Parallel and Perpendicular Line Grouping in a 3-d Scene from a Single View

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A method for hypothetising 3D configurations from an image, using a perspective model, is described. To guarantee the robustness, a probabilistic approach has been chosen. The vanishing points are detected using a sampling based on a statistical study of the uncertainty and the lines are classified using a maximum likelihood test. Classes of lines are hypothesised to be perpendicular in a similar way. The hypotheses are hierarchically built and each of them is assigned a score, using a Bayesian approach.

This paper presents a method for grouping lines that are hypothesised to be parallel or perpendicular in a 3-dimensional scene from a single image. This is achieved by exploiting both perspective transformation and general geometric information on the type of the scene.

This work is part of a project which is concerned with the location of a robot in an environment using a CAD description, from a single view. In this application the views are indoor scenes of nuclear plants.

The scene mainly consists of parallel and perpendicular lines with privileged directions i.e. vertical and two horizontal directions corresponding to the wall limits. This is the case for most indoor scenes. The knowledge of the main directions directly deduced from the image is of a great interest before applying a matching strategy with the CAD database because it considerably prunes the search space [7].

A full and flexible interpretation of the line directions is required, in order to have alternative solutions in case of failure of one hypothesis in the matching stage, and to guarantee robustness by minimising the number of parameters required by the algorithm. We use a probabilistic approach allowing us both to deal with uncertainty and produce a significance score to each hypothesis.

A number of methods for grouping parallel lines has been proposed [2,3,8,9]. All are based on the search for vanishing points in a perspective model by exploiting the property of parallel lines in the 3D space i.e. the perspective projection of which are concurrent lines in the image. First of all, as Magee and Aggarwal [8], the vanishing points are detected by accumulating the intersection points of a number of pair of straight lines in the images, but using another type of accumulation space. In order to reduce the search to a bounded close set, Barnard [3] proposed projecting the lines of the image onto a Gaussian sphere and to use the Hough transform paradigm to detect the vanishing points. The projection of points onto a Gaussian sphere is equivalent to a resampling of the image plane. The method described here also resamples the space but keeps the uncertainty of the intersection points roughly constant over the space. This also leads to a bounded close set, but with the same mapping along x and y axes. The local maxima are then detected, each of them corresponding to a vanishing point class (VP class).

At the same time, the parallel lines in the images are detected. The line orientations are accumulated and the local maxima are detected, each of them corresponding to a parallel direction class (PD class). A line may belong to one or more VP classes and to a PD class. These classifications are used to evaluate the significance of a VP class and to keep consistent alternative hypotheses in case of failure. At this stage all the classes with their associated lines and scores for belonging to that particular class, and a score for each class are recorded.

Next, each class is hypothesised to be perpendicular to another class, this hypothesis is either rejected or assigned a score determined by a likelihood test. Then consistent triplets of perpendicular directions (PPD triplets) are searched for. Therefore the algorithm provides a hierarchical sets of hypotheses from the low level (PD classes) up to a high level (PPD triplets) (figure 4). At every level, a score is associated with all the hypotheses, based on a likelihood test result. Assuming an hypothesis is true, all parameters are processed in a Kalman filter to increase the accuracy, in a similar way to the automerging process [1].

In this paper, some approaches to the problem are reviewed, the method is described and then the main advantages and disadvantages are analysed. Finally results on real scenes and a test card scene are described.

PREVIOUS WORK

The search of vanishing points consists of finding a small neighbourhood in the space crossed by a sufficient number of straight lines. The space is sampled and the number of straight lines crossing each cell is computed, using the Hough paradigm. Barnard [3] proposed projecting the lines onto a Gaussian sphere centred on the optic centre. The new search space is bounded. The search for the accumulation points is achieved by sampling the sphere

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using spherical coordinates. Unfortunately the spherical sampling is irregular and different in the x and y directions. Quan and Mohr [9] use the same method, classifying the lines by looking for vanishing points. Once a vanishing point is found with its associated lines, the lines are eliminated and the algorithm is performed again. This approach removes the possibility of having different hypotheses for the same straight line, which is a limitation for the interpretation of the scene. Dickson [6] proposed a triangular sampling of the Gaussian sphere which is very attractive because it is isotropic. However the computational efficiency has yet to be proved.

Magee and Aggarwal [8] only accumulate the projection of the intersection points of any pair of straight lines in the image, on a Gaussian sphere. It allows low_pass filtering before accumulating, so eliminating many impossible vanishing points (eg. a vanishing point cannot be on one of the segments generating it). The accumulation is achieved using the arc distance between two points.

