Use of the Radon Transform as a Method of Extracting Symbolic Representations of Shape in Two Dimensions

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The perception and representation of shape plays a prominent role in computer vision research. Vision alone cannot deliver computationally realistic descriptions of shape; the image must first be processed to extract symbolic representations of the sort computers can manipulate. The present work evaluates the use of the Radon transform as a means whereby the transition from edge image data to symbolic representation may be realized. The potential of the method is developed and illustrated by consideration of the criteria fundamental to an efficient representation of shape. Geometric properties and spatial relations made explicit by the transformation are listed and used to encode representations of shape. A methodology is suggested for the extraction of the salient features of a model representation, the use of those features in object recognition and the recovery of the viewer object correspondence.

1 . Introduction

The evolution of computational models for shape can be traced through successive trends in pattern recognition. In the 1960's, shape was represented by idealized templates. This method is inefficient because each object requires a unique template to represent each possible orientation and location of the object. Some years later shapes and patterns were being represented on the basis of global feature measurements, e.g., moments[1], [2] or Fourier coefficients[3]. These methods, while they are computationally efficient and reasonably insensitive to noise and quantisation, have the major disadvantage that the computed value of a global feature for the visible portion of an occluded object bears an arbitrary relationship to the value that would be computed for the whole object.

Other models of shape depend on knowledge of the boundary of the object, for example, chain coding[4]. In this technique, the boundary of the object is represented by short segments. In the most basic of implementations, each segment will have associated with it, one of eight possible orientations. Thus the method has the advantage of generating shape primitives that may be used to construct a description of any curve to some finite resolution dictated by the length and possible orientations of the line segments. However, the weakness inherent in this and other curve approximation schemes is that, since the representation is one dimensional, the shape of the interior region is not made explicit. The method makes explicit the relationships between adjacent points along the contour but not where and in what ways the contour doubles back on itself. This doubling back has a marked effect on our concept of shape[5].

The different methods can be categorized according to whether they are region based or contour based representations. It is generally accepted that the capacity to decompose a shape into its irreducible components or shape primitives is a fundamental prerequisite of any representation of shape. Contour based representations generally lend themselves more favourably to the generation of perceptually meaningful shape primitives. For example, Brady and Asada[6] use a hybrid method of shape detection where a representation which uses smoothed local symmetries (a region based representation, which does not produce perceptually meaningful primitives) is supplemented by the curvature primal sketch[7]. (a contour based method).

A further class of methods of shape detection is that of parametric transformation. The Hough transform[8], a special case of the Radon transform[9], is the most widely known and used of these methods. Using this technique, shapes are characterized by the parameters associated with the curves that form the boundary of the object. The spatially extended data of the binary edge image of the object are transformed to produce spatially compact features in a suitable parameter space. A comprehensive review of the Hough transform, its applications and implementations is given by Illingworth and Kittler[10]. A major drawback of the technique is that each new shape requires that the kernel of the transformation be rewritten in a form particular to that shape. Transform spaces of a minimum dimensionality of four[11] are required to accumulate the results of such transformations. This implies that each recasting of the transformation allows the detection of only one particular shape. To date, the Hough transform method of parametric transformation remains at the level of an efficient method of template matching.

An additional parametric transform method exists whereby an image may be decomposed into

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its constituent shape primitives. It has been shown [12] that the Radon transform may be retained as the generic transformation in a parametric transformation technique that allows any analytically defined, two dimensional shape to be detected to some predetermined finite resolution. The representation is contour-based. Two basic shape primitives are generated by the technique these are the straight line and curve where a curve may be an arc of a conic section. The technique requires only two dimensional transform spaces in which to accumulate the results of the transformation and no prior knowledge of the shape under detection is required except the degree of resolution dictated by the application. The present work exploits this technique as a method of shape recognition by setting in a formal framework the obvious potential of the method. It will be shown that the representations of shape developed are independent of rotation and translation and that the viewer-object correspondence may be recovered. A language of shape derives quite naturally from the methodology. The structure of the language is hierarchical, it also has the desirable property of succinctly expressing the shape of an object where the majority of the descriptors are either binary or integer values.

