these, it is clear that there is significant correlation between the value of pixels close together, both in the spatial and temporal directions. This illustrates why the independence assumption is rejected by the tests.

The distribution of intensities in one of the data blocks from the Panasonic WV-CD50 is shown in Figure 6, along with the Gaussian distribution with the same mean and variance. The Kolmogorov–Smirnov test shows that there is a significant difference. From Figure 6 it can be seen that the difference is in the tails of the distribution. By restricting the test of distribution to exclude these regions, it was found that the Gaussian distribution is an acceptable model of the noise distribution at a pixel. For more accuracy, a contaminated Gaussian model would be better [5].

Data was collected from both camera setups over a wide range of mean intensities (by varying the lens aperture). For each of these data sets, the noise variance was calculated. The noise variance is plotted against mean intensity for the Panasonic WV-CD50 in Figure 7 and the standard deviation is plotted against mean intensity in Figure 8. It can be seen that the noise standard deviation varies with mean intensity in an approximately linear manner. Similar plots are presented in Figures 9 and 10 for the Aqua HR 600. The difference between the two curves in these figures is that the ones marked

[1] Although the same sort of A/D was used in both cases, it was not the same board.

with squares were taken with incandescent light illumination off a white paper target and the ones marked with stars were taken with diffuse sunlight illumination off a matt white plastic target. The rapid change in the data between mean intensities of 100 and 150 is due to the AGC circuit built into the camera. A linear model was fitted to the data to the left of this point via a least squares estimator. The RMS error normalized by average variance or standard deviation (as appropiate) was used as a measure of how well the model fitted the data. For one data set (squares) variance is better described by a linear model than is standard deviation. For the other data set (stars) a linear model fits better to the standard deviation. These results show that the noise variance clearly varies with mean intensity and hence can not be described by an additive model.

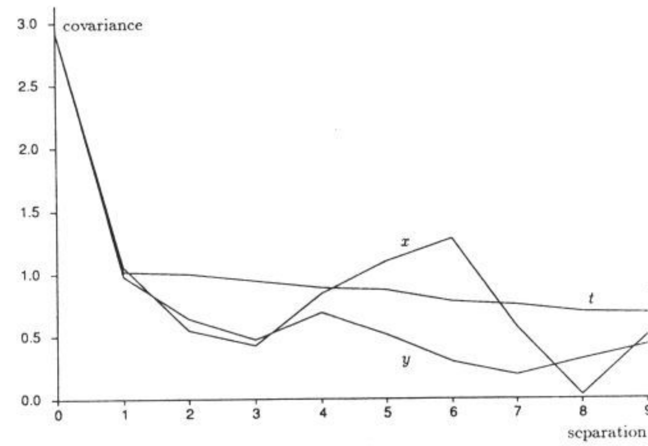

Figure 1: *The covariance along the axes for the Panasonic WV-CD50.*

CONCLUSION

This paper has presented a test of the hypothesis that the noise in an image can be modelled as an additive, independent, identically distributed, Gaussian process. Results of applying the test to two camera and A/D converter configurations were presented. These results give examples for which the above noise model is clearly inadequate. Further analysis of the data showed that there is significant correlation in the noise, especially temporally, and that the noise variance varies with mean intensity.

These results suggest that for some cameras, a multiplicative model may be a better noise model, such as used in [6]. Note that the noise would become additive in these cases if the log of the intensity is used. The significant spatial correlation could easily be explained by a 2-D "causal" spatial filter which modelled the finite bandwidth of the transmission mechanism (cables, amplifiers, etc).

Further work will be directed towards applying the test to other cameras and A/D converters. Work will also be focused on using more suitable models in the derivation of operators such as edge detectors.

