Analysis of 3D Texture

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1 Shape from Texture, Background

Several researchers have investigated the role of texture as a basis for the recovery of surface orientation. Gibson [1] was the first to address the problem of recovering the orientation of a plane covered with textural elements. He assumed that the density (number of elements per unit area) of these elements is uniform. Under this assumption he observed that the density gradient in the image specifies surface orientation. More recently [2], Witkin addresses the problem of recovering the slant σ and tilt τ of a planar surface viewed under orthographic projection. Let β be the angle which a tangent at a point of a surface marking makes with the projection of a fixed coordinate axis of the image plane and α the angle the projection of that tangent makes with the coordinate axis. Under orthographic projection then Witkin develops a geometric model which relates α to β:

$$\alpha = \tau + \arctan\left(\frac{\tan\beta}{\cos\sigma}\right)$$

The geometric model translates each value of α into a corresponding value of β for each value of (σ, τ). Thus if we have measurements of α then a Maximum Likelihood Estimator (MLE) for (σ, τ) can be computed assuming that we have a joint probability distribution function (joint p.d.f.) for β, σ and τ. In order to obtain that p.d.f. Witkin introduces his *isotropy* and *independence* assumptions:

- All surface orientations are equally likely.
- All tangent directions for markings on a surface are equally likely.
- Surface orientation and tangent direction on the surface are statistically independent.

The computation of the MLE proposed by Witkin involves a search in the 2-D space with parameters (σ, τ) and thus it is inherently slow even if this search is speeded up in the ways indicated for example in [3].

Brady and Yuille [4] develop an extremum principle that determines surface orientation from a 2-D contour. The principle maximises a compactness measure for the contour and they show that for irregular figures (or in the case of a sampled contour) their principle is roughly equivalent to Witkin's MLE. Their method though suffers from the same problems as Witkin's since it again involves a search in a 2-D space.

In a recent paper [5] Blake develops a method for estimating (σ, τ) as the orientation which makes the second moment tensor of the tangents to surface markings isotropic. Consider a tangent t_i on a point of a surface marking as an oriented line element which makes an angle θ_i with one of the coordinate axes of the image plane. The second moment tensor T of these tangents can be written as:

$$T = \begin{pmatrix} \sum\cos^2\theta_i & \sum\cos\theta_i\sin\theta_i \\ \sum\sin\theta_i\cos\theta_i & \sum\sin^2\theta_i \end{pmatrix}$$

Blake proves that the orientation (σ, τ) which makes T isotropic is the maximum likelihood orientation (as recovered by Witkin's method). Let λ_1, λ_2 be the two eigenvalues of T where $\lambda_1 > \lambda_2$. He also shows that the direction of steepest ascent of the likelihood measure K at the point where $\sigma = 0°$ is $(\cos\tau, \sin\tau)$, the eigenvector of T corresponding to λ_2. Furthermore he proves that decreasing $\cos\sigma$ by $\sqrt{2/(1+R)}$ where R is the ratio λ_1/λ_2 guarantees to increase K. These two results guarantee the existence of an iterative gradient descent algorithm to maximise K. The iterative algorithm converges to the desired orientation with an accuracy of 1° after about 20 iterations. This method although equivalent mathematically to Witkin's is inherently much faster.

Not surprisingly, in view of Brady and Yuille's result, maximum compactness is also achieved at isotropy of a suitably defined second moment tensor (not quite the same tensor as above). A similar iterative algorithm also exists in this case. However, in this paper, we concentrate on the Maximum Likelihood Estimator.

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2 Checking the Validity of the Isotropy Model

Each of the methods outlined above for the recovery of surface orientation from texture essentially assumes that the world plane is covered with textural elements whose distribution is uniform (i.e. that the texture is isotropic). Under this assumption one of the aforementioned methods gives the orientation of the world plane (σ, τ) as a Maximum Likelihood Estimator. Having obtained the two parameters (σ, τ) we can address the problem of evaluating the validity of our initial assumptions about texture isotropy. In other words we can backproject (to the plane specified by the (σ, τ) pair found) the image texture and check whether the texture so obtained has indeed a uniform distribution. One of the standard tests that can be applied for checking whether experimental data follow a particular distribution is the X^2 test.