2 . The Use of the Radon Transform

The Radon transform may be written in the convenient form suggested by Gel'fand et al.[13]:

$$\Re\{F(\mathbf{x})\} = H(p,\xi) = \int_{-\infty}^{\infty} d\mathbf{x}\, F(\mathbf{x})\delta(p-\xi\cdot\mathbf{x}) \quad (1)$$

where $\mathbf{x} = (x, y)$, $\xi = (\cos\theta, \sin\theta)$ and, $F(\mathbf{x})$ is a function defined on a domain D. In two dimensions, $\delta(p - \xi\cdot\mathbf{x})$, represents a delta function distribution situated along a line, L, with equation $p - \xi\cdot\mathbf{x} = 0$ where ξ is a unit vector in the direction of the normal to that line and p is the algebraic length of the normal. It is of particular interest to consider the case in which the general function $F(\mathbf{x})$ is replaced by a particular function $F_D(\mathbf{x})$, where

$$F_D(\mathbf{x}) = \begin{cases} 1, & \text{in } D; \\ 0, & \text{otherwise.} \end{cases}$$

This definition corresponds to the transformation of a binary image and is required by the present theory as the shapes extracted from digital images will be represented as binary edge maps or δ-function curves[14].

Gelfand et al.[13] have deduced the following properties associated with the transformation:

i) The Shifting property
If $\mathbf{c}$ is some displacement of the vector $\mathbf{x}$ then:

$$\Re\{f(\mathbf{x}-\mathbf{c})\} = H(p-\xi\cdot\mathbf{c},\xi) \quad (2)$$

ii) Transform of a Linear Transform
If $\mathbf{A}$ is a matrix and $(\mathbf{A}^{-1})^T$ is the transpose of the inverse of $\mathbf{A}$ then:

$$\Re\{f(\mathbf{A}\mathbf{x})\} = \left|\mathrm{Det}\mathbf{A}^{-1}\right| H(p,(\mathbf{A}^{-1})^T\xi) \quad (3)$$

If $\mathbf{A}$ is not a unitary transformation, $\eta = (\mathbf{A}^{-1})^T\xi$ is not a unit vector and equation 3 must be written as:

$$\Re\{f(\mathbf{A}\mathbf{x})\} = \frac{|Det\mathbf{A}^{-1}|}{|\eta|} H\left(\frac{p}{|\eta|}, \frac{\eta}{|\eta|}\right) \quad (4)$$

A shift in the image space corresponds to a displacement in the p direction only of the transform plane. A rotation, denoted by $\mathbf{A}$ in property 2, will cause a displacement in the θ direction only.

It is known[14], [12] that where the image undergoing transformation is a δ-function curve then the maxima generated in transform space correspond to the tangents to that curve at the points where curve and tangent have a common normal. In accordance with this, any curve in the image space is uniquely represented by the loci of the maxima it generates in the transform space. Isolated maximum values correspond to straight line segments in the image space. The maxima in transform space associated with arcs of conic sections in image space occur as the constituent points of a sinusoid.

It has been shown[15], [12] that convolution filters may be designed to locate the maxima in transform space corresponding to the tangents to the curves in edge image space and hence it is possible to extract parametric information, about the curves in image space, from the transform plane. However, while the Radon transform is shown to encode parametric information about shape in this directly accessible way, the information about the arcs of conic sections is spatially extended. The information needs to be presented in a more compact form if it is to be useful.

The forms of the distributions of maxima in the transform space associated with the arcs of conic sections in image space are well defined[12] and it is possible to extend the technique to include a second transformation which treats the approximately linear portions of the sinusoids as straight line segments[12]. The filtered image of the transform plane, containing only points which are maxima in the θ direction, may be used as the input to this second transformation. Isolated maxima in this second transform space correspond to arcs of conic sections in the image space. Hence, the technique can be used to decompose the edge image into its constituent shape primitives where those primitives are straight lines or arcs of conic sections.

3 . Representing shape

From the previous solutions to the problems of shape representation certain criteria emerge as being essential to a good representation of shape. A list has been compiled from various sources, most notably Marr[16], Brady[17] and Biedermann [18]. The means whereby the use of the Radon transform may be exploited with respect to the task of shape detection is developed with respect to each item on this list.

3.1 Accessibility

It is necessary to demonstrate that any proposed representation of shape is computationally realistic[16].

It is proposed that the Radon transform provides the ideal vehicle for the transition from a visual-stimulus orientated representation (the digital image) to some representation suitable for recognition and acts as the bridge between the representation of shape within the image and the internal symbolic representation of shape required for recognition.