Wei [11] proposes a calibration method based on vanishing point detection. He proves that their detection allows all the camera calibration parameters to be determined.

PARALLEL LINES GROUPING

The goal of this work is to built all the likely interpretations of the directions of straight lines in a scene. These hypotheses are hierarchically built, so that if the higher level hypothesis is rejected at the matching stage, a lower level hypothesis would be tested in a top down manner.

The difficulty of building such hypotheses comes from the accumulation of uncertainty errors throughout the acquisition and the processing of the data preventing, for example, the vanishing point to be a single point. The vanishing points are detected as the converging points of several straight lines. Then it is hypothesised that a straight line passing through a detected point P has its vanishing point in P. Let P' be the vanishing point of the straight line D. First of all, the likelihood of the hypothesis: P=P' depends on the relationship between the distance from P to D and the uncertainties of their locations. The errors of interpretation will be of two types: (1) P' is not P but is near P (e.g. nearly parallel lines), (2) the point P is near the line D by chance, P' being far from P, and the line is not at all parallel to the set of lines associated with P. The first case leads to an underestimation of the uncertainty but the second case may produce crucial interpretation mistakes. Those considerations have lead to the adoption of the following strategy. The vanishing point detection is based on an uncertainty criterion to ensure the same probability of detection over the space. Only intersection points will be accumulated allowing a prefiltering of second type false hypotheses. The classification stage will be based on a maximum likelihood test taking into account the error risks mentioned above.

Detection:

The uncertainty of the intersection point I of two straight line segments depends on a number of parameters such as the difference of the slopes and the distance from I to the segments (lever effect: a small error on the end point coordinates generates a big error on the intersection point coordinates). As the segments are distributed only within the image, if the vanishing point is far from the image, the lever effect is significant and the difference of the slopes is small. Both of these effects increase the expected uncertainty of such a vanishing point.

Let S and S' be two segments intersecting at I. Using the natural coordinates of the image (x,y), the origin being the centre of the image, it is possible to prove that (figure 1):

$$\sigma_x = K(a, D_m, D_{m'}, V)\, x^2 \qquad (1)$$

and to bound K() for x large enough (13). Inside the image, the approximation of an uniform distribution of straight lines over the neighbourhood around the intersection point hold and the expected value of the accuracy is roughly constant.

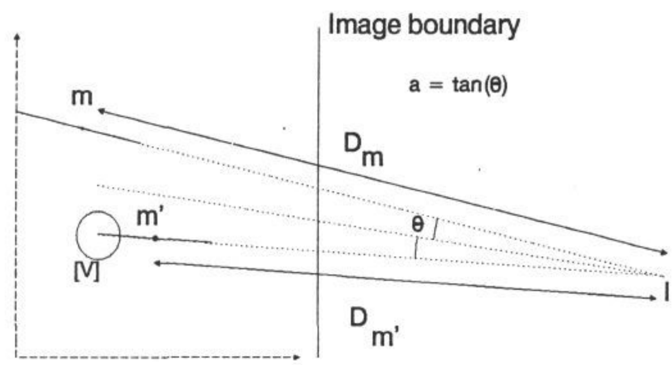

Figure 1. Influence of intersection point location on uncertainty

The intersection points of a great number of straight line segments distributed on an image, and the associated covariance matrices have been computed. The curve $\sigma(x) = \sigma_x$ is filtered (mean filter). Then, for x large enough it is approximated by a second order polynomial $\alpha x^2 + \beta x + \gamma$, using a least mean squares method. The expected standard deviation $\sigma_{x'}$ of x', x' being the resampled abscissa of I, will be roughly constant if $dx'/dx = 1/c$ for ($|x| <= x_s$) and $dx'/dx = 1/(\alpha x^2 + \beta x + \gamma)$ for ($|x| >= x_s$). This accumulator space is now considered as an image (eg. using a frame store). The number of points in a neighbourhood centred on P with a size equal to $2\sigma_{x'} \cdot 2\sigma_{y'}$ is computed by a convolution with weights equal to 1. Then the local maxima are detected.

The accumulation is achieved only on the intersection points which do not correspond to hypothesised parallel straight line segments (see further on) and of which both associated segments verified the relation: $D/l <= (k+1)/2(k-1)$, where D is the distance from the middle point of the segment to the intersection point, l the length of the segment and k the ratio of maximum distance to minimum distance from the scene to the camera.