If our texture is isotropic and planar then the orientation (σ, τ) obtained by making the second moment tensor isotropic can give us the world plane and therefore the original (unforeshortened) distribution of our texture. The X^2 test tests the validity of our isotropy and planarity assumptions. If the test fails then either the original texture is not isotropic or the surface the texture lies on is not planar (in which case the texture may or may not be isotropic).

The X^2 test [6] involves the decomposition of the outcomes of a random experiment into $\kappa + 1$ mutually exclusive sets (orientation buckets in our case), say $L_1, \ldots, L_{\kappa+1}$. Let p_j be the probability that a certain outcome of the experiment falls in bucket j where $j = 1, \ldots, \kappa + 1$ and assume that p_j depends on r unknown parameters $\theta_1, \ldots, \theta_r$ (in our case the number of parameters is two: σ and τ) so that $p_j = f_j(\theta_1, \ldots, \theta_r)$. In n independent repetitions of the random experiment let N_j denote the number of outcomes belonging to set L_j (i.e. N_j is the number of line elements whose orientation falls between the bounds of bucket j). Let then $\hat{\Theta}_1, \ldots, \hat{\Theta}_r$ ($\hat{\sigma}$ and $\hat{\tau}$ in our case) be MLE of $\theta_1, \ldots, \theta_r$. Then,

$$Q_k = \sum_{j=1}^{\kappa+1} \frac{(N_j - n\hat{P}_j)^2}{n\hat{P}_j}$$

has a limiting distribution that is the X^2 distribution with $\kappa - r$ degrees of freedom (DOF) where $\hat{P}_j = f_j(\hat{\Theta}_1, \ldots, \hat{\Theta}_r)$. The number of DOF actually depends on the way $(\hat{\sigma}, \hat{\tau})$ were obtained. In our case $(\hat{\sigma}, \hat{\tau})$ are obtained from the orientation measurements themselves as opposed to being obtained from the number of line elements belonging to each bucket. Therefore as is shown in [6] the number of DOF will be bounded between $\kappa - r$ and κ (notice that in our case $r = 2$ but we are free to choose any number greater than three for $\kappa + 1$ –the number of buckets).

We have performed several experiments with the X^2 test using artificially generated textures for which the orientation of the line elements has either a uniform distribution or a different one. In figure 1 we see a texture comprised of line elements whose orientation is uniformly distributed in $(0, \pi]$. Notice that only the orientation of

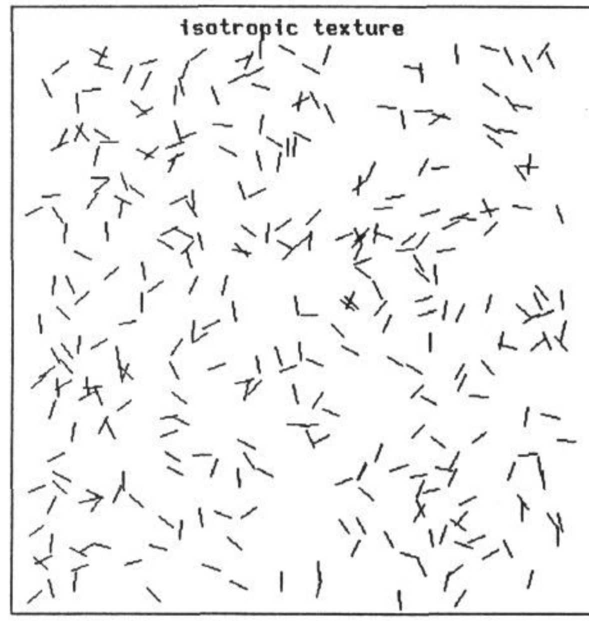

Figure 1: Isotropic texture. The orientation of the line elements is uniformly distributed in the range $(0, \pi]$.

the line elements is considered here and not their length. The X^2 test confirms the null hypothesis that the texture is isotropic (this is for a 5% significance test). In figure 2 the distribution of orientation is biased (the probability of finding a line element with orientation around 0° or 180° is larger than that of finding a line element with any other orientation).