The development of fast, efficient implementations of parametric transformation methods of shape detection has received much attention in the recent literature[19], [20], [21], [11], [22], [23] and it has been shown that it is possible to implement such techniques in a time commensurate with real time applications.

3.2 Decomposition

The power of any representation derives from an allowance of free combinations of components. A set of primitives or irreducible components should therefore be associated with the representation. Where the set of primitives is small, particularly fine discriminations need not be made in the detection process[18].

The methodology developed in [12] and outlined in section 2. can be used to decompose a binary edge image into its constituent shape primitives where those primitives are of two basic types, straight lines and arcs of conic sections. By way of illustration, the binary image of a hand drawn curve is shown in Fig. 1. The binary data are transformed and an intensity map of the resulting transform plane is shown in Fig. 2. The result of filtering the transform space to locate the maxima associated with the curved shape primitives is shown in Fig. 3. The transform space is then filtered in order to locate the maxima associated with the straight line shape primitives the results of this operation are shown in Fig. 4. A second transformation process (of the filtered transform plane shown in Fig. 3 is then required and a second filtering operation. The results of these two operations are shown in Fig. 5 and Fig. 6.

The method generates efficient shape primitives that are sufficient to decompose a binary edge image of a curve into its constituent straight lines and, (to some predetermined resolution)[12], arcs of conic section.

3.3 Geometric and Spatial Relations

It is not sufficient to decompose the image into its constituent shape primitives, in addition, the representation should also make explicit the geometric and spatial relations between those shape primitives

Once the transformation and filtering processes are complete, the parameters associated with the shape primitives are known. These parameters are labelled p and θ for the first transformation and v and λ for the second transformation. Inspection of the parameters will yield the geometric properties and relative spatial arrangements of the shape primitives.

Parallelism between pairs of straight lines may be deduced by inspection and grouping of all of the shape primitives having the same value of θ.

Knowledge of the relative lengths of the primitives may be obtained from the transformation in the following way. Each point on a straight line will contribute a value of 1 to the value of the maxima in transform space associated with that straight line. The value of the maxima may thus be used as a measure of the length of the line. Similarly for the curved shape primitives, the value of the maxima in the second transform space associated with a curved shape primitive in the image space will be a measure of the relative length of that curved shape primitive.

The symmetry between particular shape primitives may be determined by examination of the transform parameters.

Right and left facing properties are made explicit by the parameters of the transformation. The parameters describing the lines which characterize the left and right facing property of the shape will be different and thus the representation can distinguish between them.

Concentricity may be deduced in the second transform plane where curved shape primitives having the same value of λ will have concentric centers of curvature.

3.4 Stability

It is important, at the low levels of computation, to ensure that access to the representation of an object does not depend on absolute judgements of quantitative detail.

The processes leading to the formation of a representation concern a transition from events in the real world which are described by noisy, continuous data to symbolic descriptions of the sort computers can manipulate. This may involve thresholding responses whereby essential as well as incidental information may be lost. Thus a good representation of shape will have the capacity to deal with modest degradations of the input information such as might occur due to noisy, extraneous or degraded data. For example, in the case of edge detection, information about some part of an edge may be lost due to low contrast thus causing an apparent discontinuity in the edge data. With respect to the Hough transform technique, the existing work is such that the robustness of the technique in the presence of noise and extraneous or incomplete data is well tested and the efficacy of the method proven[24], [25]. Thus the question of stability has been adequately addressed previously and the results of those studies are directly applicable to the present method of parametric transformation.

3.5 Similarity

To be useful for recognition, the similarity between two shapes must be reflected in their descriptions but at the same time the representation should also encode subtle differences[26].

No measure of similarity can be deduced until a symbolic representation of shape is made available. The first criterion to be addressed is that of computability. While sophisticated high level languages may help the human being to articulate what is required from a representation of shape it should be remembered that if a representation cannot be reduced to some form of number system it cannot adequately form an internal computational model of shape. Furthermore, if the number system of the representation is binary, the representation becomes optimally computable as it can be implemented, for example, as an array of hardwired logic gates. The next most advantageous number system is one that requires only integer representations and operations.

The basic data or more properly, the axioms, of the representation are the shape primitives. These are represented by triplets of numbers which define the parametric angle, radial distance and strength of the maxima in transform space.