Two lines with the slopes a and a' are hypothesised to be parallel if $(|a-a'|/\sigma_{|a-a'|}) < v1$. Most of those pairs have a finite intersection point with no real significance. As both classifications are performed at the same time, this intersecting point is not accumulated. The direction accumulation and local maxima detection is performed in a similar process to the VP class detection.

Classification:

In the following we assume that the noise follows a Laplace Gauss law with a null expected value. The objective of classification is to find sets of 3D parallel lines but those lines are mixed with lines of any direction interacting with them. P is assumed to correspond to a real vanishing point. Two hypotheses are considered, H1: the line has its vanishing point near P and belongs to the correspondent set of parallel lines, H2: the line passes near P by chance. Let r1 and r2 be two types of risk. The first type is the risk of *considering H2 true when H1 is true and the second type is* considering H1 true when H2 is true. It is possible to favour either one or the other approach according to the consequences of the decision.

The classification of the lines is achieved by a maximum likelihood test formulated for a decision variable V: distance from the straight line D to the point P for the VP classes and the difference of the slopes for the PD classes. The distribution of the decision variable V associated with a particular class VP or PD is analysed in the neighbourhood C centred on 0 with length L (figure 2). If H1 is true, V is assumed to obey a Laplace Gauss law. If H2 is true, V is assumed to obey a uniform law inside C. The probability density model used is:

$$P(V) = \left(P(H1|C)\exp(-V^2/2\sigma_v^2)/\sqrt{2\pi}\sigma v + P(H2|C)/L\right)P(C) \qquad (2)$$

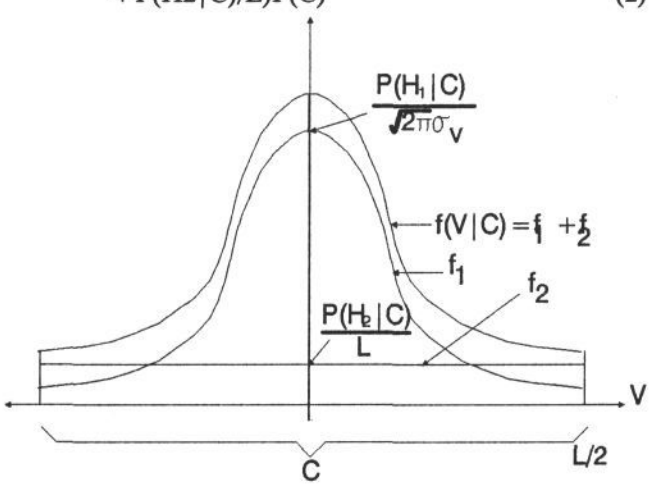

Figure 2. Probability density decomposition

The maximum likelihood test is:

$$T = P(H1|C)L\exp(-V^2/2\sigma_v^2)/P(H2|C)\sqrt{2\pi}\sigma_v \qquad (3)$$

P(H1 | C)/P(H2 | C) depends on the correctness of the initial hypothesis i.e. the explored scene mainly consists of parallel lines. Weiss [12] proposed a method for maximising the likelihood function in a slightly different situation, allowing the determination of the parameters of the uniform density. The vanishing point location is roughly known, the maximum likelihood test is just ensured to be higher than 1. A large enough r1 is chosen, which decreases r2. A reference case is defined such that the probability density is associated with the uncertainty value computed in the detection stage. T can be evaluated in the following way. For a PD class line:

$$k1\exp(-(a-a')^2/2\sigma^2_{|a-a'|})(1+a^2)^{1/2}/\sigma_{|a-a'|} \qquad (4)$$

For a VP class line:

$$k2\exp(-D^2/2\sigma_D^2)(1+a^2)^{3/2}/\sigma_D\sqrt{a^2(dx'/dx)^2+(dy'/dy)^2} \qquad (5)$$

where k1 and k2 are such that T > = 1 in the reference case.

Two iterations are performed, the first iteration uses the VP coordinates (PD slope) and the covariance matrix obtained in the detection stage to get a better approximation of these coordinates (slope). The second iteration classifies the lines such that T > = 1 and provides the final estimation of the parameters of the class (VP coordinates or VP slope and covariance matrix), using a Kalman filter (figure 3).

The score of a line associated with a class is $sc = T/(1+T)$, which is slightly different to P(H | V), as P(H1 | C)/P(H2 | C) remains unknown.

