# An empirical study on the effects of spatial discretization error in a stereo vision system

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*The spatial discretization error in an image causes triangulation error, camera calibration error in a stereo imaging system. Camera calibration error in turn causes difficulty in applying the epipolar line constraint for matching stereo images as well as causing triangulation error. We derive an empirical relationship between the uncertainty volume of triangulation at the vergence point of convergent stereo viewing and the average triangulation error in such a set-up. Given the stereo imaging parameters it is possible to estimate the average triangulation error in the stereo system. We have studied by simulation the effects of spatial discretization error on the camera calibration, triangulation accuracy and the calculation of epipolar line. We have also examined the effect of the spatial discretization error on triangulation, camera calibration and calculation of the epipolar lines at different viewing angles. The results obtained enable us to choose a compromise viewing angle such that the effects of spatial discretization error may be reduced in a near optimum way. The results also indicate how camera calibration accuracy may be improved.*

When setting up a stereo imaging system, there are several factors or design criteria to be considered. These include the reliability of stereo matching and the accuracy of the recovered 3D data. A stereo system must be able to match images correctly and obtain accurately triangulated 3D data. Due to the discrete nature of the imaging system, images are discretized into a mesh. An image captured by such a square/rectangular mesh will suffer from spatial discretization error of ±0.5 pixel in both horizontal and vertical directions. This causes camera calibration error, triangulation error and errors in the subsequent calculation of epipolar lines using the camera matrices. The knowledge of the consequences of spatial discretization error in a stereo system can be useful for the design of a stereo system in which the effects of such error should be reduced as much as possible.

## COMPROMISES IN A STEREO SYSTEM

The absolute magnitude of the spatial discretization error depends on the resolution of the discretized image, i.e. the pixel size and the number of pixels in the horizontal and vertical directions. For a stereo imaging system with a given resolution, the spatial discretization error generates an uncertainty polyhedron when two image points from two different images are used to triangulate a 3D point(Figure 1). The size of this polyhedral volume is then the uncertainty volume within which the triangulated 3D point lies. The spatial discretization error in stereo imaging systems has been studied by Rodriguez and Aggarwal[1] and Blostein and Huang[2] using a stochastic approach. Both studies considered the case in which the images are in the same imaging plane and the optical axes of the two images are parallel. The triangulation error is inversely proportional to the product of baseline × focal length. At a given focal length, a wider baseline reduces triangulation error but increases both geometric and photometric distortions in images which in turn makes matching of these images more difficult. The region covered by the fields of view of both images is reduced as the baseline increases. Only the overlapped region will give 3D measurements. The separation between two views should be small in order to facilitate matching and to recover a larger proportion of the 3D scene. There is therefore a conflict in different requirements for setting up an ideal stereo imaging system.

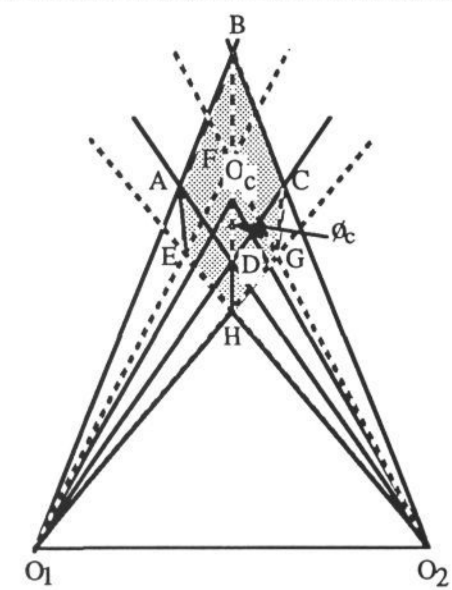


*Figure 1. The uncertainty volume; $O_1O_c$ is the viewing distance; $\angle O_1O_cO_2$ is the viewing angle.*

In this study, a convergent stereo system is chosen to produce a large overlap area while keeping a wide baseline. There is still a conflict between mutual and self obscuration of the objects in the scene and triangulation accuracy. The problem of obscuration of the objects depends on circumstance but is generally more frequent if a wide viewing angle, i.e. the angle subtended by the optical axes of the camera or cameras at two or more viewing positions, is chosen. This angle is also called the angle of separation. The problem results in matching errors and loss of 3D data.

Convergent geometry is used to maximize the image overlap on the objects of interest. The system is more efficient in this way than the equivalent parallel system. We use the

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Centre of View(COV) for convenience, i.e. both cameras point at a single point on the region of interest half way between the two cameras. Results for a system where the optic axes of the cameras do not cross exactly should be similar.