If the parametric angle is represented in degrees and quantized in integer steps, the radial distance is similarly quantized and it is remembered that the unit density δ-function formulation of the transformation means that only integer values can occur as maxima values, then the data triplets may be adequately represented using (for an image with dimensions $\leq 512 \times 512$) only 3 digit integers. Other data which are essential to the representation such as the number and type of shape primitives may also be stored as 3 digit integer values.

It is necessary to assign relevant attributes to the data. An attribute assigns a particular property to the data. For example the length of a line. It is convenient here to emphasize the difference between recognition and particular identification of an object. Highly precise parametric information about the shape primitive requires that a tremendous amount of computational effort be expended. Such information is quite distinct from the information required at the most basic levels of recognition. The distinction exists because, as human beings, we cannot *see* parametric information, we require external aids to measure it. It is possible to recognize an ellipse simply by looking at it but it would require a ruler to obtain estimates of the values of the parameters describing its major and minor axes. For the purposes of recognition, such information is usually unnecessary and is only required when two shapes are sufficiently similar as to be identically classed except for the instances of minor differences in relative dimensions.

Thus, the process of recognition may be partitioned in an hierarchical manner whereby absolute values are not required until the very latest stages. For example, a line may be described as long with respect to the relative lengths of the other shape primitives and it is sufficient to assign or not assign such a property to a shape primitive, i.e., a line is either long or not long. In addition, such an attribute as length is a particular concept, i.e. there is no dependence between particular items of data. This means that such an attribute may be expressed as strings of binary digits. For example a rectangle may be labelled with respect to length as:

$$Length = (0,1,0,1) = 0101$$

where it is understood that each digit represents the assignment of the attribute *long* to an ordered list of the shape primitives which form the 4 sides of the rectangle.

It is necessary to assign relational concepts and structural properties to the data. These are expressed by linking the data in some structured way. Again it is only necessary to use boolean truth values in the assignment of those relational attributes. For example, the line labelled a is either parallel to the line labelled b or it is not. A relational concept may thus be represented by the elements of a two dimensional matrix. Where two elements are related by the concept then a truth value is obtained. Where no such relation exists then a zero or false value is obtained. It is noted that, using such a representation, the ordering of the shape primitives, once fixed, is not significant.

Once a symbolic representation of shape has been made available, it then becomes possible to evaluate and compare representations and to determine the similarities and differences between particular instances of image data.

3.6 Saliency

The representation should be such that gross or salient features associated with shape can be made explicit and used to bring essential information to the foreground allowing smaller and more easily manipulated descriptions to suffice. The ability to extract two or three salient primitive components means that objects can be quickly recognised even when they are partially occluded or their images are extensively degraded[18].

Salient features may be extracted by simply determining the pair or triplet of shape primitives having the greatest or near greatest values of transform maxima. The method first takes centrally positioned model images, transforms those images; filters the transformed images for maxima and stores, as triplets of data, the information associated with those maxima. The data triplets are ordered on the strength of the maxima. The topmost pair or triplet of data sets from this ordered list become the salient features for that particular object.

Knowing that each description of an object a set salient features does not help to distinguish one particular object from another. The angle between the salient shape primitives is different in the case of each model image and, because angular differences are invariant under the operations of translation, rotation and scaling, the difference between images may be expressed by these angular differences, $\Delta\theta_{ij}^{model}$, between the salient shape primitives.

A description of the object is constructed in the following way. The angular differences, $\Delta\theta$, and the differences between radial distances, Δr, are calculated for each member of the ordered list with respect to each other member of the list. The results are stored as a matrix where the upper triangle of the matrix contains information about the angular differences, $\Delta\theta_{ij}^{image}$, and the lower triangle contains information about the differences in angular distance, Δr_{ij}^{image}. The matrix so formed is refered to as $D^{image}(\Delta\theta_{ij}, \Delta r_{ij})$. For angular differences $> \pi/2$ the value $\pi - \Delta\theta$ is stored. A similar constraint is applied to the values of $\Delta\theta$ extracted from the images. This is necessary when the transform plane is represented in the ranges $[0, \pi]$ and $[-p, p]$.

Once all the salient features are known they may be used to identify a particular image. Details of a relevant study are given in [12].

3.7 Invariance

Information that is the basis of recognition should be invariant to the operations of translation, rotation and scaling.