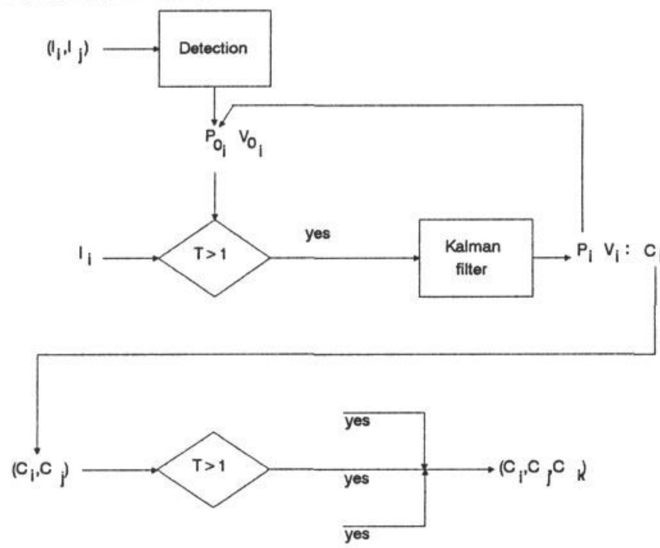

Figure 3. General system structure

Significance of the classes:

A local maximum may occur when two sets of very close parallel lines intersect (eg. sets of close pipes). This local maximum has no significance as it represents the intersection of only two directions. When two lines of the same VP class belong to the same parallel class they are considered as elements of one direction. A vanishing class is now a set of directions, assumed to be independent. Two hypotheses are considered: (H') the point P does not correspond to any main vanishing point and the distribution around P is locally uniform, (H) the point P corresponds to a main vanishing point and the probability density is as described above. The score of a class is P(H | V1,...,Vn), Vi being the decision variable associated with a line i classified in the VP class. Using Bayes theorem, in its odds form: O(H | V1,..., Vn) = L1....LnO(H) where: Li = P(Vi | H)/P(Vi | H'). The score is equal to:

$$sc = 1 - P(H')/\left(P(H') + P(H)(P(H1|H)\Pi Li + P(H2|H))\right) \qquad (6)$$

where $Li = P(Vi|H1)/P(Vi|H2) = 2Ti/\sqrt{2\pi}es$, e is the exponential constant and s the surface of the gaussian on the neighbourhood considered. P(H) is the a priori probability to have detected a real vanishing class, which is estimated as a function of the value of the corresponding VP in the accumulator and the total number of intersection points or it may be simply set to 0.5. P(H1 | H) is the a priori probability for a line to be correctly classified near a vanishing point (P(H1 | H) = P(H1 | C)). If nbd is the number of directions involved in a VP class, P(H1 | H) is approximated by (nbd-2)/nbd. The score sc is null for a VP class formed by 2 directions, very small when one of three directions is associated with a very small score, and increases rapidly with the number of reliable directions, which corresponds to the intuitive idea of the significance of a class [13].

In the same way, a parallel class may represent a direction parallel to the image plane if it consists of a number of parallel lines not too close to each other (eg. a semi continuous line has no significance by itself). If two parallel lines are such that they could be associated with the same VP class, then they are considered as elements of one subclass. A PD class is a set of subclasses of which the scores are the supremum of the scores of the elements. The PD class score is obtained as above but P(H1 | H) is approximated by (nbd-1)/nbd and P(H) depends on the ratio of the number of lines in the PD class to the total number of lines.

Assuming our image interpretation is correct, a better representation of the lines is computed, using the vanishing point coordinates, and information such as semi-continuity of a number of straight line segments, in a Kalman filter process, in a similar way to the auto-merging process [1].

PERPENDICULAR LINE GROUPING

Let f be the focal length, two vanishing points F1 and F2 correspond to perpendicular directions if

$$OF1.OF2 = -f^2 \quad (7)$$

Therefore a triplet of perpendicular directions can be represented, on the image, by a triangle of which the vertices are the vanishing points Fi associated with each direction. The orthocentre of the triangle should be the projection of the optic centre and the scalar products OFi.OFj be equal to $-f^2$. The purpose of this work is to find in the image such consistent configurations.

Two VP classes are hypothesised to be perpendicular if the likelihood test associated with the decision variable: $V = OF1.OF2 + f^2$ succeeds. The score is the test value scaled between 0 and 1. If one of the class is a PD class then V is the scalar of OF with the direction of the class.