The epipolar line constraint used in stereo matching can be calculated directly from the camera matrices obtained from camera calibration at each viewing position. Some researchers calculate the epipolar lines from the decomposed camera matrices which express the information about the imaging geometry of the stereo system explicitly[3,4]. In trinocular stereo[5,6], the intersection of epipolar lines due to a point in each image is used to verify potential matching candidates and the resultant matching ambiguity is then much less than in a binocular system. Epipolar lines become more than just a search constraint in a trinocular or 3+ view stereo system and the accuracy of the camera calibration required is also greater.

## EFFECTS OF SPATIAL DISCRETIZATION ERROR IN THE STEREO SYSTEM

The uncertainty volume resulting from triangulating two discretized points in two images has an unusual shape. The shape and size of this volume will both change with the viewing angle. A method of approximating this volume has been developed[7]. We also developed a method of estimating the average triangulation error in the stereo system from the uncertainty volume at COV when given the pixel size, focal length, viewing distance and viewing angle. The magnitude of the triangulation error and the size of the uncertainty volume are both dependent on the given parameters. Due to space restriction, we omit the derivations but give the final empirical formula here.

$$E(.) = V(.)f^2/(D^2c) \tag{1}$$

where E is the triangulation error,

V is the uncertainty volume,

f is the focal length,

D is the viewing distance,

c is a function of the pixel size.

c is a constant for a constant pixel size as in the case when the same type cameras or the same camera are used. Simulations have been carried out to verify (1). However, we are primarily interested in how the uncertainty volume and the triangulation error change with the viewing angle.

Although we can assume the distribution of spatial discretization error over the entire image to be a Gaussian distribution with zero mean, none of the existing camera calibration techniques uses the entire image to calibrate the stereo system. Most camera calibration techniques use only a small number of calibration points. The accuracy of the camera matrix is then very much subject to the spatial discretization errors of these calibration points. We are interested in improving the accuracy of the camera matrix obtained using the direct linear transformation method[9]. A perfect camera matrix defines a perfect epipolar line. The perfect camera matrix **T** represents a linear transformation of the form

$$\mathbf{xT} = \mathbf{u}. \tag{2}$$

Since calibration uses image points that may deviate from their true point of projection, the camera matrix obtained by direct linear transformation is as follows.

$$\mathbf{u}_{obs} = \mathbf{uE} \tag{3}$$

where **E** is an error matrix.

Therefore, $$\mathbf{xTE} = \mathbf{u}_{obs}, \tag{4}$$

$$\mathbf{xT}_{cal} = \mathbf{u}_{obs}, \tag{5}$$

where $\mathbf{T}_{cal} = \mathbf{TE}$.

The derived camera matrix $\mathbf{T}_{cal}$ is multiplied with an error term **E**. All the terms in the transformation matrix are now $T_{cal_{ij}} = T_{ij}+\delta_{ij}$. $\mathbf{T}_{cal}$ is the camera matrix used for triangulation and calculations of epipolar lines. The effect of calibration error on the calculation of the epipolar line is very complex. Simulations have been carried out for different viewing angles in order to understand the effect that calibration error has on the constrained epipolar line search and the calculation of the intersections of the epipolar lines .

## SIMULATION DETAILS AND RESULTS

### Triangulation accuracy

The uncertainty volume at COV was calculated for a range of viewing angles(5°-90°), viewing distances and focal lengths for a constant pixel size. Sets of camera matrices were generated for the 19 viewing angles for different viewing distances and focal lengths. This is done by specifying the viewing geometry and space to image transformation explicitly. The generated matrices were used to simulate the triangulation error. Sets of a total of 3500 random 3D points within the field of view were generated. They were projected onto the images using generated camera matrices. All the projected image points were rounded to the nearest integer coordinates to simulate spatial discretization. These rounded image points were used to triangulate the 3D positions. The Euclidean distance between the triangulated 3D position and the true 3D position was taken for each pair of corresponding image points. The average distance was taken for each angle of separation. Graphs relating V and E to viewing angles, V versus E at different D/f and the function c for a constant pixel size are plotted. The results are shown in Figures. 2-6. The Percentage decrease of the uncertainty volume and the triangulation error at $i^{th}$ viewing angle ($d_i$) is obtained by $|d_i - d_{i-1}| / d_i * 100\%$ , where i is the $i^{th}$ position.

