The Computation of Deformation and Rotation in Stereopsis

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Recent work [12] has proposed that tracking phase differences through the resolutions of bandpass filtering created via Hilbert transform pairs can be used to obtain unrotated stereoscopic disparities both horizontally and vertically. These ideas are now extended to incorporate the deformation arising from a stereoscopic transformation. From an ordered arrangement of directionally selective quadrature filters with both finite orientation and spatial frequency bandwidths, we create a further Hilbert transform pair from the energy responses of filters similarly located in space, but with different orientation preferences. From this response an estimate of both orientational disparity and orientational diffrequency is finally computed. The latter term is shown to approximate the anisotropic deformation arising from surface slant. We show using this representation that the surface slant can be approximated by differences in spatial frequency and that the diffrequency and deformation hypotheses are equivalent.

1 Introduction

The effective recovery of depth information from two stereoscopic views of the same scene has proved to be difficult because of the correspondence problem. Notably, little attention has focused on the local computation of higher order properties of the disparity field. However, Brint and Brady [3] recently proposed that (edge based) curvature primitives can be used to disambiguate edge matches to establish correspondence. In this work we assume that correspondence given a fixation point has been established, and we re-examine the local geometric properties of the disparity field with the goal of computing surface slant.

The recovery of surface slant has been subject to considerable psychophysical experimentation. Rogers and Graham [16] showed that there exists an anisotropy in the human perception of surface slant for surfaces oriented about a horizontal axis (shear), which were twice as apparent as surfaces oriented about a vertical axis (compression) for a given magnitude of slant. This has led to the suggestion that the human visual system retains differences in orientation as a cue for surface slant. Koenderink and Van Doorn [10] studied the first order behaviour of the disparity vector field, and concluded that the deformation between stereoscopic transformations defines the magnitude and direction of surface slant. We note that the magnitude of the deformation equals the disparity gradient, and the anisotropy can be explained by the presence of rotation (curl) owing to shear. The disparity gradient has received much attention in edge based stereoscopic correspondence algorithms [15]. Unfortunately, edge based approaches cannot locally obtain the slant of surfaces because the equations are underdetermined. This can be regarded as an aperture problem in stereopsis [12]. Koenderink and Van Doorn proposed a mechanism to compute deformation from orientationally selective filters. However, they did not take into account the orientational and spatial frequency sensitivity of 2-D bandpass filters. Their scheme is therefore not readily implemented in practice. In addition, Blakemore [1] has suggested that horizontal differences in spatial frequency may also be used as a cue for surface slant about the vertical axis. We will show that several such cues may be combined in the type of processing that we propose.

2 The disparity field

Koenderink and Van Doorn [10], noted that the stereoscopic transformation of an image is analogous to the transformation of a planar continuum subject to translation, rotation and deformation owing to stress and strain forces. We assume differentiability of the image function which is enforced by applying Gaussian filtering operators and that the mapping from one image configuration to the corresponding stereoscopic pair is continuous and one to one. We therefore assume that the image field is almost everywhere analytic except at the fixation point and that locally, partial derivatives of the local disparity field exist up to order two. We define corresponding

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points on the left (x_{il}) and right (x_{ir}) image plane:

$$x_{il} = F(X_i) \quad x_{ir} = F(X_i + b_i)$$

with respect to spatial coordinates (X_i) and the relative stereoscopic displacement between imaging devices (b_i). Since the stereoscopic displacement is constant and assumed small relative to the world coordinates of the imaged point:

$$\frac{x_{il} - x_{ir}}{\Delta X_i} \approx \frac{dx_{il}}{dX_i} = F'(X_i) = F'(F^{-1}(x_{il})) \qquad (1)$$