A triplet of classes is hypothesised to be a PPD triplet if the three 3D directions are perpendicular to one another. Its score is computed in a similar way to the class score. H corresponds to the hypothesis PPD is a triplet of perpendicular lines in the 3D space and H' is not H.

$$sc = (1-P(H')/(P(H') + P(H)\Pi Li)\Pi sci = sc0\Pi sci \quad (8)$$

where $Li = P(Vi \mid H)/P(Vi \mid H')$ and sci is the class score. In absence of any a priori knowledge of H, P(H) = 0.5.

Even if the calibration parameters are only approximatly known, such hypotheses are possible by taking into account the uncertaincy of these parameters. Few hypotheses are available and the tests described above provide a good filter even with imprecise parameters. In this case, P(H) may be increased in order to favour the class score rather than sc0. The calibration parameters may be estimated on testcards with the same method [11].

The result of the algorithm is a tree (figure 4). At the top of the tree are the PPD triplets (usually one or zero) formed by pairs of perpendicular directions, themselves represented by sets of 3D parallel lines grouped in main directions (close lines grouped together). At the bottom are the straight line segments detected in the image. Horizontal links exist as a straight line segment may belong to different classes. Each node has a score value.

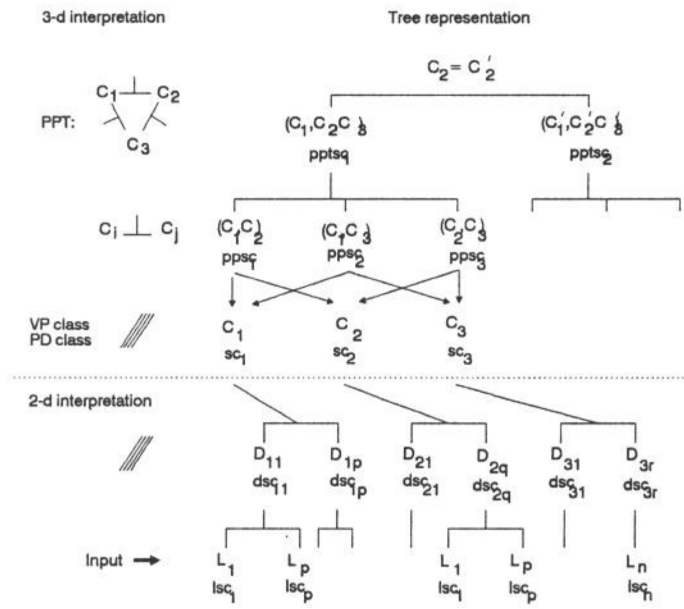

Figure 4. Hierarchical tree description of straight lines in a scene.

ANALYSIS OF THE METHOD:

The advantage of that method is that it provides a full and flexible interpretation of the image in term of 3D straight line orientation. Both parallel and vanishing classification provides complementary information. The VP and PD class detection stage is based on a statistical study of the uncertainty. This allows a nearly constant quality for the VP detection, and the determination of the reference parameters which are useful in the classification stage. This last stage is based on an uncertainty criterion different to a neighbourhood criterion (eg: a line close to P may be discarded while a further line may be accepted). Another important advantage of this method is the absence of parameter choice apart from the risks r1 and r2 which reflects both the wish for a complete interpretation and the validity of the initial hypothesis, ie. presence of a number of parallel lines in the images. The other parameters are physical e.g. the calibration parameters and the initial covariance of the end points which are assumed to be known.

The probabilist model uses a locally uniform probability density to represent H2. A wrong interpretation of a class will probably be produced by another regular structure present in the scene which is badly represented by the uniform density. In this case more a priori knowledge would be necessary to improve the model. The model used is not reliable enough to provide a value of P(H1 | C)/P(H2 | C), and therefore it does not provide the exact probability values. Further work is required to improve the model and the estimation of the density P(V | H2).

Only the intersection points of the m longest segments with all the n segments are accumulated, so the detection stage complexity is O(mn). The classification stage complexity is O(pn), p equal to the number of classes.