### The effect of spatial discretization error on camera calibration and hence triangulation and epipolar lines

A set of random 3D points were projected onto the image for different viewing positions and the image points were rounded to the nearest integer. Subsets of these image points were used for obtaining the camera matrices by camera calibration. The viewing positions were calculated from each pair of generated camera matrices and those matrices obtained from simulated camera calibration. The Euclidean distance between the true camera position and the calibrated position was taken. Figure 7 shows the average distance of the recovered camera position from the true position due to calibration error for 20 sets of data points. Figure 8 shows the triangulation error for two of the four sets of camera matrices obtained from simulated calibration. These camera matrices are used to study the effects of calibration error on the triangulation error and the calculation of epipolar lines. The two sets of camera matrices are named as follows.

calb.mat -Derived using 12 random points.
cald.mat -Derived using 48 random points.

(see Figures 8 and 9)

The procedure used for simulating triangulation accuracy when subjecting to camera calibration error was the same as that in the previous section. The only difference was that the camera matrices obtained from simulated calibration were used instead of the generated ones.

The simulation of constrained epipolar line search looked at the displacement of the points where the epipolar line enters and leaves the image, i.e. at $(0,y_1)$ and $(511,y_2)$. The generated camera matrices were used to obtain graphs representing the ideal situation in which the only error was due to spatial discretization and the camera matrices were somehow free from the effect of spatial discretization error. A circle of radius 0.5 pixel was drawn around the image centre and the cone due to the circle was projected onto other images. The projection of this cone in other images give rise to the epipolar band due to spatial discretization only. The same procedure was repeated using camera matrices obtained from simulated calibration to simulate the situations in which the camera matrices were subject to calibration error. The widths at left and right image border were taken for both cases. The difference in widths at different viewing angle were plotted. The results are shown in Figure 9.

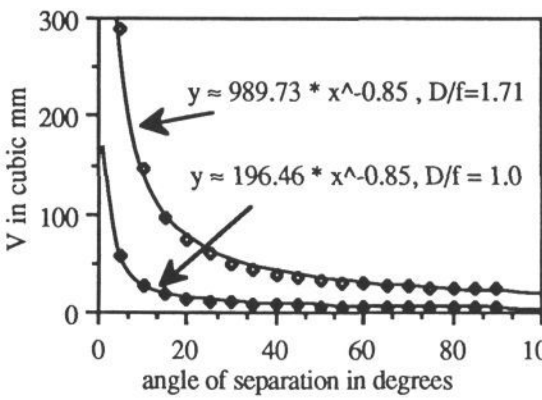


*Figure 2. The uncertainty volume at COV vs. viewing angle*

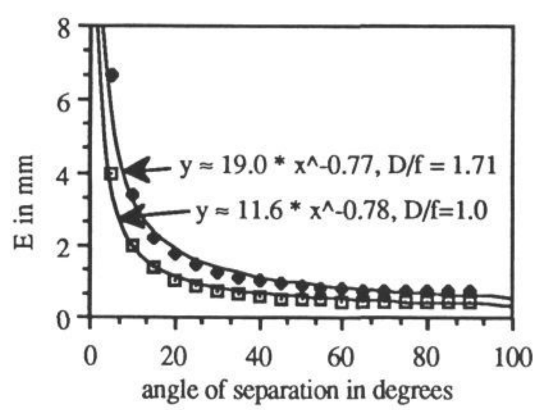


*Figure 3. The average triangulation error vs. viewing angle*

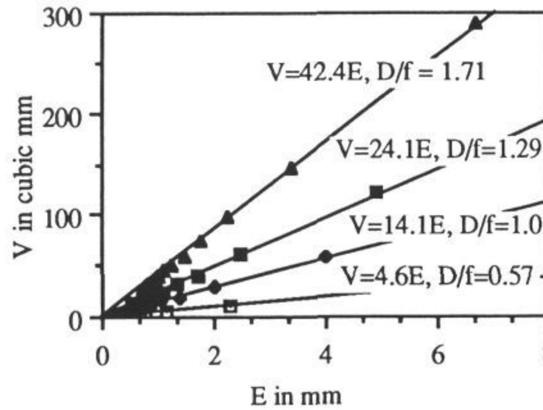


*Figure 4. The uncertainty volume at COV vs. average triangulation error for different D/f's.*