If we assume that the texture of figure 2 is a *projected* isotropic texture (i.e. a texture which has a uniform distribution in the world plane –whose orientation is *unknown*) we can apply Blake's algorithm to recover σ and τ of the world plane. Then the texture can be backprojected to the plane specified by (σ, τ) obtained by the algorithm. After application of the algorithm (20 iterations) we obtain $\sigma = 52.25°$ and $\tau = 93.2°$. The backprojected texture is shown in figure 3. Applications of the X^2 test for the backprojected texture, using variable numbers of buckets $\kappa + 1 = 4, \ldots, 12$, fails in all cases.

As pointed out by Witkin the imaging process (the orthographic projection) systematically transforms the original texture by a transformation which is given by his geometric model. Failure of the X^2 test for the last case of the preceding paragraph means that there is no projection (given by (σ, τ)) which would make the texture isotropic.Therefore, our original texture cannot have been both planar and isotropic. Additional information is then needed for the recovery of either the original tex-

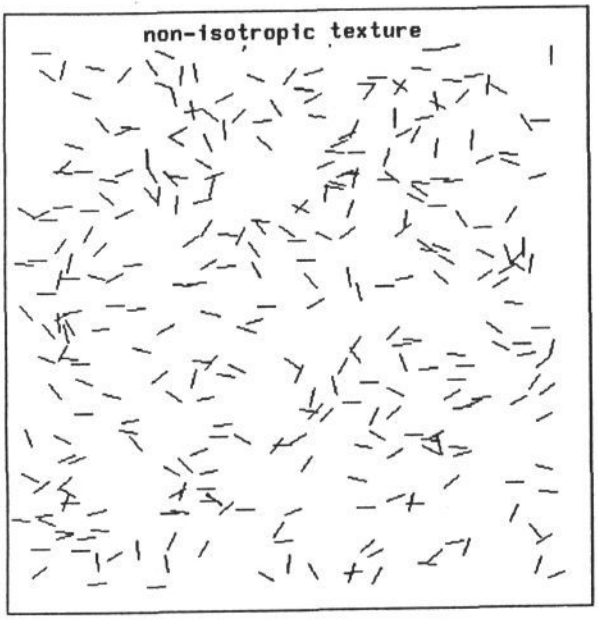

Figure 2: Non-isotropic texture. Notice that most of the line elements have an orientation around 0° or 180°.

ture distribution and/or the surface orientation.

3 Analysis of Real Textures

Our algorithms for first, obtaining surface orientation and second, checking the validity of our assumptions about the distribution of the texture in the world plane work for a specific category of natural textures. More specifically they work when the textural elements are elongated so that they can essentially be represented as line elements with a given orientation. The question then arises of how do we first, extract the textural elements from an image and second, how do we obtain a line for each one of them.

In a recent paper [7] Voorhees and Poggio address the first problem. Their method involves first finding the edges in the image with the use of the Canny operator. Then they show how one can obtain the two parameters used for thresholding the edges with hysteresis. In the results presented here, however, thresholds were set experimentally rather than automatically. In figure 4 we see the image of a plane parallel to the image plane covered with rice grains dropped on the plane at random[1]. In figure 5 the plane is creased in the middle to incline two planes with respect to one another and to the image plane. Each plane is inclined at roughly 50° to the image plane.

In figures 6 and 7 we see the edges extracted from figures 4 and 5 respectively after thresholding with hysteresis.

