3-D Cues from a single view: detection of elliptical arcs and model-based perspective backprojection *

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We describe the use of geometric reasoning and projective geometry to infer 3-D information about circles which project elliptical curves onto one 2-D image. A robust and efficient method to extract elliptical arcs starting from chains of line segments obtained by the polygonal approximation of edge curves is presented. Then an algorithm that solves analytically the problem of the perspective inversion of an elliptical arc from its projection onto the image plane is outlined. Finally experimental results on real images of mechanical parts are reported; therefore, current drawbacks and future extensions of this work are discussed.

An appropriate tool for describing the relations between image and scene objects is projective geometry. This method allows to recover the three dimensional structure of the objects starting from one or more 2-D images, without explicit range measurements. Quite a number of industrial tasks ranging from object identification and positioning to beacon-based navigation can be based on such computationally efficient approach.

The first section is dedicated to the problem of ellipse detection, that is a classical problem in Computer Vision and Pattern Recognition.

The second section presents the the perspective inversion phase. The approach refines the one already exposed in [1] and is strictly related to [2] and [3].

Finally, experimental cases related to both the ellipse detection and the backprojection phase are presented and discussed.

1 Detection of elliptic arcs

The task of extracting elliptic arcs from real images is complicated by the presence of noisy structures or *outliers* generated by highlights and shadows, and by the complexity of the scene which can be formed by several partially occluded objects (e.g. the bin-picking task). In the literature it is possible to find quite a number of methods for elliptical fitting but the problem of dealing efficiently with real images has been often ignored.

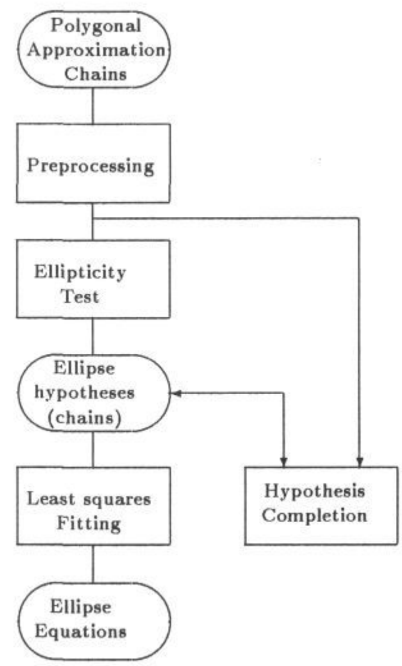

Figure 1: Flowchart of the ellipse detection algorithm

Our approach is concerned with the accurate selection of candidate structures in order to attain better performances, even in computation time, in the fitting phase. The process is highly modular and pyramidal: the output of each elaboration step is a higher level, more synthetic representation of the input data; moreover, during the elaboration, uninteresting structures are discarded and, eventually, only a subset of the initial input data are maintained. It is also possible to consider a priori, application specific knowledge to tune accurately the higher level stages of the process. The overall diagram of the process is outlined in Fig. 1.

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1.1 Preprocessing

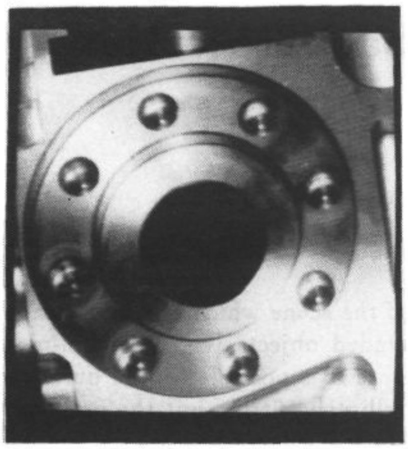

Figure 2: Image of a mechanical part

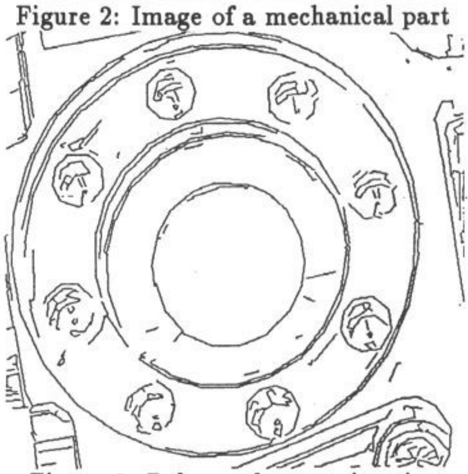

Figure 3: Polygonal approximation

The low level modules of the process has been developed in ESPRIT-P940 project *Depth and Motion Analysis* [4] and will soon be available in hardware. This will allow us to have the required input data at 5 Hz and to consider the work as part of an actual real time vision system. The edge map of the input gray-level image is extracted by means of a non-maxima suppression of the gradient module algorithm followed by hysteresis thresholding. From the edge map a linking module generates chains of 8-connected points.