We assume that $\frac{dx_{il}(0)}{dX_i} = 0$ which is equivalent to considering the local disparity field about the point of fixation (fusion). Equation (1) may be approximated by a first order Taylor expansion:

$$\frac{dx_{il}}{dX_1} = A(a_{ij})x_{il} + \ \ldots$$

We suppose one of the principle axes of $\mathbf{X}$ aligned with the translation of the camera. If we consider the above equation applying to a planar surface (image) then conversion into polar form (r, θ) via the Jacobian: $\mathcal{J} = \frac{\partial(x_{1l}, x_{2l})}{\partial(r,\theta)}$ gives:

$$\frac{d\theta}{dX_1} = \frac{1}{2}[(a_{21} - a_{12}) - (a_{11} - a_{22})\sin 2\theta - (a_{12} + a_{21})\cos 2\theta] \qquad (2)$$

$$\frac{dr}{dX_1} = \frac{1}{2}r[(a_{11} + a_{22}) + (a_{11} - a_{22})\cos 2\theta + (a_{12} + a_{21})\sin 2\theta] \qquad (3)$$

it is clear from equations (2,3) that the first order approximation to the disparity field involvès both extensions and rotations between local image points about the fixation point. Therefore, the disparity field contains components corresponding to rigid body rotations (curl), isotropic expansions (div) and anisotropic deformations (def).

3 Rotational Invariants

Central to the expansion of the local stereoscopic field, lies the concept of vergence, or fusion at a fixation point. A technique to obtain translational disparities with 2-D filtering operations has been proposed by Langley et al [12], but rotational differences were not considered. We consider two techniques. Firstly, we obtain only the rotation component of the stereoscopic field using a method that requires a minimal number of filters to estimate orientation independent of quadrature phase. Secondly, we consider a technique that permits the computation of both rotation and deformation.

3.1 Rotation

We begin by considering a direct method for obtaining the rotation component about the fixation point as the differences in local orientation that is independent of quadrature phase. This can be effectively obtained by using the interpolative properties of Gaussian derivatives and their Hilbert transforms. Freeman and Adelson[6] considered the second directional derivative of the Gaussian which unfortunately requires two additional filters to compute orientation. We use the natural tensor representation[9]:

$$\begin{bmatrix} x.x & x.y \\ y.x & y.y \end{bmatrix} \text{i.e} \begin{bmatrix} l_x^2 + \hat{l}_x^2 & \hat{l}_{x.y}^2 + l_x l_y \\ \hat{l}_{y.x}^2 + l_x l_y & l_y^2 + \hat{l}_y^2 \end{bmatrix} \qquad (4)$$

from which the eigenvectors give:

$$\tan 2\theta_l = \frac{[(\hat{l}_{x.y}^2 - \hat{l}_{y.x}^2 + 2l_x l_y)]}{(l_x^2 + \hat{l}_x^2) - (l_y^2 + \hat{l}_y^2)} \qquad (5)$$

where l_x refers to the operation $\frac{dG(x,y)}{dx} * I(x,y)$ with $G(x,y)$ as a Gaussian smoothing kernel, $I(x,y)$ an image function, and subscripts refer to the directional derivative. $\hat{l}_x$ refers to the Hilbert transform of the directional derivative of the Gaussian. It is well known that the orientational energy response of this kernel[4] with an isotropic envelope varies as a function of $\cos(\theta - \eta)$ where η refers to the orientation of the signal. The energy response of its Hilbert transform therefore varies as a function of $\sqrt{\frac{1}{2}(1 + \cos(2(\theta - \eta))}$. The real part includes a constant term which requires a further filter to estimate. Thus we note that we require the minimum of five filters to obtain a phase independent estimation of orientation (two odd-symmetric three even symmetric). In equation (4) we have applied six to avoid interpolation. Since orientation is energy based, we define orientation over $\theta \in [0, \pi]$. We also note in this case, that $\hat{l}_x^2$ may be replaced by $|l_{xx}|$ but with a loss in quadrature. The rotation ($O_{\bar{\theta}}$) may then simply be found from a careful phase subtraction (since $\theta \in [0, \pi]$): $O_{\bar{\theta}} = \theta_l - \theta_r$ which can be used to approximate the magnitude of the disparity gradient (Γ)[15]:

$$\Gamma = \frac{\sin(\theta_l - \theta_r)}{\sqrt{\frac{1}{4}\sin^2(\theta_r + \theta_l) + \sin^2\theta_l \sin^2\theta_r}} \ ; 0 < \theta_l, \theta_r < \pi$$

but only in the absence of deformation. This technique unfortunately exibits singularities such as the intersection of orthogonal lines. This can be overcome by the introduction of elliptical gaussian envelopes but at the expense of the geometrical properties of the filter in estimating orientation.