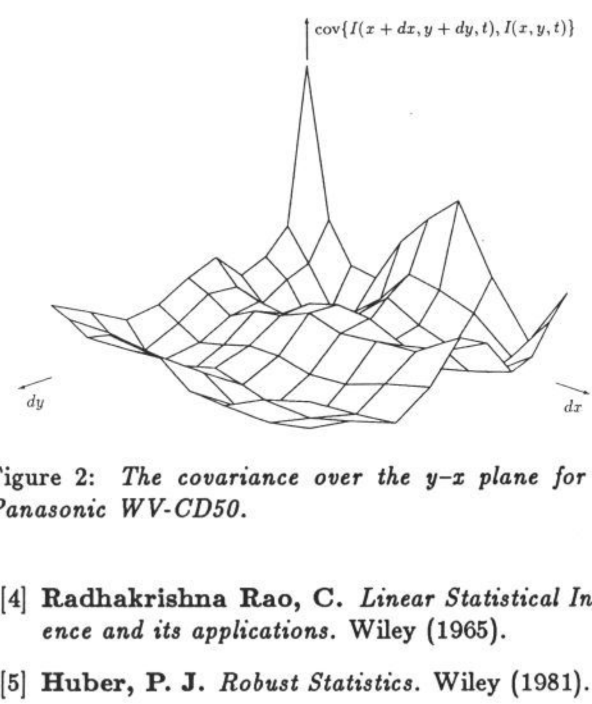

Figure 2: *The covariance over the y–x plane for the Panasonic WV-CD50.*

References

[1] **Shio, A.** "An Automatic Thresholding Algorithm based on an illumination independent contrast measure" *Proceedings IEEE International Conference on Computer Vision and Pattern Recognition* (1989) pp 632—637.

[2] **McKendall, R. and Mintz, M.** "Models of Sensor Noise and Optimal Algorithms for Estimation and Quantization in Vision Systems" *Penn. State Technical Report* (1987).

[3] **Kreyszig, E.** *Introductory Mathematical Statistics.* Wiley (1970).

[4] **Radhakrishna Rao, C.** *Linear Statistical Inference and its applications.* Wiley (1965).

[5] **Huber, P. J.** *Robust Statistics.* Wiley (1981).

[6] **Haralick, R. M. and Lee, J. S. J.** "Context dependent edge detection and evaluation" *Pattern Recognition* Vol 23 No 1/2 (1990) pp 1—19.

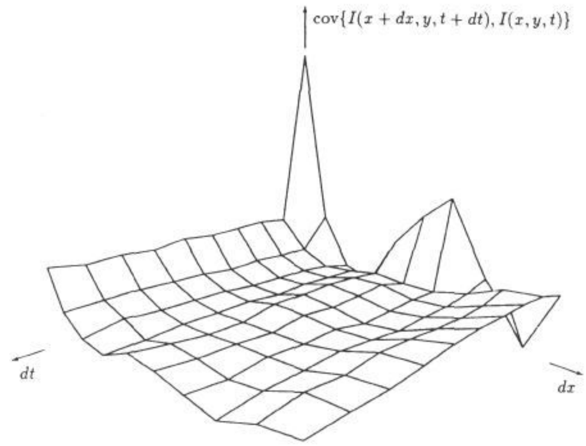

Figure 3: *The covariance over the t–x plane for the Panasonic WV-CD50.*

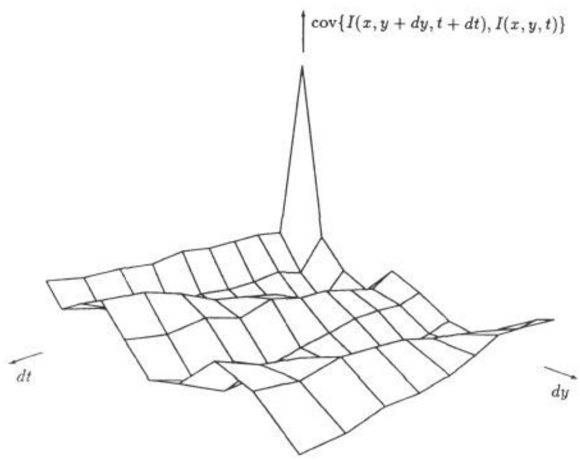

Figure 4: *The covariance over the t–y plane for the Panasonic WV-CD50.*

Figure 5: *The covariance along the axes for the Aqua HR 600.*

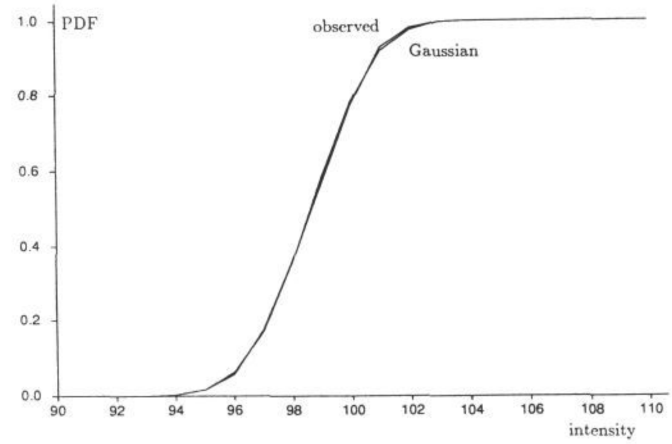

Figure 6: *The distribution of intensities in one data block for the Panasonic WV-CD50, mean = 99.1, standard deviation = 1.71, and the Gaussian with these statistics.*