It is a generally accepted principle that the symbolic description of an object will be represented in an object centred co-ordinate system but that the image input will be represented in a viewer centred co-ordinate system. This is a classic computer vision problem: matching an internal description of shape with an instance of image data when the two are described with respect to two different co-ordinate systems. To be useful, a representation of shape should offer invariance under the operations of translation, rotation and scaling and should make explicit the viewer-object correspondence.

Using the present technique, the viewer-object orientation correspondence may be recovered in the following way.

Once the object has been identified by its salient features, it is then possible to match particular shape primitives in the image with their corresponding shape primitives in the computer's internal model of the identified object in the following way:

The angle of rotation between the viewer centred co-ordinate system and the model centred co-ordinate system may be deduced using the parameters associated with the two sets of salient shape primitives, model and image, labelled as shown in Fig. 8.

Salient Features		
	θ	p
Model	θ_1	p_1
	θ_2	p_2
Image	ψ_1	r_1
	ψ_2	r_2

Fig. 8 Labelling of parameters associated with salient features

It is not possible to relate these parameters directly by inspection. Before the angle of rotation, ϕ, may be recovered four possibilities must be considered:

$$\phi_1 = (\pi + \theta_1) - \psi_1$$
$$\phi_2 = (\pi + \theta_2) - \psi_1$$
$$\phi_3 = (\pi + \theta_1) - \psi_2$$
$$\phi_4 = (\pi + \theta_2) - \psi_2$$

If $\phi_i = \phi_j$ then the corresponding values of θ_l and ψ_k may be used to pair the features.

Two ambiguities exist. The first is an ambiguity of π which occurs if the transform plane is represented in the ranges $[0, \pi]$ and $[-p, p]$ and the angle of rotation is such that ψ_2, $(\psi_1 < \psi_2)$, passes through $\theta = \pi$ and reappears at $\theta > 0$ with the sign of p reversed. If no two values of ϕ_i and ϕ_j are equal then $\mathrm{Min}(\phi_n) + \pi$ is applied and the correspondence sought.

A further π ambiguity occurs if the angle of rotation $> \pi$. This is resolved by checking if $\mathrm{Max}(\phi_n) > \phi_i$, where ϕ_i is the angular parameter describing one of the pair of corresponding salient features. In this case $\phi_i \mapsto \phi_i + \pi$ and this is the angle of rotation.

Determination of the angle of rotation allows particular shape primitives in the edge image to be paired with their corresponding primitives in the computer model of the object. Once this is done the shift parameters are calculated using the simultaneous equations:

$$r_1 = p_1 + a\cos(\theta_1 + \phi) + b\sin(\theta_1 + \phi) \quad (5)$$

$$r_2 = p_2 + a\cos(\theta_2 + \phi) + b\sin(\theta_2 + \phi) \quad (6)$$

where the (θ_i, p_i) are the parameters associated with the model shape primitives and the r_i are the parameters associated with the corresponding image shape primitives and ϕ is the angle of rotation.

The results of a relevant study are detailed in [12].

4 . Conclusions

The Radon transform is used to decompose a binary edge image into its constituent shape primitives where those shape primitives are straight lines and arcs of conic sections. The technique makes explicit certain geometric properties and spatial relations between the shape primitives which are then used to code a representation of shape. Properties not made explicit by the representation are sequence of connectivity and regions. While these seem to be important concepts in the human perception and understanding of shape, their absence in the present context remains a mute point.

The proposed representations may be adequately coded using either binary or integer values. It is possible to envisage that, using this method of shape representation, it may be implemented using arrays of hardwired logic gates each requiring only nearest neighbour connections.

The present work has, for reasons of simplicity, given illustrations that refer to straight line shape primitives. Examples of the use of the method when applied to images containing curved shape primitives are given in reference[12].