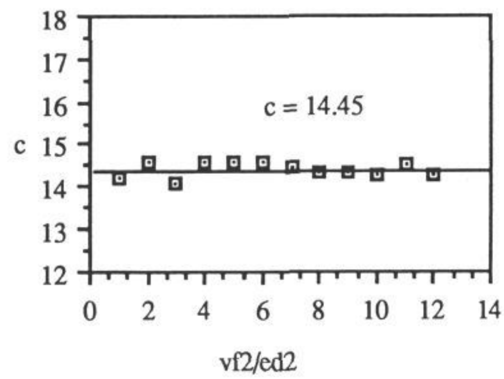


*Figure 5. The function c(pixel size) vs. $Vf^2/ED^2$*

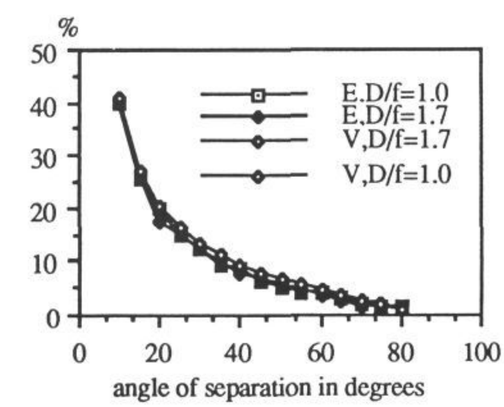


*Figure 6. The percent decrease of the uncertainty volume at COV and the average triangulation error with increased viewing angle*

# DISCUSSION

## Triangulation accuracy

The variation of the uncertainty volume and the average triangulation error with different viewing angle is approximately logarithmic (Figure 2,3). It is possible to approximate such variation by a function of the form $y=ax^b$. For the range of viewing distances and focal lengths simulated, **b** is more or less constant but **a** changes with D/f. However, the logarithmic functions are very crude approximations. The change in the uncertainty volume and the triangulation error is logarithmic only between 5 and 70 degrees of viewing angle. The results obtained using camera matrices obtained from simulated camera calibration show the same kind of variation of triangulation error with increasing angle of separation (Figure 8). The results shown in Figures 4 and 5 also verified equation (1) that gradient of the line relating V and E depends on the ratio of (D/f ). The c for a given pixel size used in the simulations is a constant and c=14.5(Figure 5).

## Camera calibration error

Figure 7 shows that the actual calibration error, indicated by the distance between the recovered camera position and the true camera position, depends very much on the calibration points used. On the whole, increasing the number of calibration points improves the accuracy of calibration. However, the improvement decreases with increasing the number of calibration points logarithmically. This agrees with Tsai's observation that merely increasing the number of calibration points will not improve the calibration accuracy very much[8]. The result in a way indicates that the estimation accuracy of the camera parameters depends on the average spatial discretization error of all the calibration points. In theory, we need to use a large number of calibration points so that the average spatial discretization error will be close to zero. Figure 7 shows we can obtain a reasonably accurate estimate of the camera parameters using as few as about 50 calibration points. We may therefore use only a reasonably small number of calibration points for camera calibration because there is no incentive to use more points which will inevitably increase the camera calibration time.

## Epipolar line search band

The distortions of the epipolar band due to camera calibration error, as shown in Figure 9, also indicates a logarithmic variation with viewing angle. This is however in pixels not in millimeters. This suggest that in order to reduce the error in the calculation of epipolar lines, accurate camera matrices and a wide viewing angle are required.

## Implications of the results

The simulation results suggest a wide angle of separation as much as 90° should be used to minimize the effect of spatial discretization error on the stereo imaging system. A wide viewing angle will make the matching of images more difficult. The final decision on the "optimal" angle of separation depends on the weighting given to each design criterion, i.e. matching ease or triangulation accuracy. In this case, more emphasis is placed on the accuracy of triangulation and calculation of the epipolar lines so that the matching algorithm can rely on the epipolar line for search and verification of potential matching candidates in a 3+ view system. This also implies that an accurate camera calibration is essential. The percentage drop of the uncertainty volume by increasing angle of separation above 40° is less than 10% . An angle separation wider than 40° will not reduce both the triangulation error and the error in epipolar line calculation significantly. Hence, one may use this as a criterion to suggest a compromise viewing angle between 35° and 45°.