RESULTS:

Preprocessing is performed leading to a polygonal approximation of the edges. The edges are detected by the Canny operator [5], followed by a polygonal approximation algorithm developed at INRIA Sophia Antopolis [4]. The chosen line representation in the image is: $y = ax+b$ if $a <= 1$ and $x = ay+b$ if $a > 1$. The resampling parameters of the accumulator space are: $x_s = 80$, $c = \delta 5.3$, $\alpha = \delta 0.0002$, $\alpha = \delta 0.1$ and $\gamma = -\delta 4$, depends on the covariance matrix V and the required resolution (the same parameters are used for y). The accumulator space is an image 256•256, and V is such that $vx = 1$, $vxy = 0$ and $vy = 1$ which leads to an expected uncertainty $1/\delta$ equal to 3.5. If an image point P has more than 6 intersection points in a neighbourhood with a radius 3.5 centred on P (intersection of 4 lines), P is hypothesised to be a vanishing point. The values v1 and v2 are chosen to be 1.8 and 1.3 corresponding to the first type risk values 7% and 20%.

We have processed different images of an indoor scene of a nuclear plant and also an image of a testcard. A number of difficulties exist in these images. The first one is the non exact parallelism of the 3D lines (e.g. in the nuclear plant image the boundaries of the door not completly open), which produces two close local maxima in the accumulator space. The horizontal lines in the image of the test card pass through both vanishing points. Possible classification of one line into several classes allows that problem to be solved. Horizontal links between the same element of different classes will help a further interpretation to solve the ambiguity. The second difficulty is the presence of a line (e.g. leg of a tripod in the nuclear plant image), near a vanishing point P by chance. The uncertainty of the distance from this line to the point P is very small because of the proximity of the segment ends and the point P. Then D/σ_D is larger than v2 and the line is rejected. The line will not be rejected if a neighbourhood criterion is used instead.

On all images, the classified lines were correctly classified. All main directions were found and perpendicular directions were correctly hypothesised. The scores of the classes gives a good idea of their relative significance.

Figure 5 shows the result of the interpretation as a hierarchical tree.

```
number of VP classes: 18
number of PD classes: 6
perp. triplets and scores: 10 19 23 0.69
                            9 19 23 0.46
                           13 19 23 0.41
class number: 10
    x, y and cov matrix: 235.66 112.71 3.71 1.78 -0.09
    number of lines: 10
    number of directions: 6
    score: 0.96
    perp classes and associated scores:     19 0.83
                                            20 0.75
                                            23 0.55
                                            24 0.72
    straight line segment number: 10 1
        xa,ya,xb,yb: 15 103 117 107
        map, a, b and cov: 0 0.039 102.41 0.0002 1.34 -0.01
        score value: 0.83
        alt. PD class: 23
        alt. VP class: 8 9
```

Figure 5. Extraction of interpretation tree result.

CONCLUSION:

A method for interpretating lines in an image as parallel lines and perpendicular lines in the 3D world has been described. The method is based on a probabilistic approach, allowing a multiple classification for each line. An important point is the fact that the choice of parameters is limited to the risks of first and second type, which are easily controlled. The method leads to a hierarchical interpretation tree, from triplets of perpendicular directions to parallel lines.

From the detected directions and the location of each segment in the image, a sketch of the 3-dimensional configuration will be hypothesised which will be used to initialise the matching with the model.

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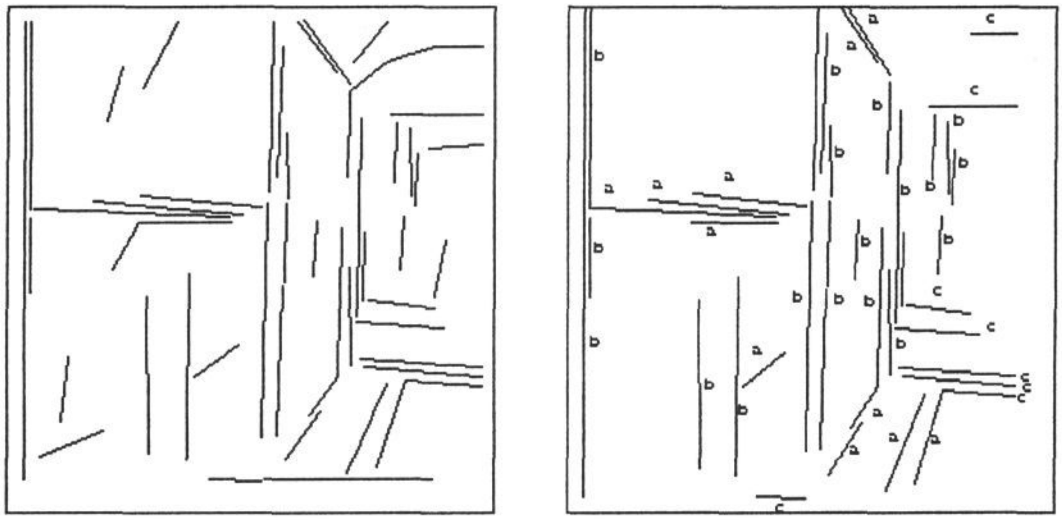

Figure 6. Image of a nuclear plant showing the detected straight lines and the highest scoring PPD with the lines in the three directions indicated by a,b,c (this is the same image as for figure 5).