[1] After dropping the grains, they were glued in position!!

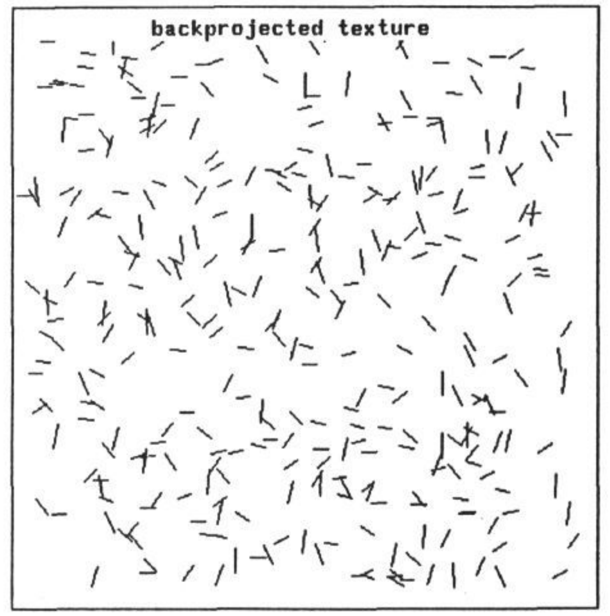

Figure 3: Backprojected texture of the texture in figure 2. Notice that the transformation leads to "a more uniform" distribution.

In order then, to extract individual texture elements we recursively link each connected (in an 8-neighbour sense) edge point. Having obtained each textural element as a linked "swarm" of image points we find the ellipse that would best fit this swarm of points. Ellipse fitting is done with a scatter matrix approach [8]. Each textural element is thus represented as a line whose orientation is the orientation of the major axis of the ellipse.In figure 8 we see the lines extracted from figures 6 and 7.

We can now apply the iterative algorithm for the data of figure 8. For the data of figure 8a we have the additional problem of segmentation. A suitable approach would be to apply the iterative algorithm for overlapping windows over the whole image and then if the (σ, τ) pairs found for those windows are the same conclude that the texture lies on the same plane. Such a technique might also bear on the problem of analysing curved textured surfaces. For the left half of figure 8a we get with the iterative algorithm $\sigma_l = 57.08°, \tau_l = -9.93°$ and for the right half $\sigma_r = 62.65°, \tau_r = -17.35°$. Veridical values for σ, τ were approximately 50° and −5° respectively. The errors of around 10° are as currently unexplained, though conjecture would be that it is simply statistical error due to limited sample size. Further work is needed to understand this. If we backproject figure 5 to the plane specified by (σ_l, τ_l) we get the texture in figure 9. Note that, as expected orientations do appear to be isotropically distributed. For this last texture application of the X^2 test with 5 orientation buckets gives for the left half a value of 10.43 which as we can see from standard tables for the X^2 distribution falls between the values for 2 and 4 DOF respectively (this is for a 1% significance test). Therefore there is some agreement of the

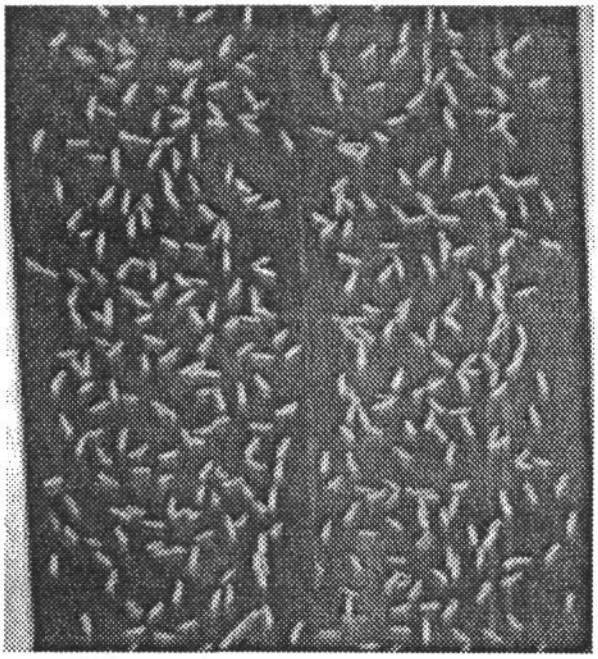

Figure 4: Rice grains are dropped on a plane whose orientation is parallel to the image plane. The orientation of the rice grains is approximately uniform.