At this level of representation, we have a list of curves described by their coordinates; if elliptical best fitting is exhaustively performed over all the chains of edges, the method turns out to be computationally cumbersome and error prone. In fact, the present representation of image contours contains only implicit geometric information (i.e. it is difficult to discriminate a curve from a straight line just reading a list of coordinates). Our choice has been to perform geometric reasoning on a more explicit representation obtained by polygonal approximation of the chains of edge points.

Polygonal approximation is a two step process: each chain is scanned and breakpoints are selected when a change of curvature above a certain threshold occurs, then a linear regression algorithm is carried out and the best least-squares segments approximating the curve between two breakpoints are created. The output is a list of chains of consecutive line segments described by initial and final coordinates and the direction cosines.

As we can see in Fig. 3, the representation retains the pictorial contents of the edge map, but is more concise and the geometric structure is more explicit.

1.2 Selection of candidates for elliptical approximation

This module processes a polygonal approximation representation of the image and selects the chains of segments that are more likely to have been generated by ellipses. The module exploits "Gestalt-like" perceptual heuristics in order to both enhance the quality of the chains that have been corrupted by noise and discard the chains that clearly cannot be approximated by an ellipse.

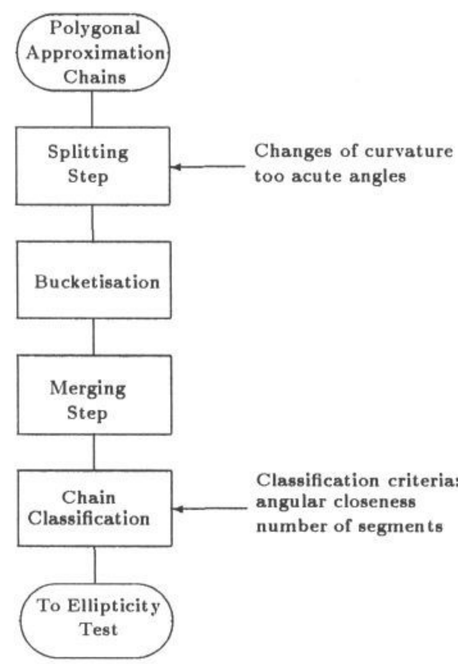

Figure 4: Flowchart of the pre-processing phase of the ellipse detection algorithm.

As outlined in Fig. 4, there are two processing stages: first input chains of segments are examined and split wherever an abrupt change in curvature occurs, then different chains which seem to belong to the same image ellipse are merged.

The splitting points are selected applying a *smoothness constraint*: the angle between two adjacent segments is calculated and, if it is smaller than a predefined threshold or a change in the current curvature sense of the chain (defined by the sign of the cosine of the angle) occurs then the chain is split. At the same time, straight lines are identified as segments whose length exceeds a threshold and extracted by the image representation, even if they occur inside a chain which is consequently split.

The objective of the second stage is merging chains that seem to have been broken accidentally: the approach consists on considering couple of chains whose extremes are sufficiently close and applying the above described smoothness constraint to the chain composed by concatenating the two candidates. If the curvature continuity test is affirmative a new chain is created rearranging the segments according to a unique orientation (i.e. if we merge two starting points the orientation of one of the two structures must be inverted).

A major problem arised is the computational effort required by the exhaustive matching of all the extremes of the chain. An elegant solution is the *bucketisation* of the extremes that allows to restrict the test to the extremes falling into the same bucket or image partition.

Finally every chain is classified according to a set of parameters such as:

- length,
- number of segments,
- degree of convexity,

in order to rank the ellipse hypotheses and diversify the subsequent processing.

This selection module greatly improves the overall performance of the system as compared with an early version lacking this stage, particularly when dealing with complex scenes and poor quality images.