3.2 Rotation and deformation

To obtain estimates of both rotation and deformation, we propose to add a higher level of processing based upon the energy responses from orientationally selective quadrature filter pairs. We form the analytic signal $\vec{Z}(x_1, x_2, \omega, \theta)$:

$$\begin{aligned}\vec{Z}(x_1, x_2, \theta, \omega) &= \Psi(x_1, x_2, \vec{\omega}_g, \vec{\theta}_g) * I(x_1, x_2, \vec{\theta}_i, \vec{\omega}_i) \\ &= L(x_1, x_2, \theta, \omega) + j\hat{L}(x_1, x_2, \theta, \omega) \quad (6)\end{aligned}$$

where $\Psi(x_1, x_2, \vec{\omega}_g, \vec{\theta}_g)$ represents a quadrature filter pair with both spatial frequency and orientational preferences. L and $\hat{L}$ are directional Hilbert transform pairs formed by bandpass filtering.

$\theta = | < \vec{\theta}_i, \vec{\theta}_g > |$ and $\omega = | < \vec{\omega}_i, \vec{\omega}_g > |$ indicate the sensitivity of the envelope of $\vec{Z}(x_1, x_2, \theta, \omega)$ to the 2-D Fourier components in the local neighborhood of the image at $I(x_1, x_2, \vec{\theta}_i, \vec{\omega}_i)$. At an orientation (θ_g) the directional instantaneous frequency $(f_{\theta i})$ may be found from the directional derivative:

$$f_{\theta i} = \frac{1}{2\pi} \mathcal{I}m \frac{d \ln \vec{Z}}{d\vec{r}_g} \quad (7)$$

where $\vec{r}_g = [\cos\theta_g, \sin\theta_g]^T$. That horizontal differences in spatial frequency can be interpreted as a slant about the vertical axis (wall plane) was first proposed by Blakemore leading to his *diffrequency* hypothesis. This is the fundamental operation of the phase-locked loop implemented by Miller [13] in hardware. This can be easily seen assuming linear phase and the expression used to interpret disparity $D(x_1)$ from the method of phase differences in 1-d [17, 8, 19]:

$$D(x_1) \approx \frac{2}{\omega_l + \omega_r}(\omega_l x_1 - \omega_r x_1 + \Phi_l - \Phi_r)$$

where ω_l and ω_r refers to the instantaneous angular frequency used to obtain disparity in the left and right image respectively [12]. The horizontal disparity gradient becomes:

$$\frac{dD(x_1)}{dx_1} \approx 2\frac{\omega_l - \omega_r}{\omega_l + \omega_r} \quad (8)$$

which with simple signals is also valid for narrow band disparity modulation. We also make the observation that in the temporal domain:

$$\omega_t \approx \frac{\Phi_{t_1} - \Phi_{t_2}}{t_1 - t_2} = \frac{\delta\Phi}{\delta t}$$

and therefore in this representation stereopsis is indeed a discrete form of velocity $(\vec{v})$ since in 1-d:

$$\vec{v} = \frac{\omega_t}{\omega_x} \approx \frac{\frac{\delta\Phi}{\delta t}}{\omega_x}$$

Equation (8) suggests a mechanism for obtaining the stretch ratio between the left to right stereoscopic transformation in a specified orientation by applying equation (7) to both images. In principle it is possible to use the directional instantaneous frequency processed in a circle to compute the divergence and deformation between stereoscopic transformations. This turns out to be numerically unstable because of singularities in phase space [7]. This form of analysis is analogous to computing the stress forces on a planar surface. In the same manner as the *rosette* formation is commonly used in material science using strain guages, we can explore spatial arrangements of filters to obtain deformation. At present, we restrict ourself to local operations to illustrate the basic idea.