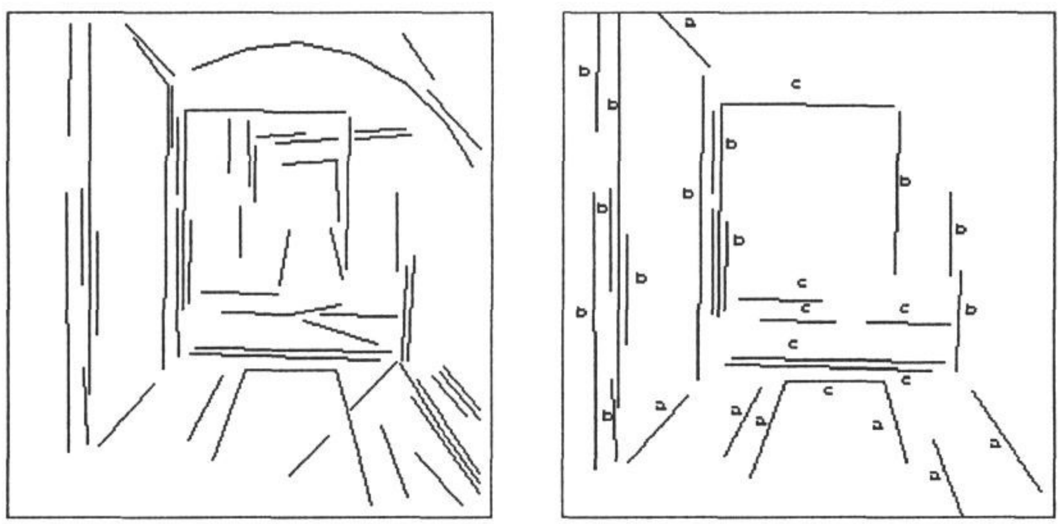

Figure 7. A different image of a nuclear plant. again showing best PPD triplet.

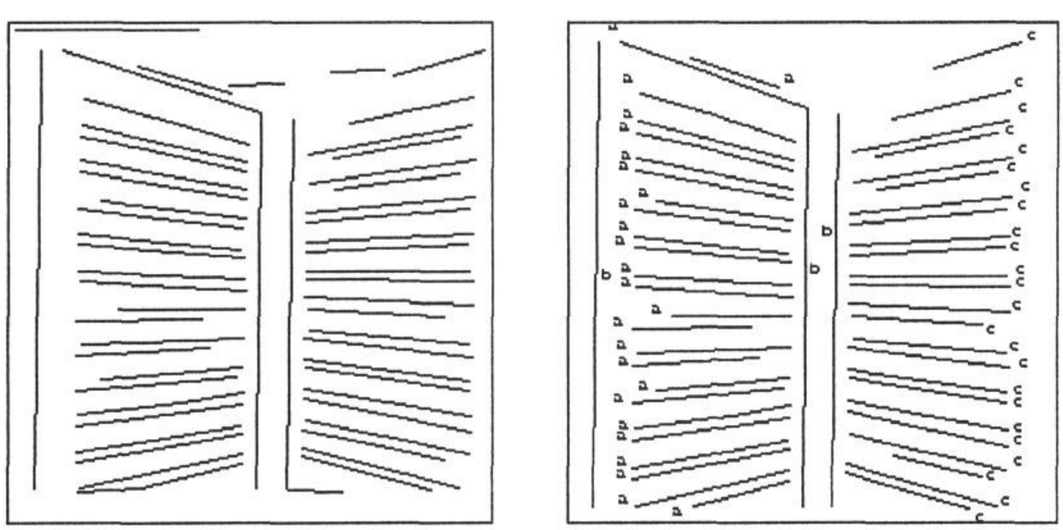

Figure 8. Image of a test card again showing result of detection of best scoring PPD triplet.