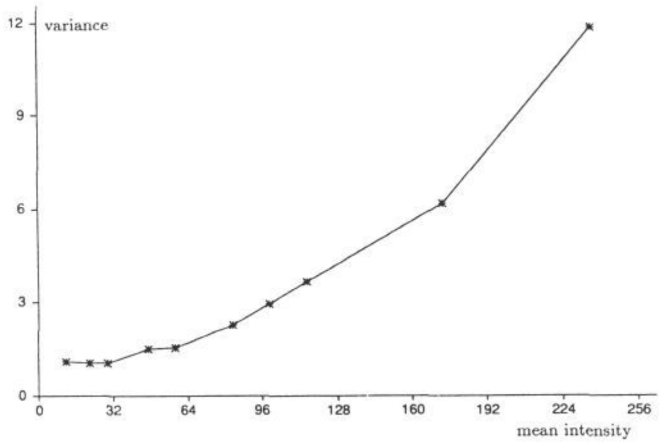

Figure 7: *Intensity variance plotted against mean intensity for the Panasonic WV-CD50.*

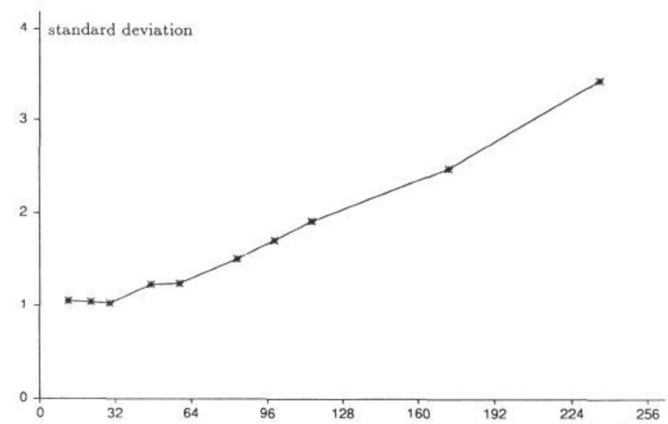

Figure 8: *Intensity standard deviation plotted against mean intensity for the Panasonic WV-CD50.*

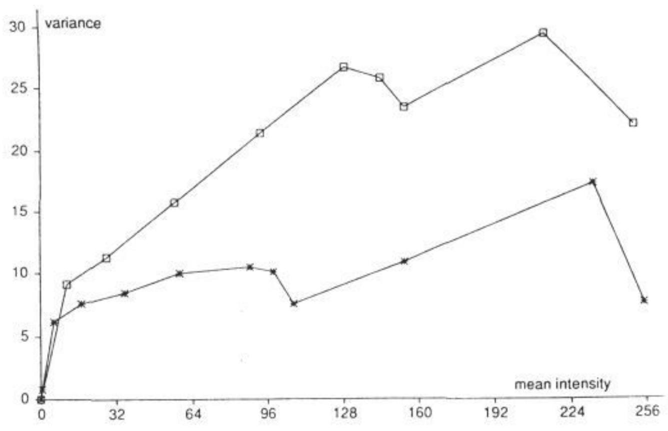

Figure 9: ***Intensity variance plotted against mean intensity for the Aqua HR 600.***

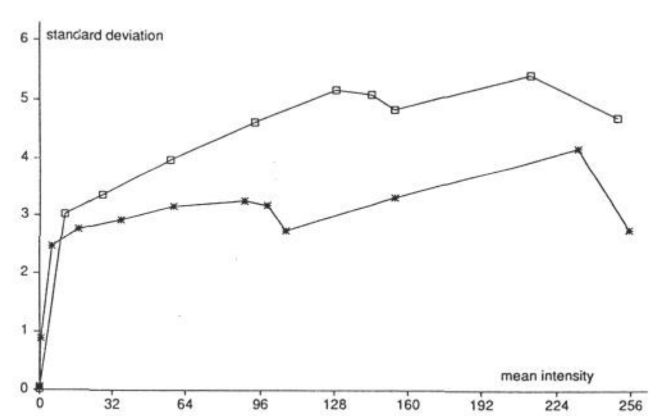

Figure 10: ***Intensity standard deviation plotted against mean intensity for the Aqua HR 600.***