To avoid the computation of instantaneous frequency, we use the envelope from equation (6) and assume both image sensors are fixating at the same point in the world at the origin $x_i(0)$. We form:

$$|\vec{Z}_l(\theta, \omega_l; x_i(0))| = |\Psi[\vec{\theta}_g; \vec{\omega}_g, x_i(0)] * I_l[x_i(0)]|$$

which is the energy response from 2-D filters with similar spatial frequency preferences applied at a single point but different orientations. This defines an energy function from a circle to an interval on the real line $[0, \pi]$. A similar representation is also computed for the right image. Under stereoscopic transformation, the response from the directionally selective filters will encode the 1st order slope of the surface by an orientational modulation. We create an additional Hilbert Transform pair defined in the left image by:

$$P_l(\theta) = |\vec{Z}_l(\theta, \omega_l; x_i(0))| * \chi(\theta, v_g) = s(\theta) + j\hat{s}(\theta)$$

where $\chi(\theta, v_g)$ is a 1-d bandpass quadrature filter with centre angular frequency v_g. Thus we are decoding the disparity signal by the group delay in the envelope[18] under stereoscopic transformation. This also implies that the first order characteristics can also be obtained from a direct quadrature phase relationship. We consider the phase in response to Gaussian noise [2]:

$$v_g\theta + \Phi(\theta) = \tan^{-1}\frac{\hat{s}(\theta)}{s(\theta)}$$

The instantaneous frequency is defined by [5]:

$$f_i(\theta) = \frac{1}{2\pi}\mathcal{I}m\frac{dP_l(\theta)}{d\theta} = \frac{1}{2\pi}\frac{s(\theta)\dot{\hat{s}}(\theta) - \hat{s}(\theta)\dot{s}(\theta)}{s^2(\theta) + \hat{s}^2(\theta)}$$

Thus we may apply a quadrature pair of filters and their derivatives to obtain the local instantaneous frequency [12]. Because $|\vec{Z}_l(\theta, \omega_l; x_1, x_2)|$ is bandpass, $P_l(\theta)$ is also:

$$P_l(\theta) = E_l(v_g, \omega_l, \theta)\exp[v_g\theta + \Phi(\theta)] \quad (9)$$

Notice the envelope $E_l(v_g, \omega_l, \theta)$ is sensitive to the directional spatial frequencies present in the original image. We expand the angular part of equation (9), ignoring second and higher order terms:

$$v_g\theta + \Phi(\theta) \approx \Phi(\theta_o) + v_g\theta + \Phi'(\theta_o)(\theta - \theta_o)$$

the local instantaneous frequency is then:

$$f_{il}(\theta) = \frac{1}{2\pi}(v_g + \Phi'(\theta)) = \frac{1}{2\pi}v_l$$

Now assume that $f_{il}(\theta)$ is approximately constant between θ and $\theta + \delta\theta$, as is the envelope[14]:

$$E_l(v_g, \omega_l, \theta) \approx E_l(v_g, \omega_l, \theta + \delta\theta)$$

The violation of this assumption is well known in telecommunications theory and its consequence introduces phase distortion in the carrier signal. To overcome these restrictions it is usual to amplify the frequency components present in the passband of interest. This may be achieved by the subtraction of the Laplacian. This proposal provides a direct application for cells with similar profiles present in biological systems.

To illustrate these ideas, consider the transformation due to the slant of a planar surface in the right image relative to the left. We have:

$$P_r(\theta) = E_r(v_g, \omega_l + \delta\omega, z)\exp[jv_l z] \tag{10}$$

where $z = \theta + \frac{d\theta}{dX_1}$ and the local change in spatial frequency ($\delta\omega$) occurs because of stretches explained by equation (3). The physical interpretation is a signal that is both phase (PM) and amplitude (AM) modulated. PM occurs because of equation (2) while the AM occurs because of equation (3). Since the spatial frequency term in the energy response of a 2-D bandpass filter only responds to the modulus of the departure of the fundamental image signal from the centre frequency of the filter, the transformed energy response or AM signal cannot be uniquely determined in this representation. This is a consequence of local stretches of the image field. We therefore have to rely on the orientational phase differences to demodulate the disparity signal. Consider the instantaneous frequency of the right image by differentiating the angular part of equation (10):

$$f_r(\theta) \approx \frac{1}{2\pi}\frac{d(v_l z)}{dz}\frac{dz}{d\theta} = \frac{1}{2\pi}v_l[1 + \frac{d(\frac{d\theta}{dX_1})}{d\theta}]$$