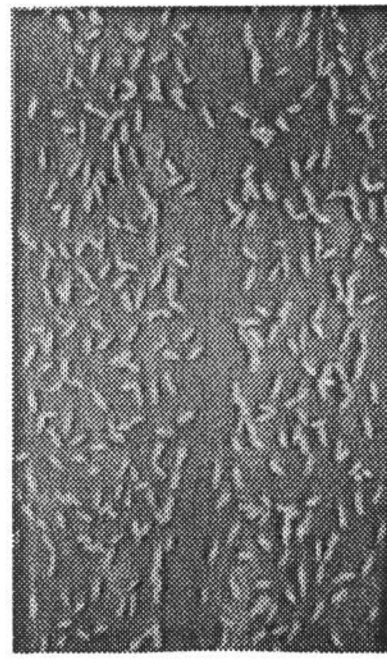

Figure 5: Notice how the texture appears compressed in the horizontal direction. Compare with figure 4.

data with the null hypothesis (i.e. that the texture on the backprojected plane is isotropic). Indeed application of the X^2 test –same number of buckets– on the data of figure 4 gives a value of 13.14 which again as can be seen from the tables falls between the values for 2 and 4 DOF respectively.

As we said in the beginning our algorithms work for a certain class of textures where the textural elements have a relatively well defined orientation. For other cases one might use additional information as for example density of textural elements per unit area [9]. In that case the hypothesis corresponding to isotropy above, is homogeneity-uniformity of density. One might also try to recover not only the foreshortening parameters of the imaging transformation but the distance and rotation ones as well, in which case computation of higher order moments may be needed in a way suggested in [10].

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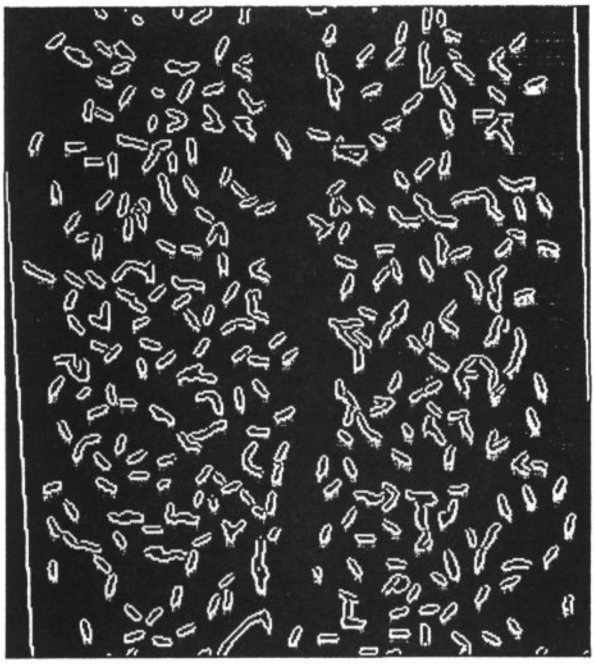
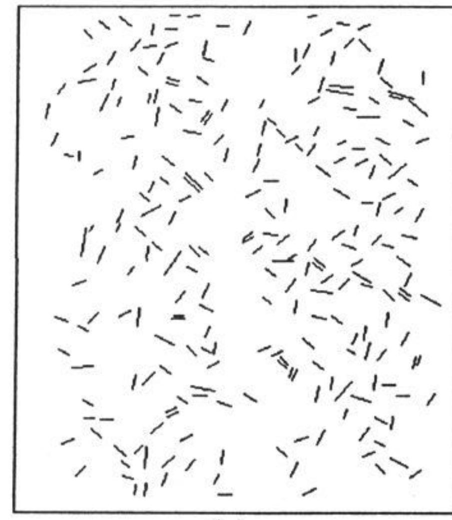


Figure 6: Edges of figure 4.