5 . Acknowledgements

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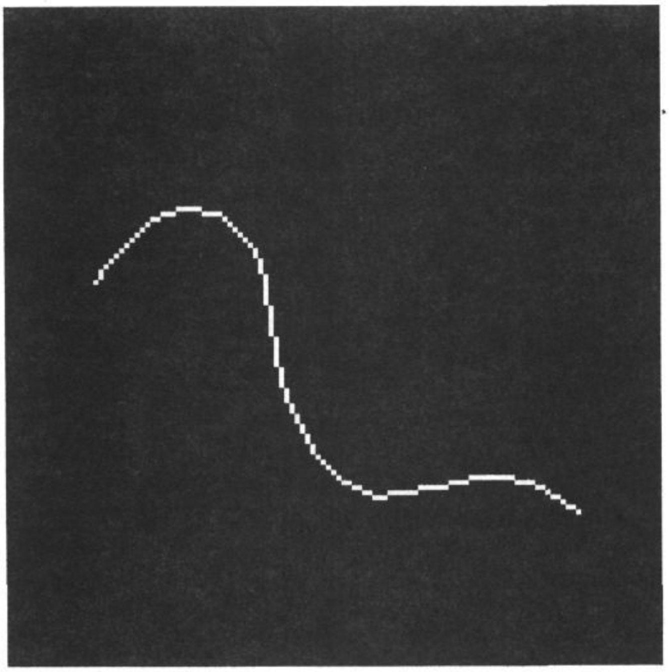