The local *diffrequency* ($f_d(\theta)$) between the left and right stereoscopic image pairs becomes:

$$f_d(\theta) \approx \frac{1}{2\pi}v_l\frac{d(\frac{d\theta}{dX_1})}{d\theta} \tag{11}$$

Using equation (9) we can estimate the orientation disparity ($O_d(\theta)$) using the method of phase differences:

$$O_d\ (\theta_o) \approx \frac{2}{v_l(\theta_o) + v_r(\theta_o)}[\Phi_l(\theta_o) - \Phi_r(\theta_o)] \tag{12}$$

We can track orientational disparities in the form of a 1-D phase-locked loop or take a weighted estimate through resolutions of filtering.

Consider from continuum mechanics, the term analogous to the *material derivative*:

$$\frac{d(\frac{d\theta}{dX_1})}{d\theta} = (a_{22} - a_{11})\cos 2\theta + (a_{12} + a_{21})\sin 2\theta$$

We can re-write this term as:

$$\mathcal{R}e\ \sqrt{(a_{12} + a_{21})^2 + (a_{22} - a_{11})^2}\exp[j2\theta - \eta]$$

where $\eta = \tan^{-1}\frac{a_{12}+a_{21}}{a_{22}-a_{11}}$. The modulus of the coefficient in the above term gives the magnitude of the slant. The phase gives the direction of the slant.

An examination of equations (8,11,12) indicates that we can estimate the above term by:

$$\frac{d\frac{d\theta}{dX_1}}{d\theta} \approx \frac{2}{f_l + f_r}f_d(\theta) \tag{13}$$

since we can consider taking a reference from either image. Therefore diffrequency can be used to approximate the slant of the surface, by using the anisotropic behaviour of deformation. However, because of numerical difficulties in the computation of instantaneous frequency, we restrict our consideration to equation (12).

4 Methods

To implement this scheme, we have applied Gabor function as our 2-D orientation and spatial frequency bandlimited filter. To preserve orientational selectivity over spatial frequency, we have applied filters whose envelope aspect ratio is 2:1 in favour of orientation.

We assume that orientational disparities will be small. The risks from aliasing which are problematic with a phase differencing technique are therefore reduced. We therefore apply a weighted estimate and cyclotorsion as opposed to Phase-locking. From the energy responses from the band of filters, we obtain the weighted mean disparity, and diffrequency estimates by convolving with the quadrature Gabor function in both images by three resolutions of coarse to fine filtering:

$$O_{est}(\theta) = \frac{\sum_{j=1}^{n} E_{lr}(\theta, v_j)O_d(\theta)}{\sum_{j=1}^{n} E_{lr}(\theta, v_j)} \tag{14}$$

where $E_{lr}(\theta, v_j)$ refers to the product of energy from the 1-D Gabor filters used to interpret disparity (fig. 1). A similar expression was applied to estimate the local diffrequency. At a given scale, we estimate the least squares mean phase difference (curl) using a similar mechanism proposed for camera vergence [11], which act in the same manner as cyclotorsion:

$$\mathbf{S_l S_r}^T[\mathbf{S_r S_r}^T]^{-1} = \mathbf{R}_{\bar{\theta}}$$

$$\mathbf{S_r S_l}^T[\mathbf{S_l S_l}^T]^{-1} = \mathbf{R}_{\bar{\theta}}^T$$

with:

$$\tan O_{\bar{\theta}} = \frac{\sin O_{\bar{\theta}}}{\cos O_{\bar{\theta}}}$$

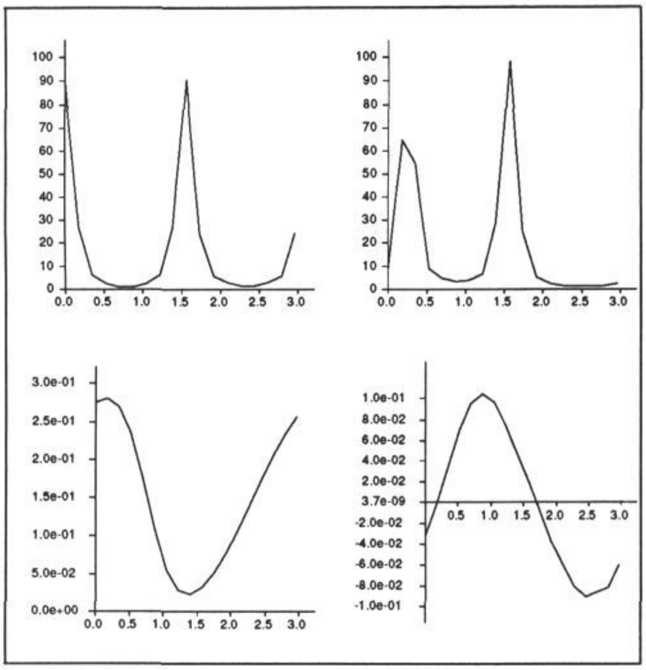

Figure 1: **Top left**: *Energy response from a band of 2-D filters plotted against orientation located at the intersection of an ideal corner.* **Top right**: *Energy response from 2-D filters similarly located at a sheared corner.* **Bottom left**: *Orientation disparity. Curl estimated at 0.148 rads, Def esimated as:* $D_{x1} = -0.0065$, $D_{x2} = 0.1224$ *rads. Actual disparity gradient at 0.254 rads, error of 0.016 rads.* **Bottom right**: *Weighted diffrequency estimates.*

as the least squares phase difference, with coefficients taken from:

$$\mathbf{R}_{\bar{\theta}} - \mathbf{R}_{\bar{\theta}}^T = 2\sin\mathbf{O}_{\bar{\theta}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

and

$$\mathbf{R}_{\bar{\theta}} + \mathbf{R}_{\bar{\theta}}^T = 2\cos\mathbf{O}_{\bar{\theta}}\, \mathbf{I}$$

Here $\mathbf{S_l}, \mathbf{S_r}$ refer to the $2\mathbf{x}M$ matrices of Hilbert transform pairs $(s(\theta) + j\hat{s}(\theta))$ and $R_{\bar{\theta}}$ contains the least squares phase difference. Disparity in this case is interpreted using the centre frequency of the bandpass filter $\chi(\theta, v_g)$. We note that the the curl also defines the magnitude of slant about a horizontal axis. Removing the mean rotation also reduces both the risks from phase aliasing and disparity error[12] because we increase the range of orientational disparity detection. The remaining disparity signal is now easy to compute. To extract the magnitude and phase of the deformation component we make the analogy to the computation of orientation [9] using tensor field filtering. We therefore consider the eigenfunctions of the tensor:

$$\sum_{\theta=0}^{M} O_{est}(\theta)[T_k - \frac{1}{2}I] \qquad (15)$$

where $T_k = [\cos\theta, \sin\theta]^T[\cos\theta, \sin\theta]$. In this representation, the eigenvalues correspond to the magnitude of the deformation, while the eigenvectors correspond to the surface slant. Alternatively, we could apply a weighted least squares fit [11]:

$$\mathbf{E\Phi D} = \mathbf{Ed}$$

where $\mathbf{E}$ is an $M\mathbf{x}M$ matrix with leading diagonal formed from the product of energy responses. $\mathbf{d}$ represents the measured disparities ($M\mathbf{x}1$), $\mathbf{\Phi} = [\cos 2\theta_i, \sin 2\theta_i]$ represents the orientation of the ith disparity measurement, and $\mathbf{D}$ is a vector containing the magnitude of deformation resolved into vertical and horizontal slant.