(a)

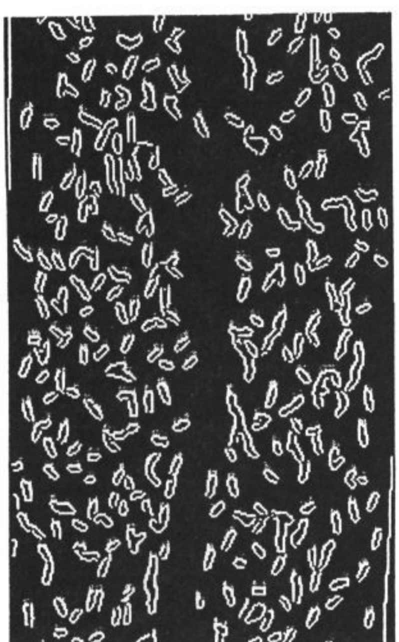
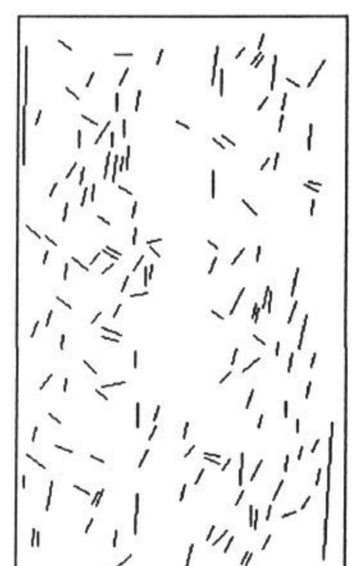


Figure 7: Edges of figure 5.



(b)

Figure 8: Lines extracted from fig. 6 (a). Lines extracted from fig. 7 (b).

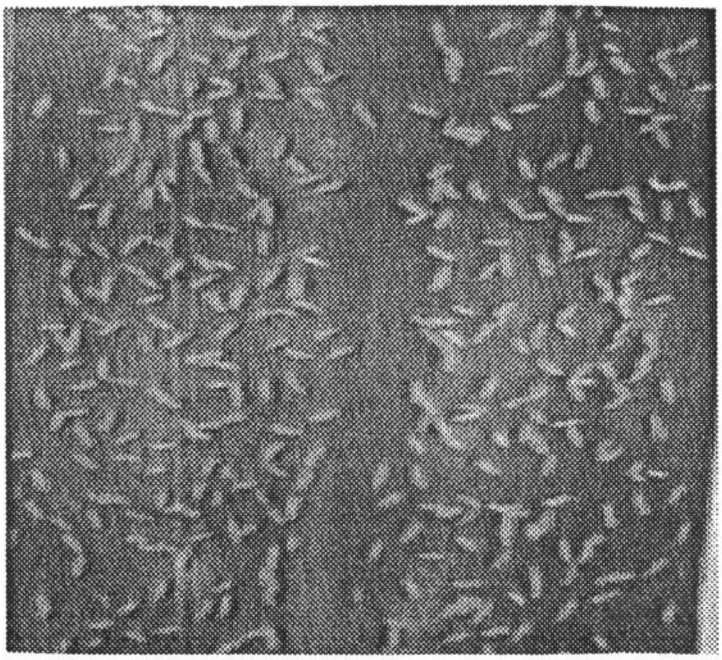


Figure 9: The texture of figure 5 as it appears on the backprojection plane as recovered by the iterative algorithm. Orientations of the rice grains now appear to be roughly isotropically distributed.