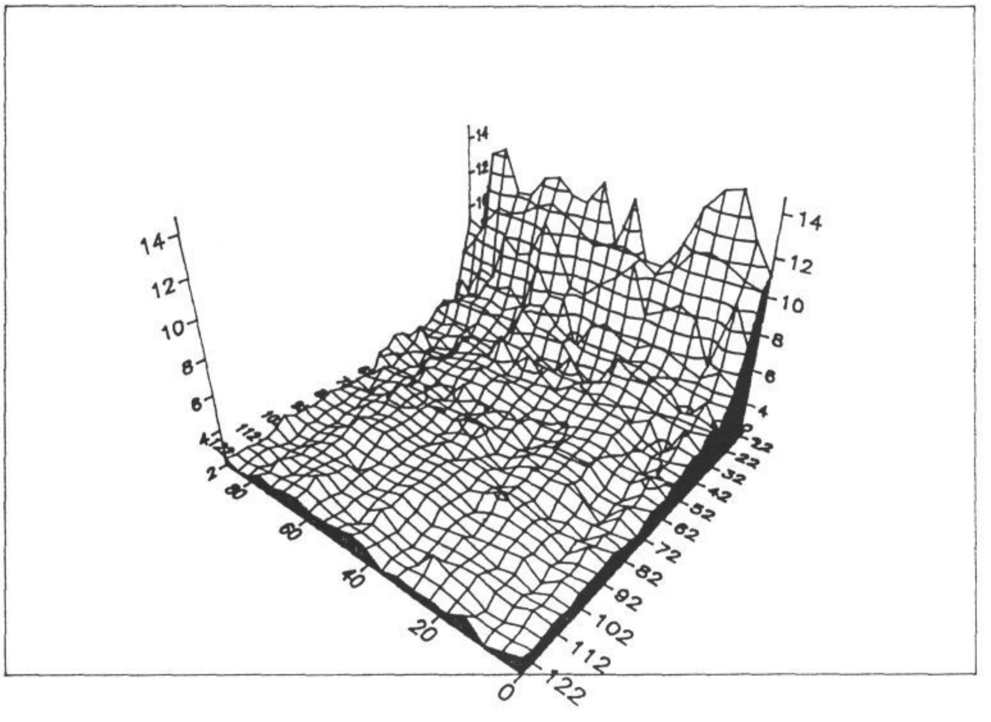


*Figure 7. 3D plot of average error in recovering the camera position using matrices obtained from simulated camera calibration. vertical axis: distance from the true camera position in 3D in mm; horizontal axes: viewing angles (0-90) in degrees, no. of calibration points used (12-126).*

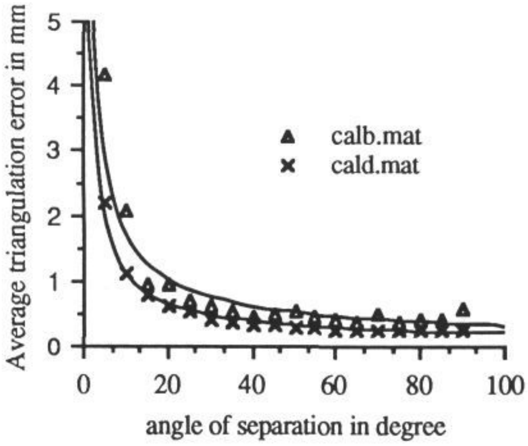


*Figure 8 The average triangulation error at different viewing angles when the camera matrices are subject to spatial discretization error.*

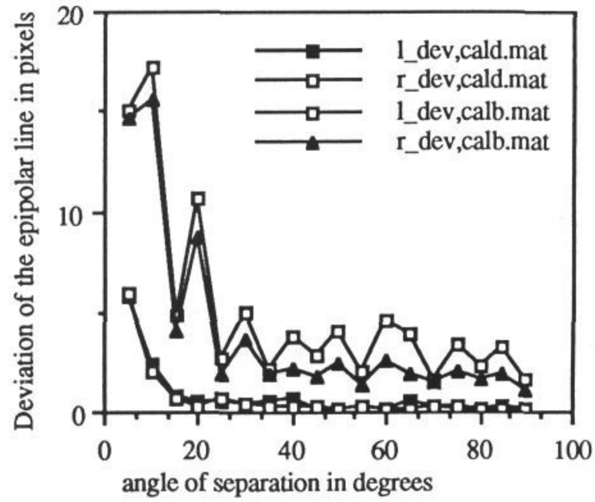


*Figure 9. The deviation of the epipolar line at different viewing angles when the camera matrices are subject to spatial discretization error.*

## CONCLUSION

The following conclusions can be drawn from the simulation on the effects of spatial discretization error :

1) Triangulation error and distortion to the epipolar line due to calibration error decrease with increasing angle of separation. The rate of decrease is approximately logarithmic.
2) Equation (1) relates the uncertainty volume and the triangulation error. Given the viewing distance, the viewing angle , the pixel size and the focal length, it is possible to calculate analytically the triangulation error from the uncertainty volume when c is also known.
3) Camera calibration error can be reduced by using more calibration points. However, above a certain number of calibration points (approximately 50), merely increasing the number of calibration points does not improve calibration accuracy significantly.
4) An "optimal" angle of separation of 35° to 45° is suggested. The improvement on the triangulation accuracy and distortion of epipolar lines is insignificant for angles larger than 45°.

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