5 Results

We present some simple data to highlight the principles that we have introduced. In figure 1 we show the energy response to an ideal corner that has been subjected to a horizontal shear. To obtain the total orientational disparity requires the summation of the rotation term to the vertical component of deformation. The energy response maxima correspond to the mutual alignment in orientation of quadrature filters located at an ideal and sheared corner. The estimated disparity and local diffrequency from subsequent postprocessing the band of filters is also shown. We also present a second example taken from a random dot stereogram that has also been subjected to horizontal shear. The Needle diagram (fig. 3) indicates the direction and magnitude of the disparity gradient in the absence of translational disparity.

6 Conclusion

We have shown, that the first order differential transformations of slanted surfaces can reduce to differences in spatial frequencies between left and right stereoscopic image pairs. Thus we have combined the theories of Blakemore [1] and Koenderink and Van Doorn [10] into an equivalent representation. Naturally, this scheme can equally be applied to visual motion.

There are two situations that require clarification given the methods proposed in this paper. Firstly, consider the case that the local image field contains strongly directional energy. The local image field will only experience rotations or divergences depending on the slant of the surface, and orientation of the signal. In the case of an isolated but elongated line, the local stereoscopic field will only amount to a rigid rotation. Secondly, consider the case of an elongated pair of lines that bisect the principle axes of the deformation component. Under these conditions, the local image field will also only be observed as a rigid rotation. We have proposed a

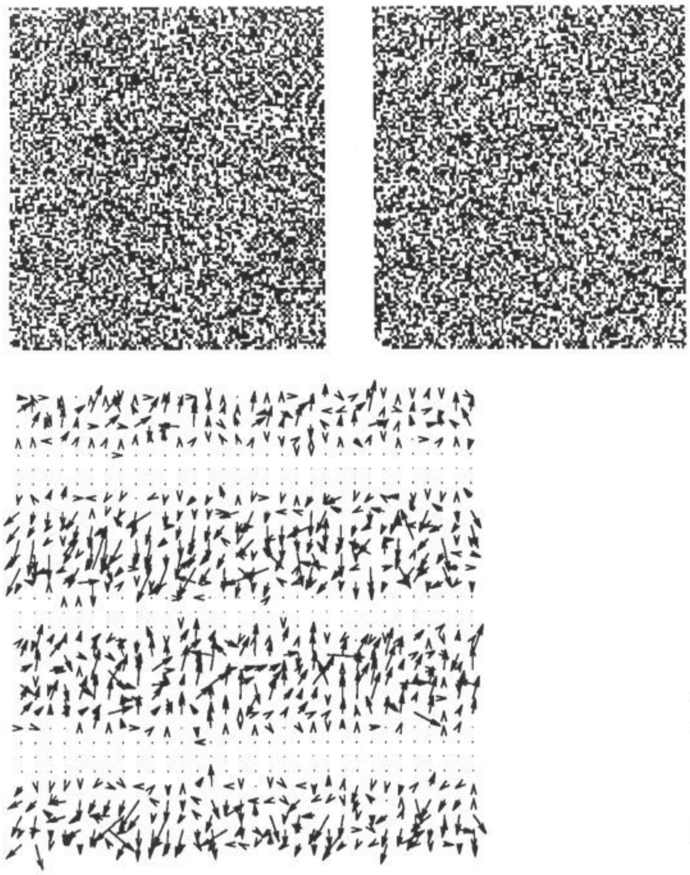

Figure 2: **Top** *Random dot stereogram subjected to a modulated horizontal shear.* **Bottom** *Disparity gradient vector field*

technique that restricts consideration to rotational differences.

These approximations to the local behaviour of the disparity vector field draw interesting conclusions regarding the architecture of the human visual cortex. In view of the large receptive field of orientationally selective filter that we apply, we do not expect a considerable change in orientational energy between neighboring points. To avoid circular convolutions at each point, we can propose that the 1-D phase locking iteration can be performed by stretching the M orientation samples at each point onto a linear array, and applying the 1-D operators over neighboring pixel locations. From this point of view, the architecture of the human striate cortex is highly suited towards the type of processing proposed in this paper.

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