1.3 Ellipticity test

Every previously preprocessed and classified chain of segments coming from linear approximation of curved edges are now to be geometrically tested in order to determine whether it can be associated to an image ellipse. The crucial point of inferring the presence of an ellipse just examining edges has been widely addressed in literature: Rosenfeld and Nakagawa in [5] use polygonal approximation of edges and compute parameters of the polygonal chains before deciding which of them are ellipses by performing heuristic tests. This approach lacks a real ellipticity test so that is not selective enough in discriminating ellipses from other geometric structures.

Hough transform techniques are largerly used for fitting the ellipse equation. Fitting ellipses directly from data points requires the determination of five parameters: in order to render more tractable the problem it is possible to reduce the dimensionality of the problem by decomposing it in two stages: finding the center and then determinating the remaining parameters [6].

An improvement of this method has been developed by Yuen et al. [7]; they rely on the following property to detect the center of the ellipse without the cited limitations: the centre of the ellipse must lie on the line joining the intersection between two non-parallel tangents to the ellipse to the mid-point of the segment which joins the two tangential points. The critical point of this approach is the unstable estimation of the tangents due to uncertainity in the computation of the gradient.

Our own method is based on the detection of the centre as well, but does not rely upon tangents to the ellipse, so that, it does not require an estimate of the local slope of the curve.

The method is based on the following theorem of projective geometry about polars and poles with respect to conic sections.

THEOREM 1.1
The pole of any diameter is the point at infinity. The polar of any point on a diameter is parallel to the chords defining the diameter. The centre of a conic is the pole of the straight line at infinity.

Therefore, the centre of an ellipse can be found as the intersection of two diameters and a diameter can be obtained by joining the mid-points of any two parallel chords, as in Fig. 5.a.

In our case the major problem which arises is how to build two parallel chords having only an incomplete polygonal approximation of the conic. We coped with it by making the following approximation, illustrated in Fig. 5.b:

HYPOTHESIS:
given two non-parallel polygonal approximation segments $\vec{AB}$ and $\vec{CD}$, being D' the intersection of the straight line $\vec{CD}$ with the parallel to $\vec{BC}$ drawn through the point A we assume that $\vec{BC}$ and $\vec{AD'}$ are two parallel chords of the ellipse.

That is acceptable if the considered segments are a good approximation of the curvature of the ellipse and if they are not too close or too different in lenght. The approximation error is given by $\vec{HD'}$.

The centre detection algorithm is now straightforward:

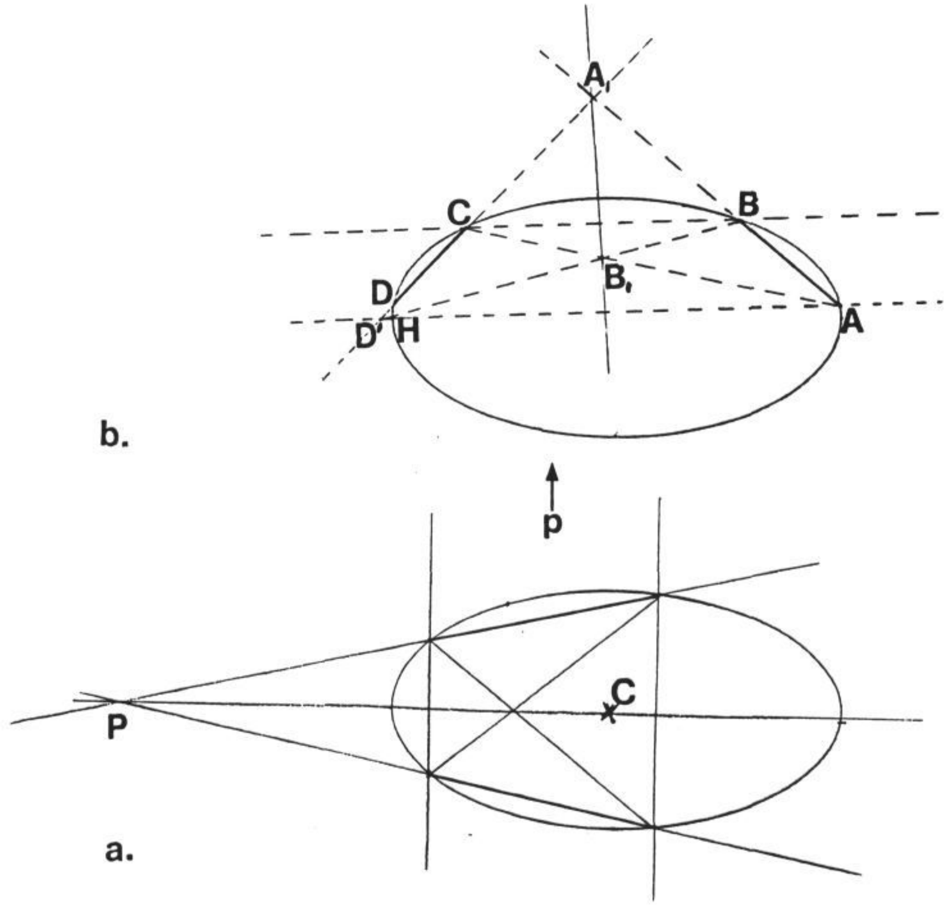

Figure 5: a. Principle of the ellipticity test; b. Implementation of the ellipticity test.

1. *being A_1 the intersection of $\vec{AB}$ with $\vec{CD}$ and B_1 the intersection of $\vec{AC}$ with $\vec{BD'}$, the centre of the ellipse lies on the straight line A_1B_1, which is a diameter;*

2. *consider a third segment $\vec{EF}$, a second diameter $A_2\vec{B}_2$ can be obtained in the same way from the segments $\vec{AB}$ and $\vec{EF}$;*

3. *the intersection of $A_1\vec{B}_1$ with $A_2\vec{B}_2$ is a hypothesis of ellipse centre.*

It is now possible to iterate the process considering all the remaining segments in order to confirm and refine or reject the hypothesis. At last, if enough evidence supports the detected centre, a new ellipse hypothesis, still linked to a polygonal approximation chain, is istantiated.

1.4 Completion and refinement of ellipse hypotheses

A complete image ellipse often happens to be partially occluded by an overlapping object or a shadow so that a subarc remains isolated and can't support a hypothesis by himself. The preprocessing stage can deal only with local gaps and imperfections but lacks a more general view of the scene. A further step in the process copes with these cases: every istantiated ellipse hypothesis and all the segments still unallocated are considered in order to merge together hypotheses generated by the same image ellipse and, moreover, to complete them with isolated groups of segments belonging to the same ellipse.

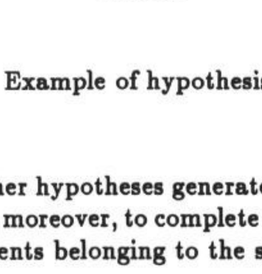

Figure 6: Example of hypothesis completion.

At this level of the process, we try to find undetected subarcs whose curvature is opposite to that of the already instantiated hypotheses. It is very important to succeed in completing an ellipse even with a small part of it but opposite in curvature because, in this case, the result of the fitting phase is much more accurate and reliable.

The unallocated segments are inserted in a histogram according to their orientation. Then the orientation of the segments belonging to the considered ellipse hypotheses are used as pointers to extract from the histograms candidates for the completion, which are eventually tested exploiting the above described geometric criteria. If the selected segment is not isolated but belongs to a chain, all the segments of the chain are tested. If ambiguous cases arise, simple maximum likelyhood heuristics are used.

As already pointed out, having ranked and ordered candidate chains and relative ellipse hypotheses, it is possible to try to complete first the best in order to minimize the error rate. An example of hypothesis completion is depicted in Fig. 6.

1.5 Fitting and validation

The equation of the ellipse is obtained by fitting it to the mid-points of each segment belonging to the chains hypothesized as elliptical arcs. A standard least squares routine, returning the coefficients of the ellipse, is then carried out. The use of polygonal segment mid-points as fitting data instead of edge points entails two main advantages: computation time is spared and the problem of uniform sampling as regards the curvature is implicitly solved. On the other hand, problems could arise when dealing with too small or very slant ellipses because in these cases the polygonal approximation is not accurate enough.

At last, a few threshold controlled tests are performed and only the most robust results are given as the whole process output. A priori knowledge based validation tests are based on the following measures:

- axes ratio,
- absolute perimeter length,
- ratio between calculated and actually observed in the scene perimeter,
- estimated least squares error.