Fig.1 *Binary image of curve*

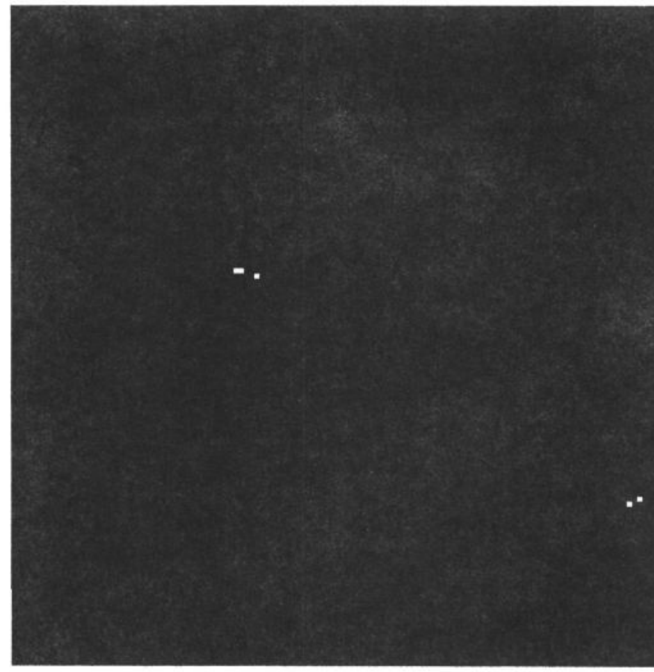

Fig.4 *Straight line filter*

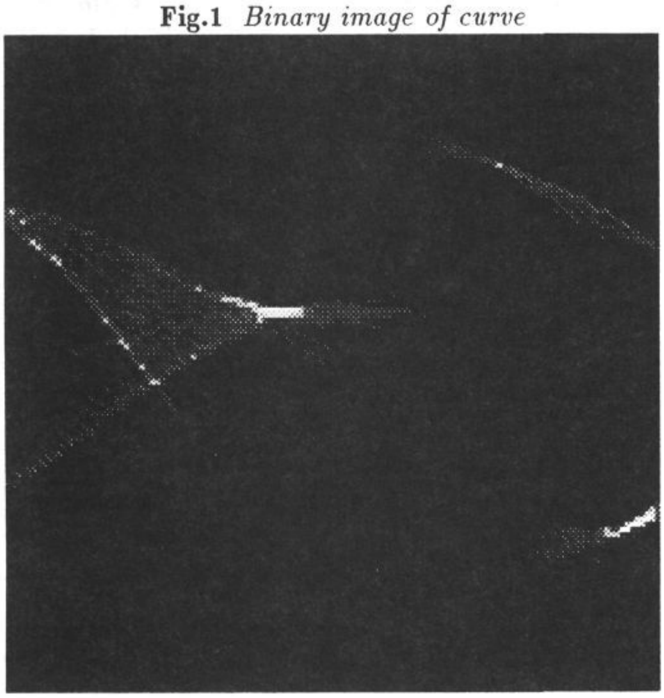

Fig.2 *First transform*

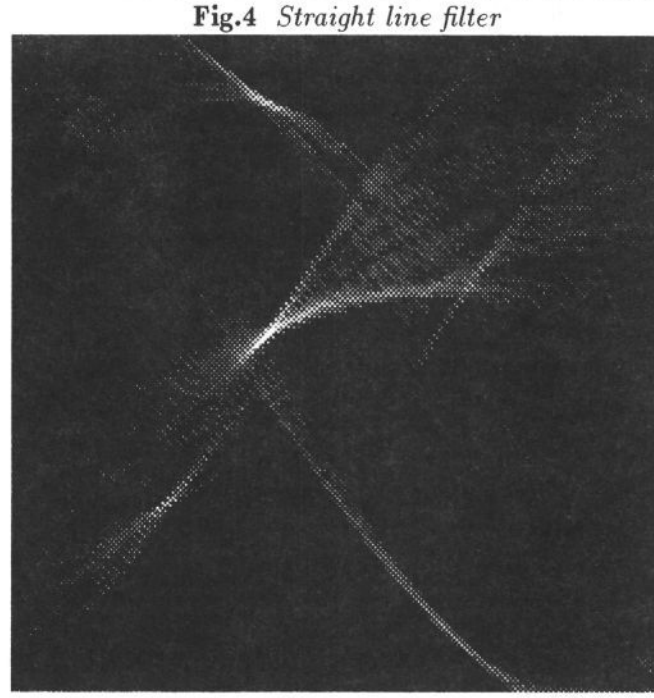

Fig.5 *Second transform*

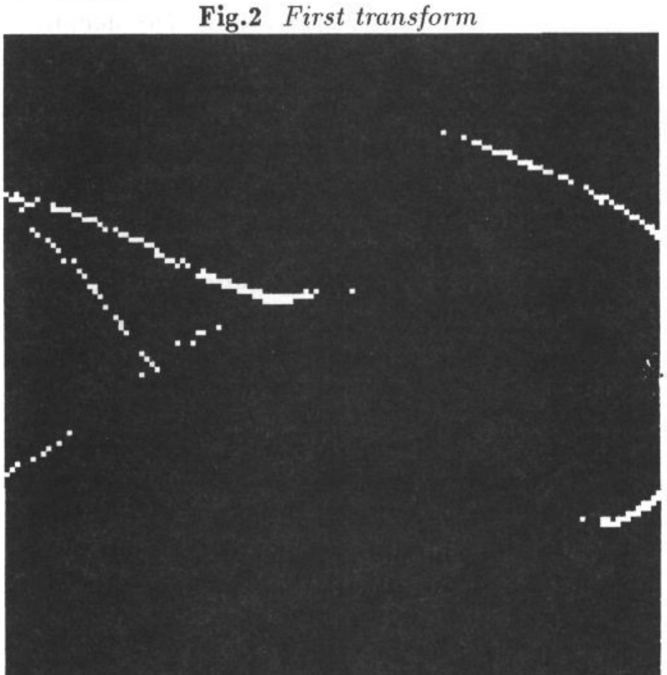

Fig.3 *Curve filtering process*

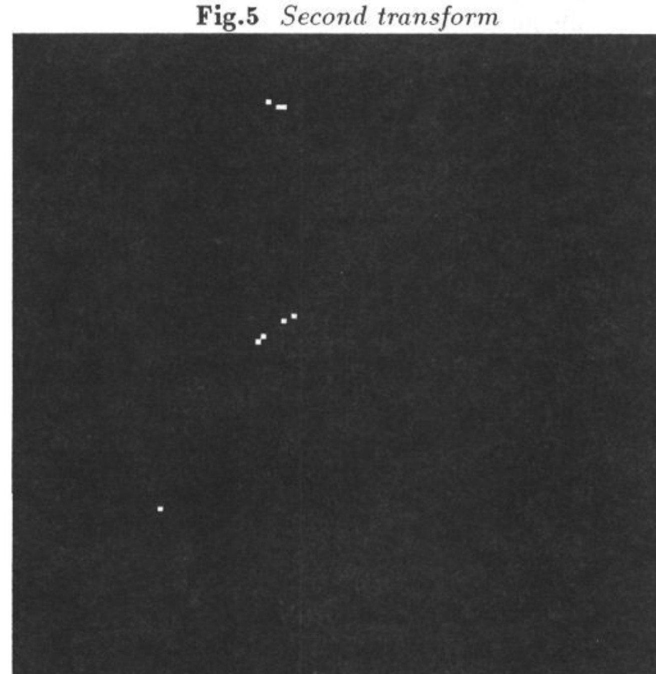

Fig.6 *Filtered second transform*