1.6 Experimental results of the ellipse detection algorithm and possible extensions

In Fig. 7 there is an example of the outcome of the ellipse detection module. The test-object is a special mechanical part used to calibrate tactile measure robots; the images include quite a number of elliptical arcs together with noisy structures generated by shadows and highlights. The results demonstrate the reliability and robustness of the algorithm which is able to detect correctly most of the image ellipses even in presence of noise, outliers and double edges, with a low false alarm rate. However the algorithm can be improved in some points:

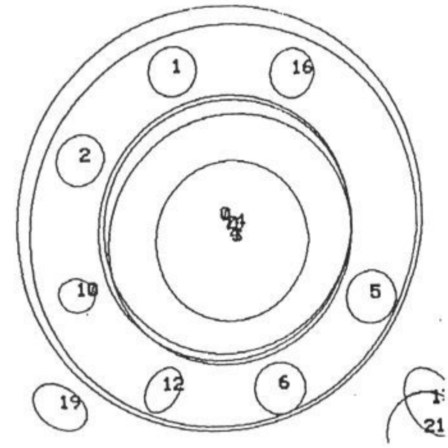

Figure 7: Results of the ellipse detector

i. as it is well known, the least squares fitting technique requires data from a large portion of the ellipse in order to achieve sufficiently accurate results, being biased to high curvature solutions. Thus the fitting module will be replaced by a bias corrected Kalman filter as in [8].

ii. In order to improve the precision of the method when there are very few data available (tiny ellipses) it is possible to recuperate the edge points associated to the involved polygonal chain and, therefore, to perform the fitting step over these data.

iii. The bucketisation step in the preprocessing module could be refined using partially overlapping buckets in order to deal with border ambiguities.

2 Perspective inversion

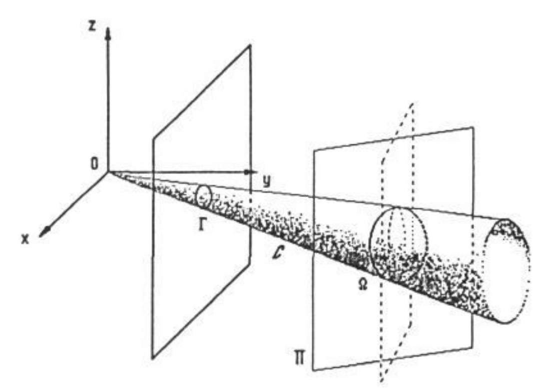

Figure 8: 3D circle and corresponding projected image ellipse.

Given a set of ellipses extracted from an image by means of the purely data-driven process described above, it is possible to infer the 3D orientation of the circle arcs which projected each ellipse onto the image plane. The only assumption is that the ellipse arc in the image comes from a circle arc.

From a mathematical point of view, the problem of inverting the perspective projection for an ellipse we know coming from a circle, is reduced to finding out those planes whose intersections with the cone over the ellipse and with vertex in the origin are circles (see Fig. 8). We can only determine the normal to the right planes, and not the distance from the origin, because parallel sections of a cone are all similar geometric entities. Only the radius value allows us to choose among the parallel planes the one which corresponds to the actual case. Avoiding special cases, there are two possible normals for every ellipse, i.e. two possible sets of parallel planes.

For a thorough description of the mathematical approach we refer to [3].

2.1 Experimental results

The proposed method has been integrated in an object recognition system oriented to merging together different sources of 3D information [9]. A model-based object recognition framework actually permits to cope with the solution ambiguity problem of perspective inversion.

Object recognition experiments have been carried out on a test set including cylinders and polyhedrals with circular holes and, therefore, some accuracy estimation, relative to the perspective inversion process alone, can be given. In particular, geometrical relations such as the angle between couples of normals and distances between centres have been compared with the expected values.

Even if a better estimation of the intrinsic camera parameters could greatly improve the overall performance, we have found a percentual mean error of 3.0% on distance measurements and less than 5 degrees on the normal angles.

3 Conclusion

In this paper a novel method of extraction of elliptical arcs from 2D images has been presented, together with a projective geometry based technique for inferring the 3D position and orientation of the circles which projected the ellipses onto the image. Experimental results of both the process stages and an outline of the underlying mathematical theories are also included.

The major drawback of the low level phase is the ellipse equation fitting: actually the least squares technique fails if the proportion of the ellipse visible in the image is not large enough. It is possible to overcome this problem by introducing a more sophisticated fitting method such as a bias corrected Extended Kalman Filter [8].

The perspective inversion techniques are inherently ambiguous, therefore cannot uniquely determine the object spatial attitude without using *a priori* knowledge or integrating together other 3D information. We have already experimented the synergy between this technique and structured light [9], whilst an extension to include 3D data coming from segment-based stereo [4] and occluding boundaries analysis is currently investigated. Another possibility to cope with the ambiguity problem could be the spatial integration of geometrical information coming from different known points of view (active vision).

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